ON REPECHAGE DESIGN IN KNOCKOUT TOURNAMENTS WITH 16 PLAYERS

Jelena Sedlar[1], Dražen Čular[2,3], DamirVukičević[4], Igor Jelaska[2]

[1]*Faculty of Civil Engineering, Architecture and Geodesy, University of Split*

[2]*Faculty of kinesiology, University of Split*

[3]*Croatian Institut of kinesiology and sport*

[4]*Faculty of Science, University of Split*

Original scientific paper

Abstract

We propose several new repechage designs for knockout tournaments with 16 players, and then analyze the justness of medal distribution of such a tournament under random draw. The players are linearly ordered by strength, and a measure of justness of a tournament under a particular draw is introduced which depends on results of three (or four) best players. Then we take an average of such measure for every possible draw, and we obtain a measure of justness of a tournament design by which we compare tournaments with repechage designs proposed.

Key words: *knockout tournament, repechage, justness, simulation*

Introduction

Sporting contests have been of much interest to researchers. Pioneering papers in this area include [3, 4, 15]. This literature mostly assumes that whenever two players play a game, there is a fixed probability of one player beating the other. Results mostly offer formulas for computing overall probabilities with which various players may win the tournament.

One of the most widely used tournament structures is knockout tournament, so it was studied from many aspects. First, it was studied weather stronger players win more knockout tournaments and the answer was negative [2, 7]. Thus, knockout tournaments begun to be studied from the seeding point of view. Various methods of seeding were proposed and studied, from standard to random [5, 6, 9, 14]. Another interesting aspect that was frequently studied is efficaciousness of knockout tournaments [1, 10, 13]. Many other aspects were studied too such as predicting a winner of a match from previous statistics, influence of prizes to results, importance of a particular match, etc [8, 11, 12].

All this research was concerned primarily with the winner of the tournament. Yet, there are many popular sporting events where not only winner, but also second and third best players are declared. Olympic games are maybe the greatest and the most famous sporting event of such nature. Therefore, it is of interest to study not only winning of the tournament, but winning of the second and third place. In a knockout tournament, if the draw is random, two strongest players can meet already in the first round.

Among the others examples, we present the situation that occurred in the taekwondo tournament of the 2008. Olympic Games when in the first round, as a direct result of a random selection of opponents, finalists in previous Olympic Games, five-time world champion, double Olympic champion S. Lopez (USA) and three-time world champion and Olympic vice-champion B. Tanrikulu (TUR) met. Winner was Lopez, who is, among other things, a consequence of heavy fighting with Tanrikulou lost already in the next round of M. Sarmiento (ITA) and ultimately through the repechage won the bronze medal. Tanrikulu as one of the best competitors did not won medal, because he did not qualify to participate in the repechage according to the then existing model, which was first introduced with the intention to reduce possible "injustice" in final result as consequence of draw.

Another example in which draw largely influenced the final ranking can be found in qualifying tournament for the Olympic Games 2012. More precisely, in the category of 80 kg in the sixteenth finals met Iranian Y. Karami, world champion in 2003, bronze medalist from the 2004 Olympics and gold at the world championship in 2009 and in the previous example already mentioned two-time Olympic and five-time world champion American S. Lopez. The victory in the grueling duel with 5:4 carried Y. Karami, who ultimately won second place, and S. Lopez was again left without lending.

This examples are pointing to the necessity of construction of newel models of repechage designs by which justness of competition system will be improved.

In this paper, we will propose several repechage designs for knockout tournaments with 16 players, and then we will analyze the justness of medal distribution of such a tournament under random draw. We will assume that the player's strength is linearly ordered, thus for every possible draw a tournament can be simulated and the result of second and third best player analyzed. We introduce a measure of justness of a tournament under a particular draw, and then take an average of such measure for all possible draws. Thus we obtain measure of justness for a tournament by which different models can be compared.

The repechage designs (i.e. tournament models) proposed in this paper are divided in categories using following conditions:

a) maximal number of games a player can play (4 or 5),

b) number of bronze medals distributed (1 or 2),

c) if there can be games in tournament after the game for gold (yes or no).

