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Dominant components of vibrational energy flow in stiffened panels analysed by the structural intensity technique

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ABSTRACT

Stiffened panels are widely used in naval architecture and ocean engineering, and knowledge about their dynamic behaviour represents important issue in the design procedure. Ordinary vibration analysis consists of natural frequencies and mode shapes determination and can be extended to forced response assessment, while the Structural Intensity (SI) analysis, assessing magnitude and direction of vibrational energy flow provides information on dominant transmission paths and energy distribution including sink positions. In this paper, vibrational energy flow in stiffened panels under harmonic loading is analyzed by the SI technique employing the finite element method. Structural intensity formulation for plate and beam element is outlined, and developed system combining in-house code and general finite element tool is described. As confirmed within numerical examples, the developed tool enables separation of SI components, enabling generation of novel SI patterns and providing deeper insight in the vibrational energy flow in stiffened panels, comparing to existing works.

1. Introduction

Stiffened panels are primary constitutive elements in all fields of engineering, as for instance: aeronautical, civil, mechanical, naval, ocean, etc (Cho et al., 2014, 2015a). The most important engineering problems encountered with stiffened panels can be classified into three main groups: bending, stability and vibration (Troitsky, 1976). Stiffeners of different cross-sections are regularly used to increase panel loading capacity and to prevent buckling, but at the same time they influence system dynamic properties and should be properly taken into account in vibration analysis.

Ordinary procedure for vibration analysis of stiffened panels includes solving of eigenvalue problem, i.e. determination of natural frequencies and mode shapes, and their comparison with excitation frequencies if required. Also, sometimes it is necessary to check whether the obtained response satisfies prescribed criteria (in terms of displacement, velocity or acceleration) or not, and in that case forced vibration analysis in frequency or time domain is done (Cho et al., 2015b). In spite of variety of available methods for vibration analysis of stiffened plates, that can be classified into the analytical ones, semi-analytical and numerical methods, remedying of vibration problems (particularly in complex ship and offshore structures) is still rather complicated task. Moreover, vibration analysis of complex ship and offshore structures is usually based on trial and error approach, and can be rather time consuming, especially if advanced computational methods involving large modelling and computational efforts are used. Therefore, knowledge about source power, dominant transmission paths and vibrational energy dissipation, can be very helpful in vibration problem elimination. Such information can be obtained by the Structural Intensity (SI) analysis, which calculates vibrational energy flow from vibratory velocity and internal force of the structure. Hence, the SI analysis provides additional information and deeper physical insight into vibration characteristics of the structure. This subject is investigated for a long time, both experimentally (Noiseux, 1970; Pavic, 1976, 1987; Verheij, 1980; Saijyou and Yoshikawa, 1996; Pascal et al., 2006; Eck and Walsh, 2012) and numerically (Gavric and Pavic, 1993; Xu et al., 2004a,b,c; Tran et al., 2007; Cho et al., 2016; Petrone et al., 2016). The usual objective of both measuring and calculating the vibratory energy

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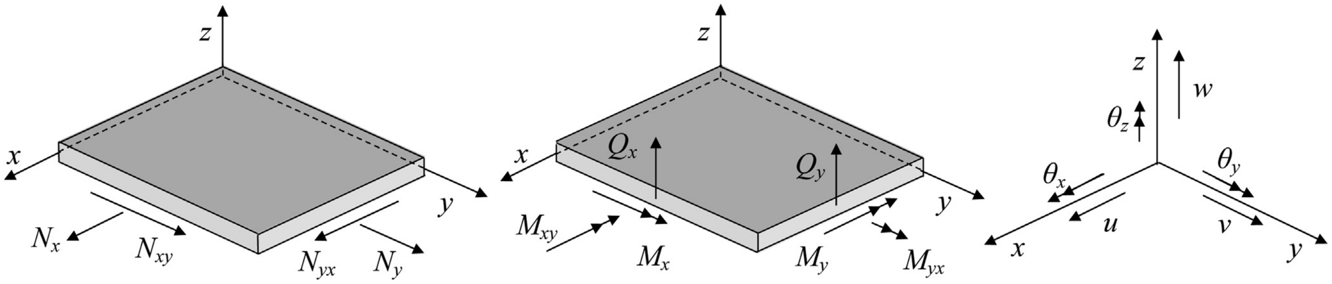


Fig. 1. Forces, moments and displacements for plate element.

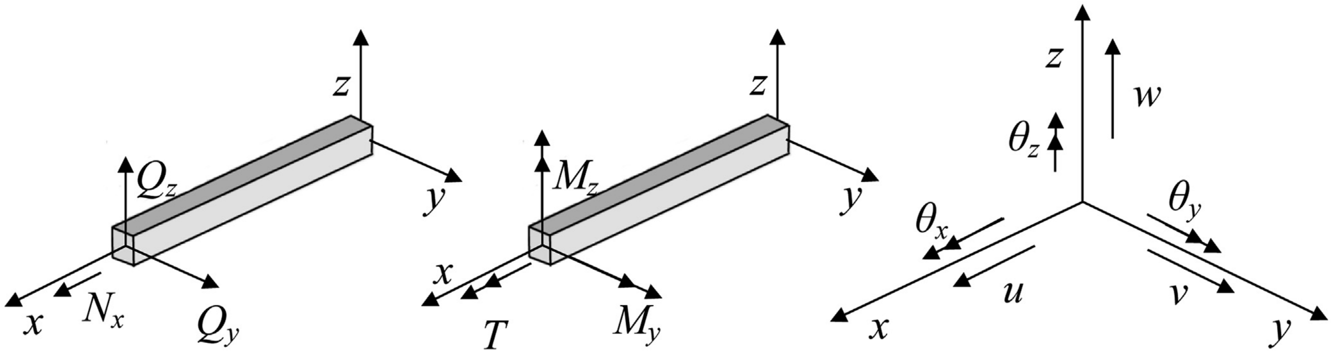


Fig. 2. Forces, moments and displacements for beam element.

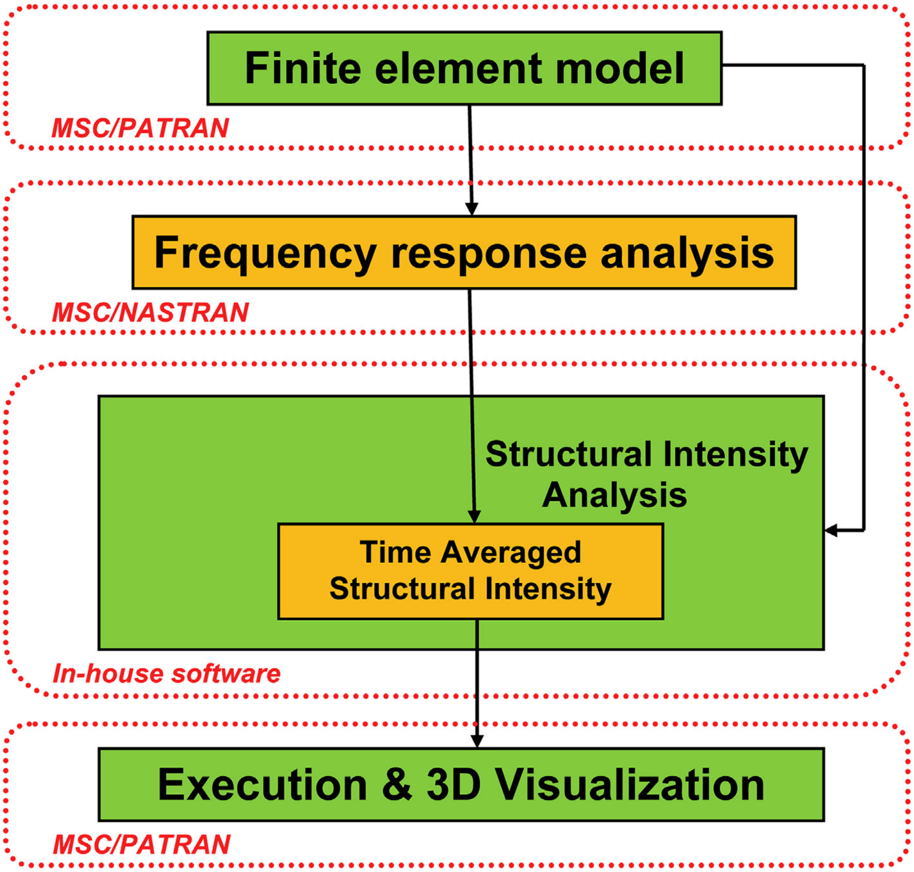


Fig. 3. Scheme of computational system for structural intensity analysis and visualization.