One additional constraint which holds for every category is that no player is allowed to have more than 2 losses. Tournaments with 4 games per player and in which final game is for gold are most common, so we start with them. We analyze several repechage designs from that category and find the best model (the results support usage of model from taekwondo tournament in the Summer 2008 Olympics which was carried out as single elimination tournament with double repechage among all losers of the contestants on the final match). Then we study several repechage designs in which games after the game for gold or in which 5 games per player are allowed to see if the medal distribution in such tournaments is improved and by how much.

In the end, let us introduce names for the categories of tournaments. Since we have 3 criteria, with two possibilities each, we will have 8 categories. The name of one category will be for example "category 4-2-No" meaning tournament models with 4 games per player, 2 bronze medals distributed and no games after the game for gold.

2. Preliminaries

2.1. Assignments

Let $C=\{c_1,\dots,c_{16}\}$ be the set of players which are labeled according to their objective strength, i.e. player player c_i wins over player c_j if and only if $i<j$. Let T denote a tournament plan among 16 players. Then T has 16 starting positions $V=\{v_1,\dots,v_{16}\}$. Let A be assignment of players C to positions V. Therefore, A is bijective function $A\colon C\to V$. Let $\boldsymbol{A}$ denote set of all possible assignments of players C to positions V, and let $|\boldsymbol{A}|$ be the number of assignments $A\in\boldsymbol{A}$. Obviously, $|\boldsymbol{A}|=16!$.

2.2. Results

Given the tournament plan T and assignment A of players to starting positions, the distribution of medals under that assignment is determined (since the order of players is strict). We shall consider tournament plans in which one gold, one silver and one or two bronze medals are given. Therefore, we introduce set of medals $M=\{1,2,3,4\}$, where 1 represents gold medal, 2 silver, 3 bronze and 4 not winning any medal. Assignment A in tournament T results in medal distribution function $d_A : C\to M$. We say that assignment A for the given tournament plan T with one bronze is perfectly just if $d_A(c_i)=i$ for $i=1,2,3$ and $d_A(c_i)=4$ for $i=4,\dots,16$. If there are two bronze medals in T than we say that assignment A is perfectly just if $d_A(c_i)=i$ for $i=1,2,3$, $d_A(c_4)=3$ and $d_A(c_i)=4$ for $i=5,\dots,16$.

2.3. Measuring justness

It is trivial to note that not all assignments of players to starting positions are necessarily perfectly just. For example, if players c_1 and c_2 meet in a first round of a knockout tournament without repechages, than second best player will not win any of the medals. Repechages are designed precisely for the purpose of correcting such injustices.

The measure of justness $\boldsymbol{J}(T)$ of the tournament plan T we obtain in a following way. For any assignment $A\in\boldsymbol{A}$ we calculate justness $J(A)$ of the outcome under that assignment so that for every of the three (or four in tournaments with two bronze medals) best players we add to $J(A)$ number of penalty points which equals the sum of differences between player's real value and the result obtained in a tournament if the result obtained is worse than his real value. Then we take an average of $J(A)$ over $\boldsymbol{A}$ to be $J(T)$. Explicitly, for a tournament plan T in which only one bronze medal is distributed, we have

$$J(T)=\frac{1}{|\boldsymbol{A}|}\left(\sum_{A\in\boldsymbol{A}} J(A)\right), \tag{1}$$

where

$$J(A)=\sum_{\substack{i\in\{1,2,3\}\\ i\geq d_A(c_i)}}(i-d_A(c_i)) \tag{2}$$

for tournaments in which one bronze medal is distributed. Similar formula holds for tournament plan in which two bronze medals are distributed, only $J(A)$ is calculated by formula

$$J(A)=\sum_{\substack{i\in\{1,2,3\}\\ i\geq d_A(c_i)}}(i-d_A(c_i)+\max\{0,3-dA(c_4)\} \tag{3}$$

2.4. Knockout tournament and canonical assignment

In a knockout tournament with 16 players, players $C=\{c_1,\ldots,c_{16}\}$ are assigned to starting positions $V=\{v_1,\ldots,v_{16}\}$, then tournament takes place according to plan in Figure 1, a).