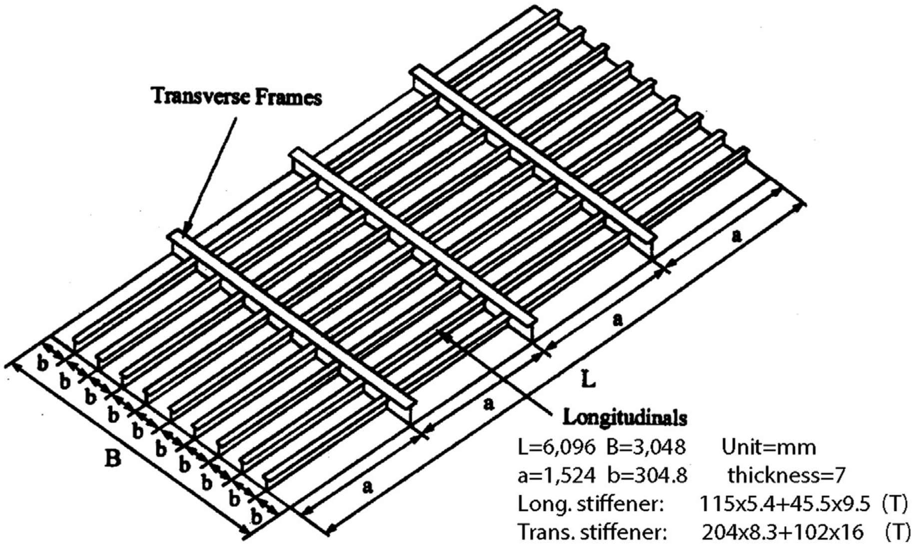


Fig. 4. Schematic presentation of the rectangular cross-stiffened panel (Xu et al., 2004c).

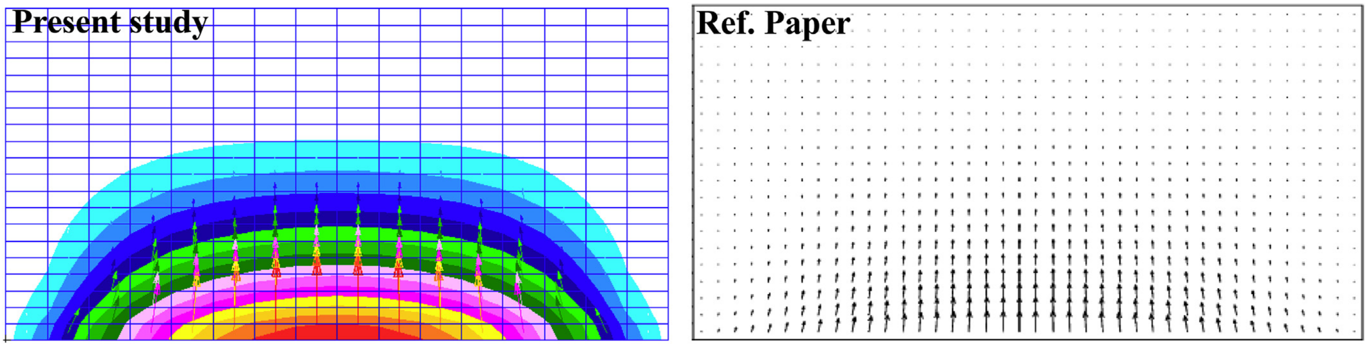


Fig. 5. Comparison of SI vectors in bare plate subjected to in-plane pressure in transverse direction.

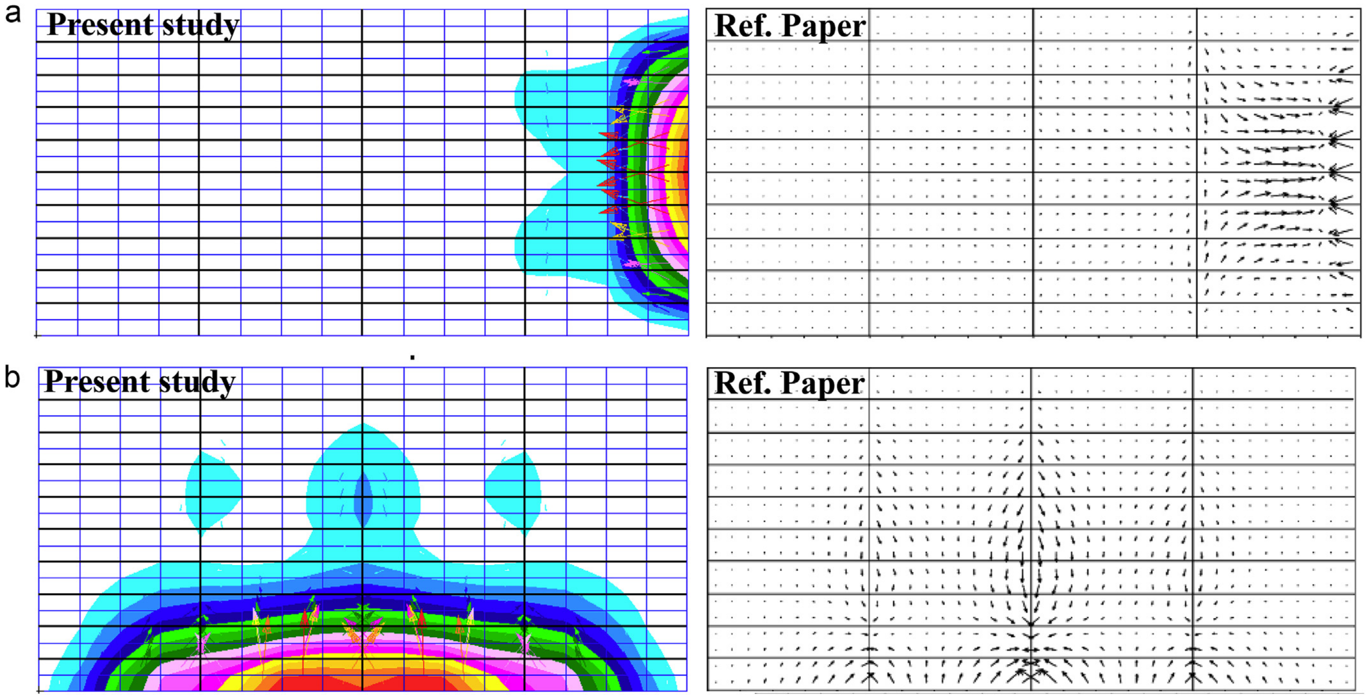


Fig. 6. Comparison of SI vectors in cross-stiffened panel; a) under in-plane pressure in longitudinal direction, b) under in-plane pressure in transverse direction.