We say that two assignments, A_1 and A_2, are equivalent if for every game under assignment A_1 there is a game under assignment A_2 in which the same couple of players meets each other. It is easily verified that this is indeed relation of equivalence, hence divides assignments in separate classes. What is important to note is that two equivalent assignments have exactly the same results in every round of the tournament, and that all equivalence classes have the same number of assignments in it.

Now, for every class of assignments we want to establish one representative which we will call canonical assignment. Let A be assignment such that $A(c_i)=v_j$ and $A(c_k)=v_l$, and players c_i and c_k meet in some game of tournament. Than we say A is canonical if and only if $i<k$ implies $j<l$. Therefore, assignment A is canonical if in every game of the tournament (under that assignment) wins a player with starting position denoted with smaller number. The example of a knockout tournament under two equivalent assignments, one not canonical and other canonical, is given in Figure 1, b) and c).

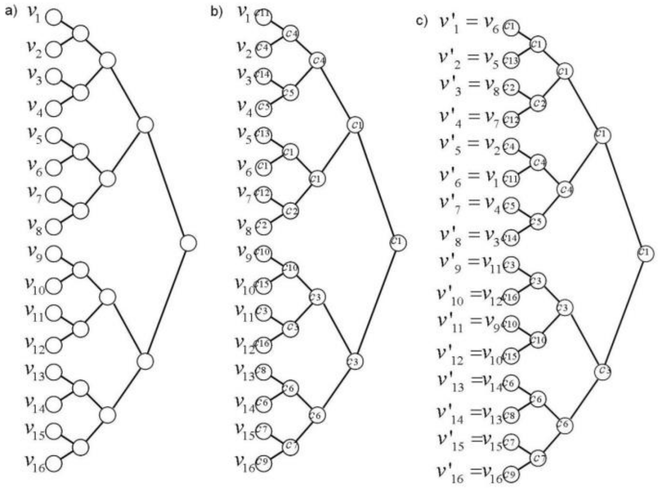

Figure 1a): Tournament plan of knockout tournament with 16 contestants, Figure 1b): an assignment A of players to starting positions, Figure 1c): canonical assignment A' of assignment A from b)

Since equivalent assignments have exactly the same results in every round of the tournament, and all equivalence classes have the same number of assignments in it, we can henceforth restrict our attention only on canonical assignments.

3. Tournament models with at most 4 games per player

Since losers in the first round of knockout tournament have played the least number of games, given our constraint it is natural that repechage starts with those players. Also, since ordering of players is linear, the best player always wins, so his result does not add to measure of justness. For the brevity sake, we introduce the notation $v_{i_1}\to v_{i_2}\to\ldots\to v_{i_k}$ for a repechage 'line' in which first game is between players assigned to v_{i_1} and v_{i_2}, then winner plays vs player on v_{i_3}, and so on until the last game in which the winner of previous games plays with v_{i_k}.

3.1.1. Category 4-2-No

Here, since two bronze medals are distributed, it is natural to design a repechage with two lines. Since the third best player is certainly (and the fourth very probably) on one of positions v_2, v_3, v_5, v_{10}, v_{11}, v_{13}, it is only natural to consider tournament plans with all the possible pairs of repechage lines, as is shown in Figure 2. Of course, bronze medals are distributed to winners of these lines.

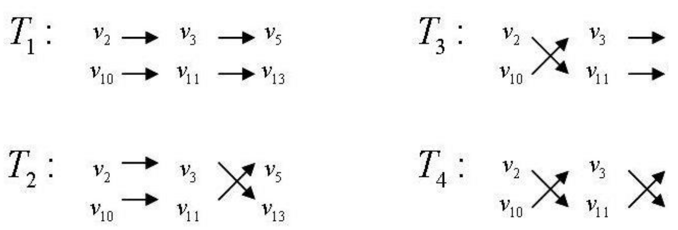

Figure 2: Repechage lines in Tournament plans T_1, T_2, T_3 and T_4

Model T_1 **:** Here, repechage consists of $v_2 \to v_3 \to v_5$ and $v_{10} \to v_{11} \to v_{13}$.

<u>*Justness:*</u> We distinguish three cases. First, if $A(c_2) \in \{v_9, \ldots, v_{16}\}$, then c_2 wins silver. In this case at most 1 penalty points are added, and that when $A(c_3), A(c_4) \in \{v_2, \ldots, v_8\}$ or $A(c_3), A(c_4) \in \{v_{10}, \ldots, v_{16}\}$

In the second case $A(c_2) \in \{v_2, \ldots, v_8\}$, therefore c_2 wins bronze. Here, we distinguish two subcases. First, if $A(c_3) \in \{v_2, \ldots, v_8\} \setminus A(c_2)$ then c_3 doesn't win medal.

If also $A(c_4) \in \{v_2, \ldots, v_8\} \setminus \{A(c_2), A(c_3)\}$, then c_4 doesn't win medal either, else c_4 wins silver. The second subcase is $A(c_3) \in \{v_9, \ldots, v_{16}\}$, where c_3 wins silver, while c_4 doesn't win medal if $A(c_4) \in \{v_2, \ldots, v_8\} \setminus A(c_2)$, and wins bronze if $A(c_4) \in \{v_9, \ldots, v_{16}\} \setminus A(c_3)$. Therefore,

$$J(T_1) = \tfrac{8}{15}\left(\tfrac{7}{14}\tfrac{6}{13} + \tfrac{7}{14}\tfrac{6}{13}\right) + \tfrac{7}{15}\left(\tfrac{6}{14}\left(\tfrac{5}{13}3 + \tfrac{8}{13}2\right) + \tfrac{8}{14}\left(\tfrac{6}{13}2 + \tfrac{7}{13}\right)\right) = 1.11282$$

By a similar analysis it is readily shown that $J(T_2) = 0.976557$, $J(T_3) = 1.044689$ and $J(T_4) = 0.999268$.

Let us note that in each of the tournaments $T_1, \ldots, T_4$ players from v_2 and v_{10} will have played at most 3 games. Using that fact we obtain the following model.

Model T_5 **:** Here, repechage consists of lines $v_2 \to v_6 \to v_3 \to v_{13}$ and $v_{10} \to v_{14} \to v_{11} \to v_5$.

<u>*Justness:*</u> Note that this model is improvement of T_2. Analyzing cases when improvement occurs it can be shown that $J(T_5) = 0.92381$

3.2 Category 4-2-Yes

Since we can't improve justness of silver medal winner, and the two bronze medals are distributed, the tournament plans considered are the same as when the result of finals is not known.

3.3 Category 4-1-No

Since in this category we don't know the winner to the end, the most reasonable tournament plan seems to be T_6.

Model T_6 **:** This model consists of $v_{13} \to v_5$ (i.e. losers of semifinals play for the bronze medal).

<u>*Justness:*</u> We distinguish three cases. In the first case $A(c_2) \in \{v_2, v_3, v_4\}$, therefore c_2 doesn't win a medal. If also $A(c_3) \in \{v_2, v_3, v_4\} \setminus A(c_2)$, then c_3 doesn't win medal either, else c_3 wins at least bronze. In the second case, $A(c_2) \in \{v_5, v_6, v_7, v_8\}$, therefore c_2 wins a bronze. If also $A(c_3) \in \{v_2, \ldots, v_8\} \setminus A(c_2)$, then c_3 doesn't win medal, else c_3 wins a bronze. And, finally, the third case is $A(c_2) \in \{v_9, \ldots, v_{16}\}$, therefore c_2 wins silver. in this case c_3 doesn't win bronze only if $A(c_3) \in \{v_2, v_3, v_4\}$ ili $A(c_3) \in \{v_{10}, v_{11}, v_{12}\}$.