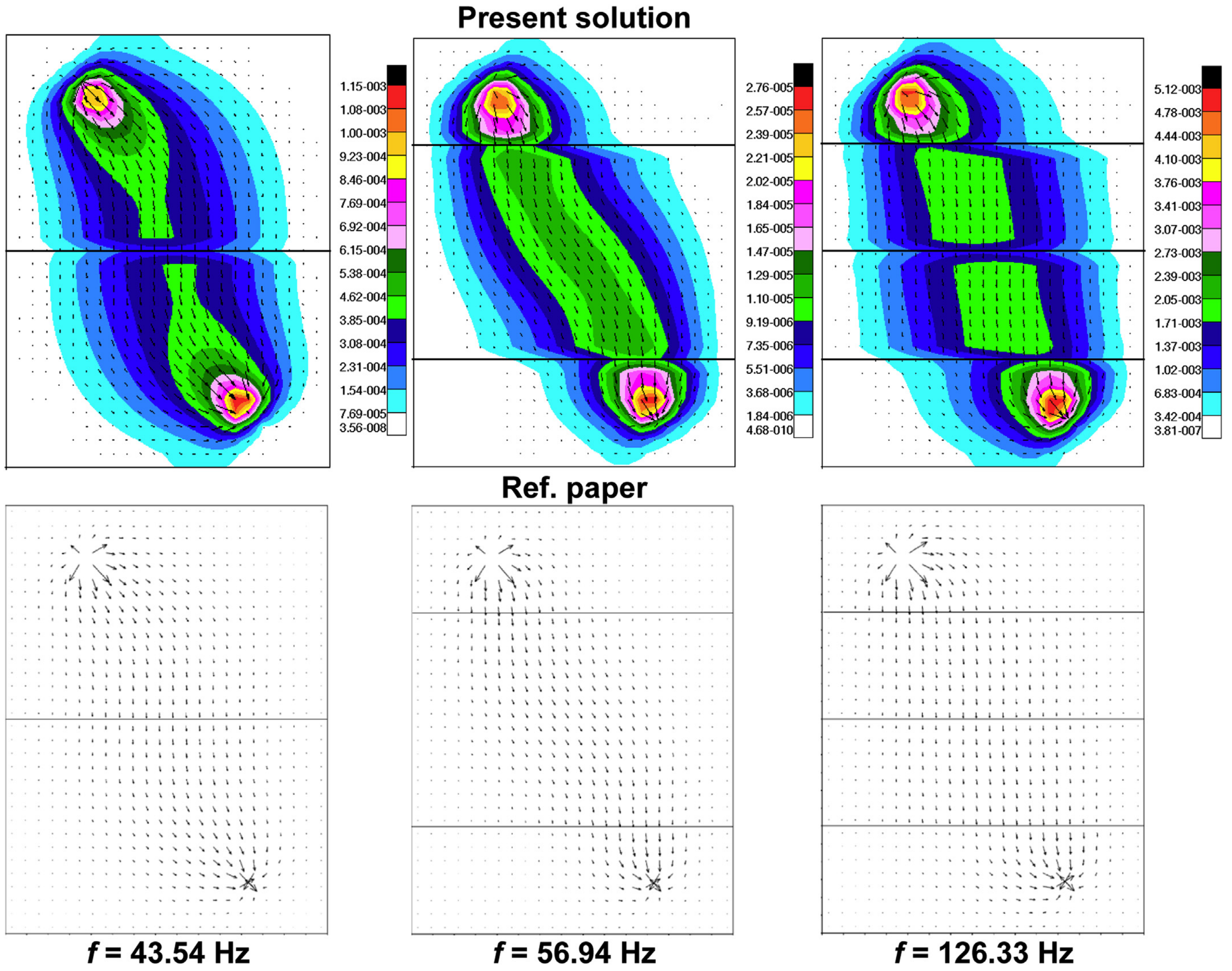


Fig. 7. Total structural intensities in stiffened panels with one, two and three stiffeners.

propagation is exact determination of vibration source location, identification of energy propagation paths and regions of vibration absorption. Since this paper is focused on the determination of structural intensities of stiffened panels by means of the Finite Element Method (FEM), more detailed literature review on numerical approaches in SI analysis is further given.

Application of FEM using the normal mode approach to SI analysis of structures is illustrated for a simply supported plate by Gavric and Pavic (1993). Xu et al. (2004a) studied the SI pattern of composite plates with openings and also presented an interesting energy transmission study of rotating hard disk systems (Xu et al., 2004b). Xu et al. (2004c) applied SI technique to analyze vibration performance of stiffened panels of marine structures under lateral pressure excitation, point force excitation and in-plane pressure excitation. The transient dynamic characteristics of plates under low-velocity impacts are analyzed by Liu et al. (2005). Structural intensity analysis of thin laminated composite plates subjected to thermally induced vibration is performed by Tran et al. (2007), where streamline technique, well-known presentation tool from fluid mechanics, is used to represent the obtained results. Park and Hong (2008) utilized SI technique to analyze transverse vibration of Mindlin plates considering the effects of shear distortion and rotary inertia, while Khun et al. (2004) performed SI analysis of plates with multiple discrete and distributed spring-dashpot systems. Energy transmission through a box-type structure built up by plates is considered by Chen et al. (2012). There are also several very recent works on vibrational energy propagation through structures, confirming importance of this topic. Navazi et al. (2016) measured energy density in high frequency vibrating plates and compared the results with the energy finite element analysis. They concluded that the kinetic and potential energy counterparts of the energy density are equal in high frequency bands. Cho et al. (2016) analyzed vibrational energy flow in stepped thickness plates by SI technique and derived a set of novel intensity paths of total intensity components. Petrone et al. (2016) presented numerical and experimental analysis on structural intensity in orthotropic rectangular plates, investigating the effects of constraints, loading conditions, damping, thickness and fibres orientation. Tian et al. (2017) presented practical application of SI approach to crack detection for jacket-type offshore platform. They have proposed the Line Spring Model (LSM) of surface crack based on plate crack structure and established relationship between the additional angle, displacement and relative crack depth. By observing the input energy, distribution, transmission and vibration performance of structural intensity the crack location can be detected. In the context of FEM application to structural intensity analysis,

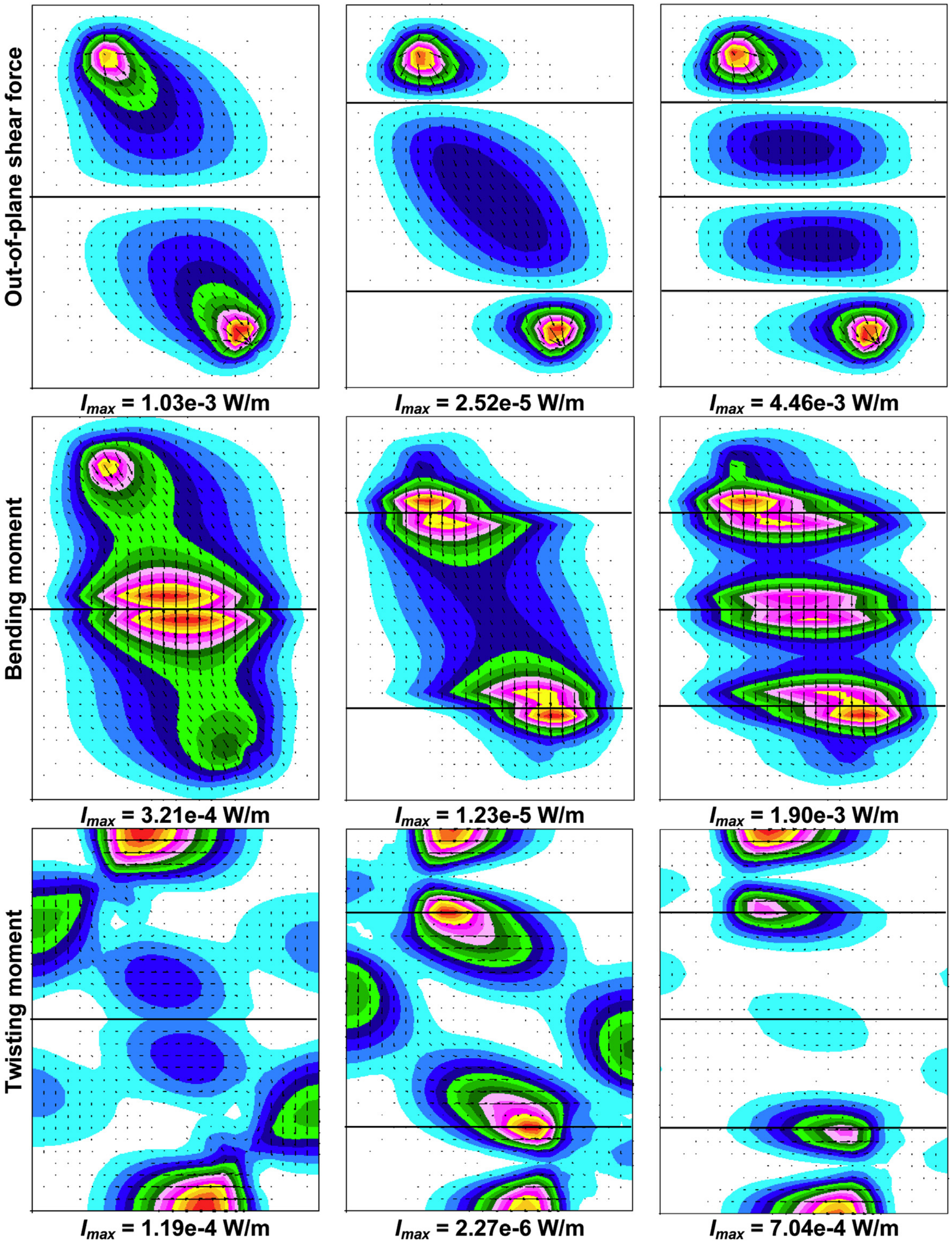


Fig. 8. SI components in stiffened panels with one, two and three stiffeners.

development of different calculation systems which at the certain assessment step involve general commercial tools like NASTRAN (Hambric, 1990; Hambric and Szwerc, 1999; Cho et al., 2016), ANSYS (Xu et al., 2004a,b,c; Tian et al., 2017), or ABAQUS (Khun et al., 2004; Lee et al., 2006; Tran et al., 2007), should be mentioned. Beside finite element method, there are several references dealing with application of the Asumed Mode Method (AMM), as for instance SI analysis of stiffened plate (Cho et al., 1998), and SI analysis of stiffened plate carrying lumped inertia and stiffness attachments, considered both by FEM and AMM (Cho et al., 2003).

In spite of the above mentioned SI analyses of stiffened panels and wide range of other SI applications, to the author's knowledge there are no references considering contribution of SI components to the total SI levels in plates and stiffeners. Namely, in all above references the total structural intensity is considered, which provides only global perspective on the energy transmission.