Therefore,

$$J(T_6) = \tfrac{3}{15}\left(\tfrac{2}{14}3 + \tfrac{12}{14}2\right) + \tfrac{4}{15}\left(\tfrac{6}{14}2 + \tfrac{8}{14}\right) + \tfrac{8}{15}\tfrac{6}{14} = 1.038095$$

3.4. Category 4-1-Yes

Here, silver medalist cannot be changed, since the finals is the fourth game for both finalists. Third best player (if not finalist) is certainly among six players that lost to finalists in previous rounds (i.e. assigned to one of the positions v_2, v_3, v_5, v_{10}, v_{11}, v_{13}).

Model T_7 **:** Here, repechage consists of $v_2 \to v_3 \to v_5$. Winner of this line wins the bronze medal .

Justness: Let $A \in \boldsymbol{A}$. We distinguish two cases. First case is $A(c_2) \in \{v_2, \ldots, v_8\}$. In this case c_2 wins bronze medal. If also $A(c_3) \in \{v_2, \ldots, v_8\} \setminus A(c_2)$, then c_3 doesn't win any medal. Therefore, assignment like this contribute to $J(T_7)$ with $\frac{7}{15}\frac{6}{14}4$. If, on the other hand, $A(c_3) \in \{v_9, \ldots, v_{16}\}$, then c_3 wins silver medal. These assignments contribute to $J(T_7)$ with $\frac{7}{15}\frac{8}{14}2$.

The other case is $A(c_2) \in \{v_9, \ldots, v_{16}\}$. Here player c_2 wins silver. If also $A(c_3) \in \{v_9, \ldots, v_{16}\} \setminus A(c_2)$, then c_3 doesn't win medal, and the assignment like that contributes to $J(T_7)$ with $\frac{8}{15}\frac{7}{14}2$. If on the other hand $A(c_3) \in \{v_2, \ldots, v_8\}$, the tournament plan is perfectly just. Therefore,

$$J(T_7) = \frac{7}{15}\frac{6}{14}2 + \frac{7}{15}\frac{8}{14} + \frac{8}{15}\frac{7}{14} = 0.933335 .$$

Model T_8 **:** Here, the repechage consists of $v_2 \to v_3 \to v_5 \to v_{13}$. This last game takes place only if it doesn't violate given constrains (i.e. if the winner of the line $v_2 \to v_3 \to v_5$ is player from v_2 since then he will have played only three games). Winner of the repechage wins the bronze medal.

Justness: With this addition to T_7, the justness of T_7 improves only in the case of $A(c_2) \in \{v_9, \ldots, v_{16}\}$, $A(c_3) \in \{v_{13}, \ldots, v_{16}\}$, while the best of the players on positions $\{v_2, \ldots, v_8\}$ must be assigned to v_2. Therefore,

$$J(T_8) = J(T_7) - \frac{8}{15}\frac{4}{14}\frac{1}{7} = 0.911565 .$$

Model T_9 **:** Here, the repechage consists of lines $v_2 \to v_3 \to v_5$ and $v_{10} \to v_{11} \to v_{13}$. If a game between winners of these lines can be played given the constraints, then its winner wins the bronze medal, else the bronze medal is given to winner of line $v_2 \to v_3 \to v_5$.

Justness: With this addition to T_7, the justness of T_7 improves only in the case of $A(c_2) \in \{v_9, \ldots, v_{16}\}$, $A(p_3) = v_{10}$, while the best of the players on positions $\{v_2, \ldots, v_8\}$ must be assigned to v_2. Therefore,

$$J(T_9) = J(T_7) - \frac{8}{15}\frac{1}{14}\frac{1}{7} = 0.927891$$

4. Tournaments with at most 5 games per player

In these cases, since there is multitude of possibilities, the analysis of the justness was done by computer, in a way that all possible canonical assignments were simulated and the justness of results calculated.