This paper is strongly motivated by the above mentioned significant work of Xu et al. (2004c) dealing with SI analysis of

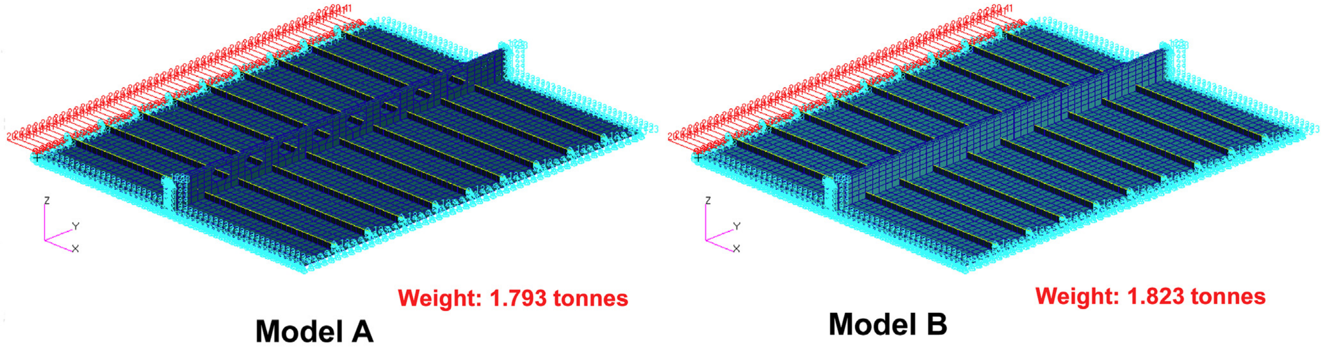


Fig. 9. FE models of stiffened panels with web frame; web frame with holes (Model A), web frame without holes (Model B).

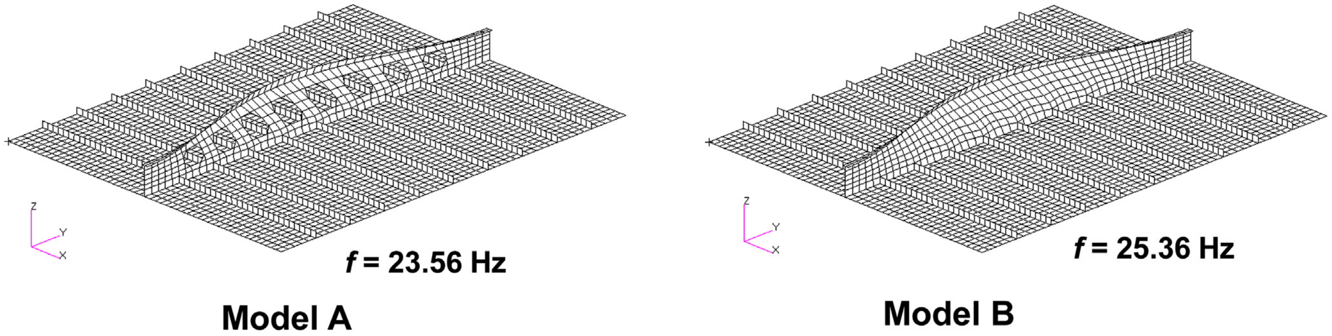


Fig. 10. Tripping vibration modes of stiffened panels with web frame.

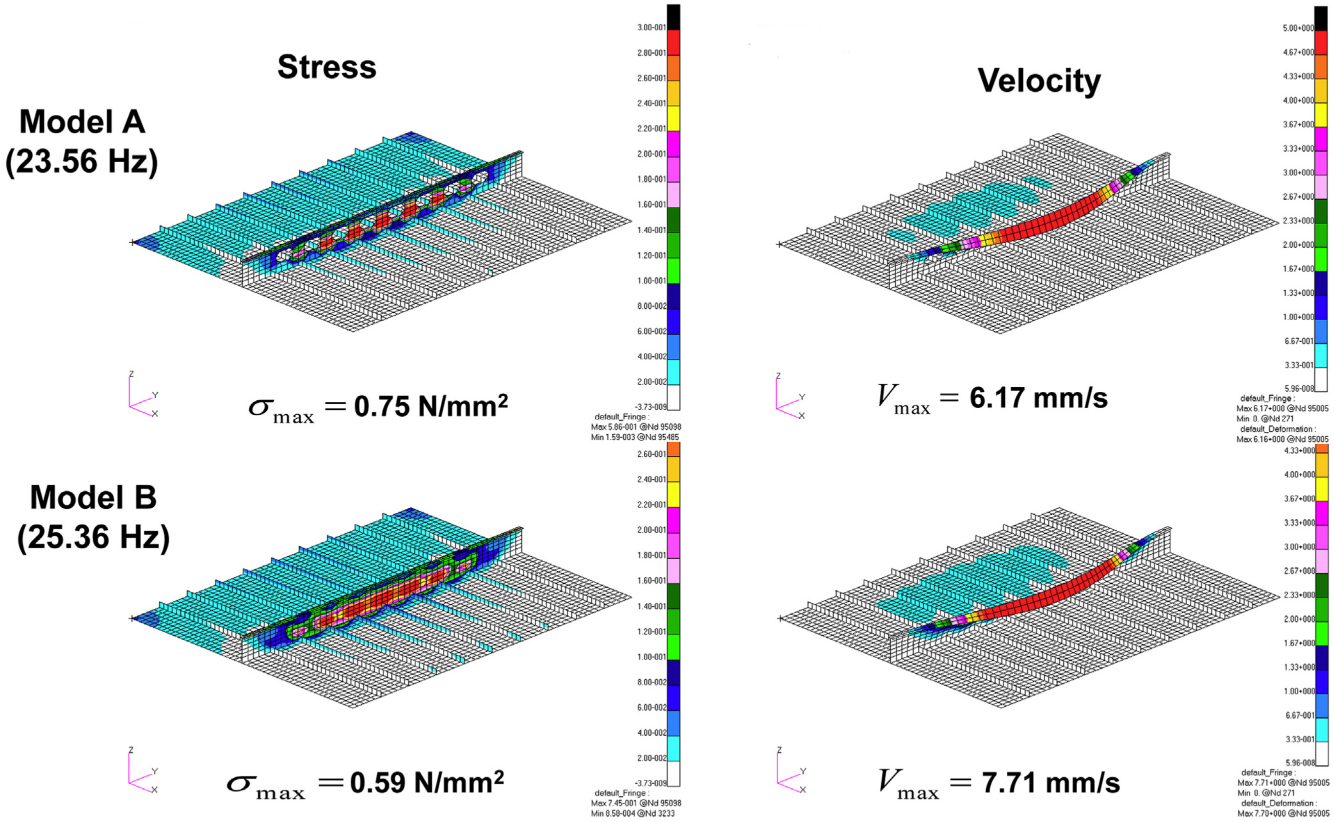


Fig. 11. Von Mises stress and velocity distributions in stiffened panels with web frame.