4.1. Category 5-2-No

Model T_{10} **:** Repechage consists of four lines, two main ($v_2 \to v_3 \to v_5$ and $v_{10} \to v_{11} \to v_{13}$), and two additional ($v_4 \to v_6 \to v_7$ and $v_{12} \to v_{14} \to v_{15}$). Then winners of these lines play in between ($v_2 \to v_3 \to v_5$ with $v_{12} \to v_{14} \to v_{15}$ and $v_{10} \to v_{11} \to v_{13}$ with $v_4 \to v_6 \to v_7$). The gold medal is given to the winner of the finals, silver to the loser of the finals, and the two repechage winners get the bronze. Computer simulation gives $J(T_{10}) = 0.832013$.

Here, an 'exotic' model can be defined, which is more just, but also more inadequate from sporting point of view.

Model T_{11}**:** Repechage consists of four lines (as in T_{10}). If the number of games for winners of main lines doesn't exceed 3, than such winner plays with winner of opposite additional line (as in T_{10}). Thus we obtain two winners of repechage, each of which plays a game with the finalist he hasn't played before (if there is such and the game doesn't violate constraints, else the finalist is unchanged). If none of the original finalists is defeated in those game, they play for gold, else gold is given to undefeated finalist and the silver to the player who defeated the other original finalist. Bronze medals are given to two winners of repechage if no finalist is defeated, or else to one of such winners and defeated finalist.

Note that these additional repechage lines imply possible injustices, correction of which will improve justness of model. If, for example, in the second case, winner of the line $v_{10} \to v_{11} \to v_{13} \to v_1$ is v_{10}, and the winner of the line $v_{12} \to v_{14} \to v_{15}$ is v_{12}, which then beats v_1 in the game for bronze, then v_{12} gets the medal, and v_{11} doesn't, which is known to be injustice, since v_{11} and v_{12} must have played a game in elimination tournament in which v_{11} was better (this is implied by the fact that assignment is canonical). Computer simulation gives $J(T_{11}) = 0.555175$

Note that although justness $J(T_{11})$ is considerably better then $J(T_{10})$, model T_{11} is less acceptable from sporting point of view, since the finals might not take place (which occurs in second and third case) and it might turn out that a player plays 'for another' (when correcting mentioned injustices, player v_{12} beats v_1 in the game for bronze medal, he is doing that for player's v_{11} benefit, and not his own).

4.2. Category 5-2-Yes

Model T_{12} **:** Repechage consists of lines $v_2 \to v_3 \to v_5$ and $v_{10} \to v_{11} \to v_{13}$. Gold is won by winner of the finals. Loser of the finals plays for silver with the winner of the opposite repechage line (i.e. v_9 plays with the winner of $v_2 \to v_3 \to v_5$). Loser of that game gets one of the bronze medals, and the other is won by the winner of the other repechage line (i.e. $v_{10} \to v_{11} \to v_{13}$). Computer simulation gives $J(T_{12}) = 0.646154$.

Model T_{13}**:** Here, repechage consists of four lines, two main and two additional (as in T_{10}). Then winners of these lines play in between (as in T_{10}). Therefore, we get two winners of the repechage. Then, the game for silver is played between loser of the finals and one of those two repechage winners which hasn't played with him already (if there is such). Gold medal is given to winner of tournament,

silver to the winner of the game for silver (or else to the loser of finals), and the bronze medals are given to the loser of the game for silver and to the other winner of repechage.

Computer simulation gives $J(T_{13}) = 0.457876$.

Model T_{14}**:** Since in model T_{13} additional repechage lines are included, the same injustices can happen as in T_{11} which can be 'artificially' (i.e. without additional games) corrected. Thus we obtain T_{14} with $J(T_{14}) = 0.411357$.

4.3. Category 5-1-No

Model T_{15} **:** Repechage consists of lines $v_2 \to v_3 \to v_5$ and $v_{10} \to v_{11} \to v_{13}$, and of the game between winners of those lines. The winner of this last game wins the bronze. By computer analysis we established $J(T_{15}) = 0.666666$.