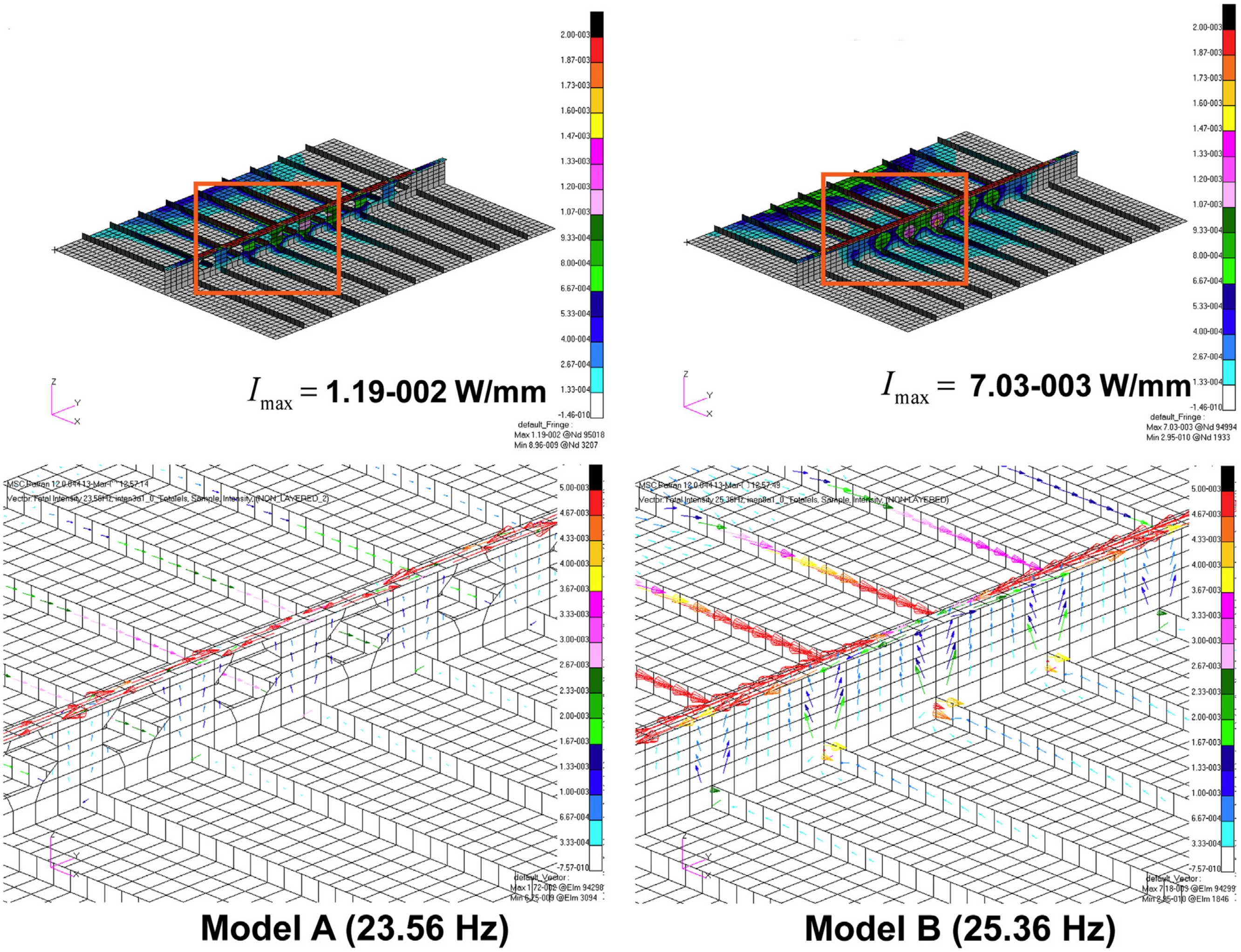


Fig. 12. Total structural intensities in stiffened panels with web frame.

stiffened panels of marine structures under harmonic excitation, simulating waves and equipment loading. They showed that stiffeners affect the energy flow, but still the power sources and sinks can be clearly identified despite the stiffener existence. Although the authors analyzed influence of plate/stiffener proportions on the structural intensity levels, exact quantification of structural intensity depending on the number of stiffeners and their placement as well as analysis of contribution of each intensity component would be helpful for better understanding of energy transmission. Moreover, in the paper of Xu et al. (2004c) energy transmission through the stiffeners is not investigated in details. This seems to be necessary because the stiffeners are often made with different holes, which are expected to significantly influence the transmission path. So, in this paper a further step towards understanding physical phenomena in vibrating stiffened panels of ships and offshore floating units was done. Expressions for structural intensity of plate and beam element in frequency domain are outlined. Developed numerical simulation system for SI calculation and visualisation in thin-walled structures is described. As a novelty, it enables analysis of each structural intensity component separately, and here is used to identify the most dominating ones for stiffened panels and to generate novel SI component transmission paths. Accuracy of the system is confirmed by comparing generated total SI patterns with those available in the relevant literature.

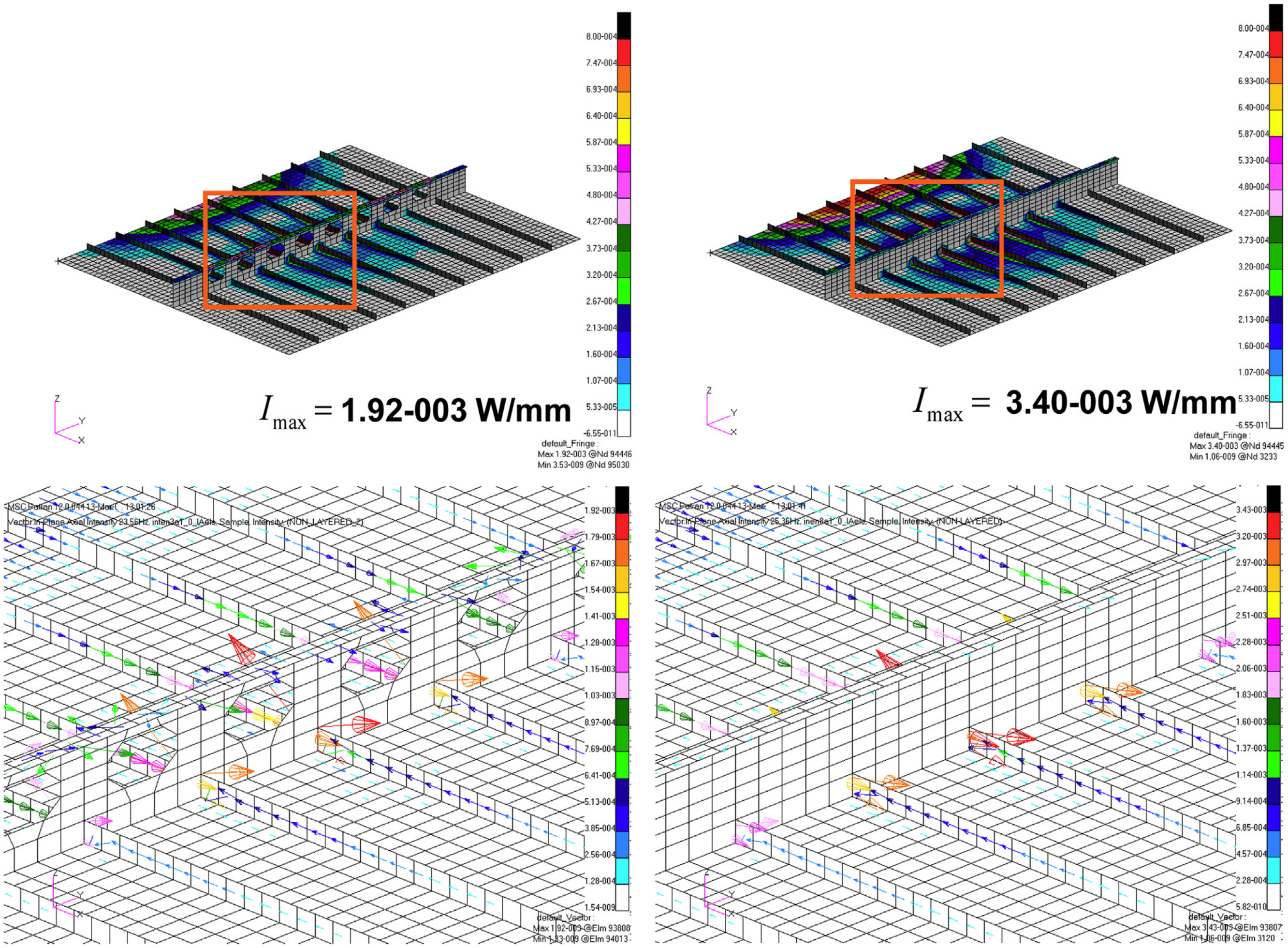
2. Formulation of structural intensity for plates and beams

The structural intensity is defined as a vibrational power flow per unit-cross sectional area of a structure under dynamic loading. To treat the structural intensity of thin-walled structures by means of finite element method, it is necessary to express structural intensity of shell and beam elements in terms of internal forces and moments, and displacements (translational and angular), obtained from forced vibration analysis. The instantaneous intensity component is a time-dependent vectorial quantity representing energy density change in a given infinitesimal volume. In time domain the i-th component of structural intensity yields (Noiseux, 1970):

$$I_i(t) = -\sigma_{ij}(t)v_j(t), \quad i,j = 1,2,3, \tag{1}$$

where $\sigma_{ij}(t)$ denotes stress tensor component at a point where i is the normal direction of the area, while $v_j(t)$ represents velocity vector in the j-direction at time t.

The instantaneous intensity for the plate mid-surface lying in the x-y plane, Fig. 1, is given by the following expressions (Cho et al., 2016):



Model A (23.56 Hz)

Model B (25.36 Hz)

Fig. 13. In-plane axial SI components in stiffened panels with web frame.

$$I_x(t) = -\left(\dot{w}Q_x + \dot{\theta}_y M_x - \dot{\theta}_x M_{xy} + \dot{u}N_x + \dot{v}N_{xy}\right), \tag{2}$$

$$I_y(t) = -\left(\dot{w}Q_y + \dot{\theta}_y M_{yx} - \dot{\theta}_x M_y + \dot{u}N_{yx} + \dot{v}N_y\right), \tag{3}$$

where M_x, M_y, M_{xy}, M_{yx}, N_{xy}, N_{yx}, Q_x, Q_y, N_x, and N_y denote bending moments, twisting moments, in-plane shear forces, out-of-plane shear forces and in-plane axial forces per unit width of plate, respectively. Also, u, v, w and θ_x, θ_y represent translation displacements and rotations respectively, while $(\dot{\ })$ denotes the time derivative of the corresponding quantity.

The net energy flow through the structure is the temporal mean of the instantaneous structural intensity, and for a single frequency case, the time averaged structural intensity during the period for a plate yields (Cho et al., 2016):

$$I_x(\omega) = -\frac{j\omega}{2}\left(w^*Q_x + \theta_y^* M_x - \theta_x^* M_{xy} + u^* N_x + v^* N_{xy}\right), \tag{4}$$

$$I_y(\omega) = -\frac{j\omega}{2}\left(w^*Q_y + \theta_y^* M_{yx} - \theta_x^* M_y + u^* N_{yx} + v^* N_y\right), \tag{5}$$

where $j = \sqrt{-1}$ is the imaginary unit and the asterisk represents complex conjugate. Internal forces and complex displacements required for SI computation can be obtained by normal mode approach or by using direct integration methods.

The instantaneous structural intensity for a beam element, Fig. 2, is expressed in a similar manner, and yields (Gavric and Pavic, 1993):

$$I_x(t) = -\left(\dot{u}N_x + \dot{v}Q_y + \dot{w}Q_z + \dot{\theta}_y M_y + \dot{\theta}_z M_z + \dot{\theta}_x T\right), \tag{6}$$

where N_x, Q_y, Q_z, M_y, M_z, T represent axial force, shear forces and torque, respectively. Also, as indicated above, while u, v, w and θ_x, θ_y represent translational and rotational degrees of freedom. Similarly to plate element, time averaged structural intensity for beam element is expressed as follows:

$$I_x(\omega) = -\frac{j\omega}{2}\left(u^*N_x + v^*Q_y + w^*Q_z + \theta_y^* M_y + \theta_z^* M_x + \theta_x^* T\right). \tag{7}$$

The structural intensity computation and visualisation for stiffened panels is performed by means of developed simulation system consisting of MSC/Patran (MSC Software, 2010a) for FE modelling and structural intensity visualization on the model, MSC/Nastran (MSC Software, 2010b), for forced vibration analysis and an in-house code for structural intensity computation, Fig. 3. The developed in-house code was developed by means of C++ and

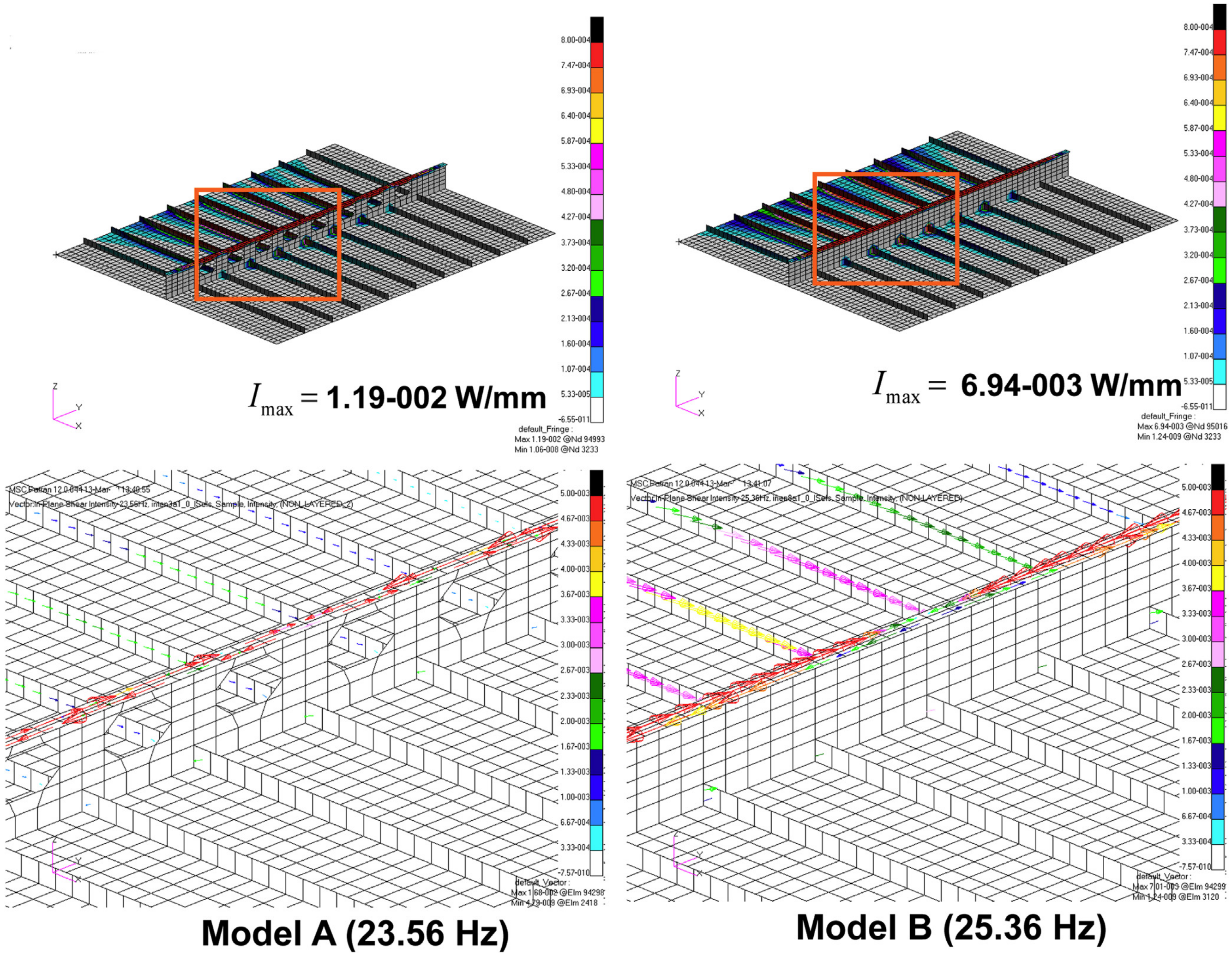


Fig. 14. In-plane shear SI components in stiffened panels with web frame.

visualization of SI was done by using PCL function of MSC/Patran. It reads necessary information (geometry data, internal forces at element center, complex nodal displacement components, etc.) from MSC/Nastran input and output files and calculates the structural intensity components after transforming element forces into global coordinate system and evaluating displacements at element centroid using nodal displacements. After that, the SI vectors in a form suitable for visualization in MSC/Patran are provided.

3. Numerical calculations and discussions

Before going into detailed calculation of structural intensities of stiffened panels, validation study is performed to confirm reliability of developed simulation system. For that purpose structural intensity analysis of the cross-stiffened panel representing typical warship bottom structure from (Xu et al., 2004c) is undertaken, Fig. 4. The panel is simply supported meaning that translational displacements are suspended along its boundaries while rotational displacement is allowed. The structure is made of the steel with Young's modulus, density and Poisson's ratio of 2.058×10^{11} N/m^2, 7800 kg/m^3 and 0.3, respectively. The excitation in this example is taken as the pressure acting on one edge of the panel in longitudinal and transverse direction, respectively, with particular aim to simulate loading caused by vertical and horizontal wave-induced bending moments encountered in seaway (Xu et al., 2004c). The magnitude and frequency of line pressure yield 1 N/m and 2 Hz, respectively, while constant damping ratio is set at 0.07. In Fig. 5, the intensity vectors are first compared for the plate without stiffeners (bare plate) and subjected to in-plane pressure in transverse direction. Validations for simplest numerical test case of simply supported bare plate subjected to harmonic point force with one and two dashpots, respectively, can be found in Cho et al. (2016), where results from Khun et al. (2004) are taken as a reference. The comparison of structural intensity vectors for cross-stiffened panel subjected to in-plane pressure in transverse and longitudinal direction, respectively, confirming the reliability of developed simulation system, is shown in Fig. 6. Namely, SI vectors obtained by developed tool are very similar to those generated by Xu et al. (2004c).

The next numerical example, dealing with a simply supported stiffened plate with different number of stiffeners is also considered by Xu et al. (2004c), where the total structural intensities are calculated only. The plate length and width yield 1080 mm and 1440 mm, respectively, while the plate thickness is set at 6.0 mm. Stiffeners are flat bars with height and thickness equal to 110 mm and 10 mm, respectively. A dashpot with a damping rate of 2000 N/m is attached to a panel at $x = 810$ mm and $y = 180$ mm for all the cases. The material properties are the same as in the previous example. An excitation force of 1 N is applied at $x = 270$ mm and $y = 1260$ mm along the z-direction of the plate with excitation

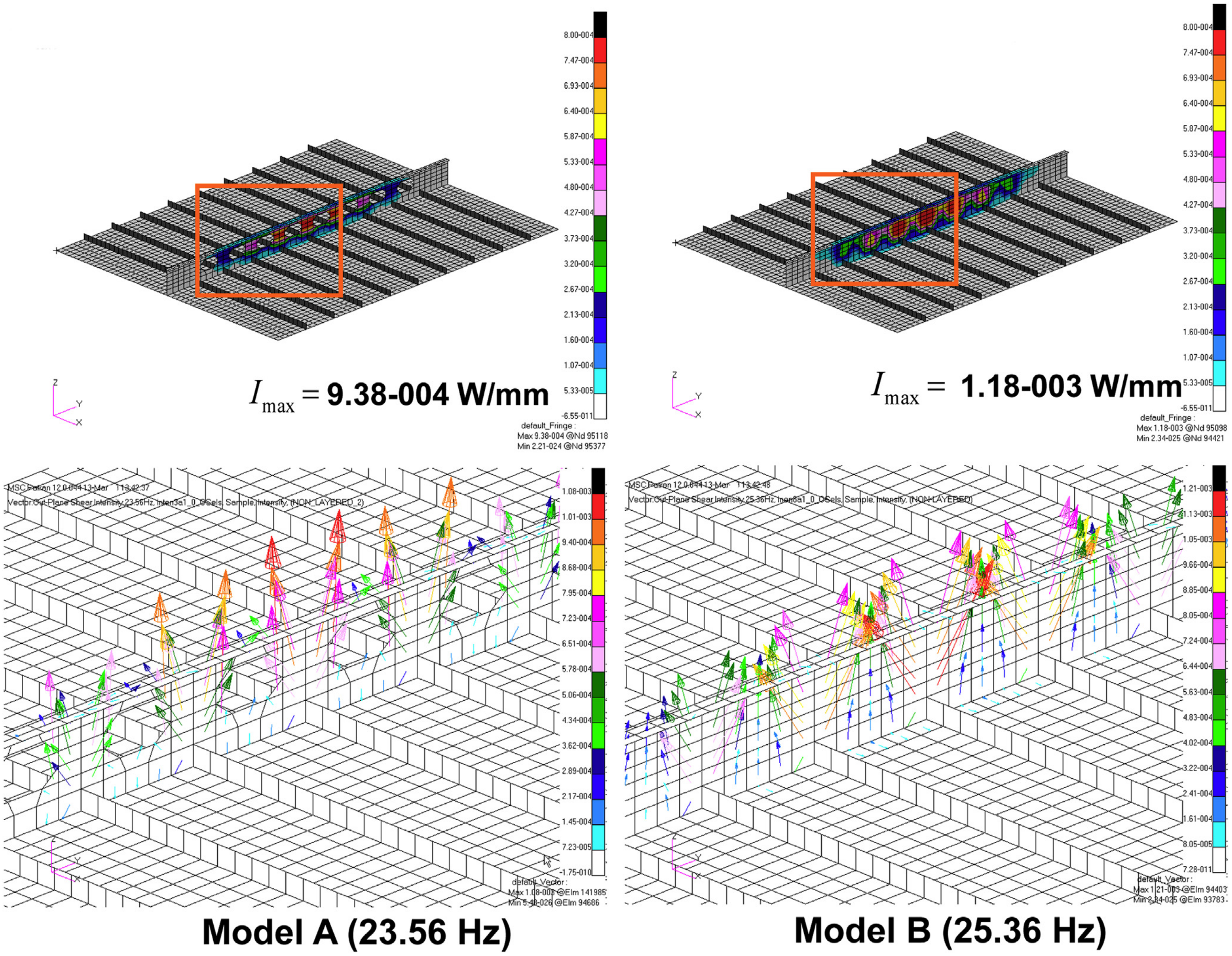


Fig. 15. Out-of-plane shear SI components in stiffened panels with web frame.

frequency close to the fundamental structural vibration frequencies for each of these cases. Natural frequencies for all cases are given in Fig. 7, where total intensities of analysed stiffened panels are presented, together with reference results by Xu et al. (2004c). The patterns are in good agreement. In the reference paper (Xu et al., 2004c), the intensity levels are not mentioned, but they are presented here in the context of determining SI components with dominant contribution to total intensity levels. Based on the results in Fig. 8 one can see that dominant contribution to total intensity is the out-of-plane shear force component.

In the next numerical example, structural intensities in stiffened panel with attached transverse web frame are thoroughly investigated. Cases of transverse web frame with (Model A) and without holes (Model B) are considered, Fig. 9, in order to investigate hole influence on transmission paths. Panel dimension are set at 5200 mm × 6500 mm × 5.0 mm in *x*, *y* and *z* direction, respectively. Equidistant longitudinals having the properties of bulb profiles of 100 mm × 6 mm, are modelled by 2-node beam elements, while for the rest of the structure the 4-node plate elements are used. The web frame is a T-profile with dimensions 450 × 7 + 100 × 10 mm, and the whole size is 390 × 210 mm with R100 in the corners. In total, Model A consists of 2960 plate element and 468 beam elements, while Model B is constituted of 3000 plate and 468 beam elements, respectively. The panels are subjected to continuous edge in-plane loading giving the total excitation force of 1.0 kN. Simply supported boundary conditions (i.e. restrained translational displacements) are applied along edges except *x*-direction of the excited edge which is set to be free. The weight of Model A and B yield 1.793 tonnes and 1.823 tonnes, respectively. Each of the models is analysed for the excitation frequency corresponding to the first natural frequency. The first natural frequencies for both panels are related to web frame tripping modes, and for Model A and B yield 23.56 Hz and 25.36 Hz, respectively, Fig. 10. As expected, natural frequency of Model A is slightly lower due to lower rigidity caused by holes in web. Stress levels in Model A are slightly higher, which is probably a consequence of geometric discontinuities, Fig. 11. Velocity fields for both models have nearly the same pattern, but the amplitude is slightly higher for Model B. Total structural intensities in both panels, which are actually directly dependent on velocities and stresses, are presented in Fig. 12. Presence of holes significantly changes the intensity field not only in web, but also in a plate, indicating that much more vibrational power flow per unit cross-section area is in the latter case (Model B) transmitted through the panel plating. Zoomed view in Fig. 12 shows that intensity vectors in longitudinals are of higher magnitude in case of Model B. The total intensity vectors in the web frame are changed due to hole presence in a way that magnitudes in the web are slightly higher for Model B, but in the flange are at similar levels.

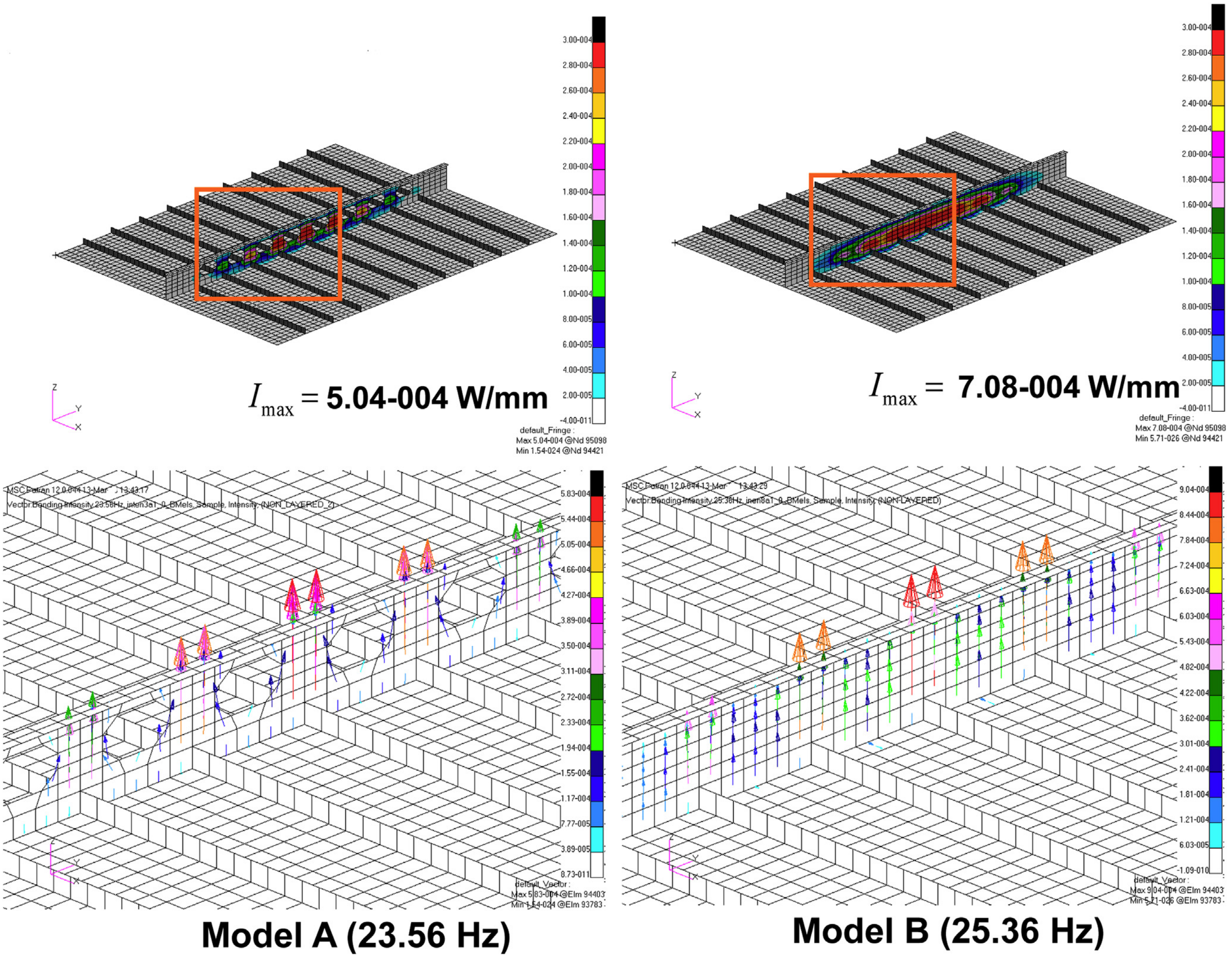


Fig. 16. Bending moment SI components in stiffened panels with web frame.

More detailed insight in this physical phenomenon is available from the SI components, which are presented in Figs. 13–17. In-plane axial component pattern in plate, Fig. 13, is much wider in case of Model B, while intensity vectors in the longitudinals are practically not influenced by the hole. In plane-shear component vectors in longitudinals are significantly larger in Model B case, while intensity field in plate is only slightly wider, Fig. 14. Global perspective on out-of-plate shear component is nearly the same for both models, Fig. 15, but zoomed view shows that the hole influence is significant. The same can be noticed for the bending moment component, Fig. 16. Twisting moment component, Fig. 17, has no significant influence to total intensity levels, but it is evident that presence of holes in the web totally changes vibrational energy flow. In Model B case, the most of the vectors is parallel with global *x-y* plane, while for the Model A high values of intensity vectors are present at the hole corners. That is probably consequence of local stress concentrations in the corners which represent points of high geometric discontinuities. For both analysed models the in-plane shear component is dominant in total structural intensity. Furthermore, the directions of SI vectors at the opening edges in Figs. 15–17 deviate from normal which implies that when the cross sectional area of the plate is reduced by the opening presence, the vibration energy is confined to flow within a narrower cross sectional closer to the opening. The indication of existence of the opening is clear in the structural intensity diagram.

4. Conclusion

Structural intensity technique utilizing the finite element method is applied to stiffened panels, subjected to different types of harmonic loading. The analysis is performed by means of in-house SI simulation system, which combines general FE tool and developed dedicated software for structural intensity calculation and visualisation. The system is validated by comparing structural intensities of bare plate and cross-stiffened panel with the available results from the literature. Energy flow phenomena are studied at some examples from the literature and set of completely novel patterns of SI components for stiffened panels are worked out. Also, thorough structural intensity analysis of stiffened panel with a web frame is performed. The web frame is studied with and without holes and hole influence on all SI components is elaborated. Although the structural intensity is here used to investigate physical phenomena of energy transmission in stiffened panels, it can be useful in vibration problems remedying in ship structures. Namely, ships are regularly sensitive to superstructure vibration caused by main engine and propeller, where structural intensity analysis enables to assess and to control dominant energy transmission paths, and in that way to reduce the amount of transmitted energy from

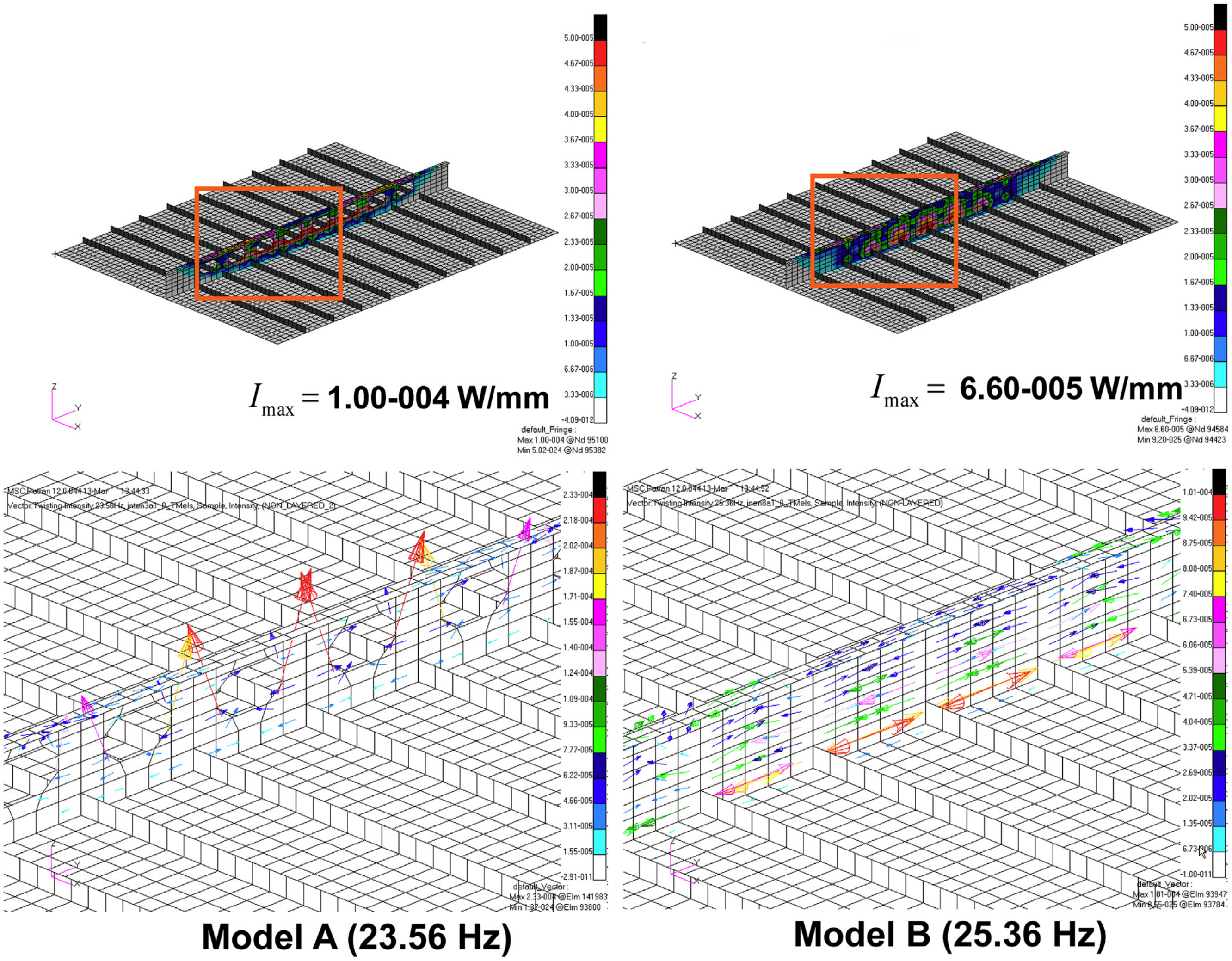


Fig. 17. Twisting moment SI components in stiffened panels with web frame.

excitation sources to superstructure. The results confirm that dominant transmission paths can be modified by changing local geometries or by introducing dampers in the dynamic system. In that case the structural intensity can offer information about optimum combination of countermeasures and their locations within the structure. In this way, a faster and more reliable remedying of vibration problem can be achieved. After formulation of structural intensity for plate and beam finite elements, and development of above described simulation system, energy flow in complete ships and offshore structures (or any other thin-walled structures) can be analyzed. Beside stiffeners, the panels in ships and offshore structures are regularly produced with openings of different shapes (rectangular, circular, elliptic, oval, etc.), to reduce structural weight, to ensure material savings or venting, or to provide passage ways, which also influence system dynamic properties and energy transmission paths. Such structural elements will be subject of further research by the structural intensity technique.

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