Model T_{16} **:** Repechage consists of lines $v_2 \to v_3 \to v_5 \to v_9$ and $v_{10} \to v_{11} \to v_{13} \to v_1$. Now, we distinguish three cases. First, if the winners of these lines are players from v_9 and v_1, than they play for the gold and silver (which is a proper finals), while the bronze is won by the player who lost semifinal to gold medalist. Second case, if player from position v_1 didn't win his repechage line, then winner of that line gets silver, v_9 gets gold, and for the bronze medal a game is played between v_1 and the winner of additional line $v_{12} \to v_{14} \to v_{15}$ of repechage. And the third case is analogous to second, but when it is player from v_9 who didn't win his repechage line.

Note that these additional repechage lines imply possible injustices (as in T_{11}), correction of which will improve justness of model. Computer simulation gives $J(T_{16}) = 0.466666$.

Note that although justness $J(T_{16})$ is considerably better then $J(T_{15})$, model T_{16} is less acceptable from sporting point of view, since the finals might not take place (which occurs in second and third case) and it might turn out that a player plays 'for another' (when correcting mentioned injustices, player v'_{12} beats v'_1 in the game for bronze medal, he is doing that for player's v_{11} benefit, and not his own).

4.4. Category 5-1- Yes

Model T_{17} **:** Here, repechage consists of lines $v_2 \to v_3 \to v_5$ and $v_{10} \to v_{11} \to v_{13}$. If winners of

those lines can play a game with each other, so that winner of that game will be able to play for silver (given the constraints) with the loser of the finals (if necessary, i.e. if those players haven't already met), than the game takes place. Gold medal is given to winner of tournament, silver medal is given to the winner of the game for silver (if such occurred), or else to the loser of finals. Bronze medal is given to the loser of the game for silver (if such occurred), or else to the winner of the game between winners of repechage lines. Computer simulation gives $J(T_{17}) = 0.428572$.

5. Conclusion

We considered several models of tournaments in which 4 games per player are allowed, 2 bronze medals distributed and in which there are no games after finals (which are most commonly used). We established that among such models with exactly the same number of games played (T_1, T_2, T_3 and T_4), model T_2 is the most just with $J(T_2) = 0.976557$. Therefore, our results support practice in which such model is used (i.e. taekwondo tournament plan in the Summer 2008 Olympics). Further, we proposed a model of tournament which is more just than that by adding only two more games to the repechage (model T_5 with $J(T_5) = 0.92381$).

Then, we investigated how much the justness of tournament model can be improved by allowing 5 games per player. There, we proposed models for which the justness is improved to 0.832013 (model T_{10}) or even to 0.555175 (model T_{11}, but this model is impractical from sporting point of view).

By further allowing games after finals, we further improved justness to 0.457876 (model T_{13}) or even to 0.411357 (model T_{14}, but this model again contains somewhat artificial rules from sporting point of view).

These conclusions refer to tournaments with two bronze medals. They are not comparable with tournaments with one bronze medal since the measure of justness differs in those two cases. Considering separately tournaments with one bronze medal distributed, we established that the justness of the most simple tournament plan T_6 is $J(T_6) = 1.038095$. By allowing games after finals this can be improved to $J(T_8) = 0.911565$ (model T_8). By allowing further 5 games per player justness can be improved to 0.428572 (model T_{17})

The justness of all tournament plans considered in this paper is given in Table 1.

1 bronze medal	2 bronze medals
$J(T_6) = 1.038095$	$J(T_1) = 1.11282$
$J(T_7) = 0.933335$	$J(T_2) = 0.976557$
$J(T_8) = 0.911565$	$J(T_3) = 1.044689$
$J(T_9) = 0.927891$	$J(T_4) = 0.999268$
$J(T_{15}) = 0.666666$	$J(T_5) = 0.92381$
$J(T_{16}) = 0.466666$	$J(T_{10}) = 0.832013$
$J(T_{17}) = 0.428572$	$J(T_{11}) = 0.555175$
	$J(T_{12}) = 0.646154$
	$J(T_{13}) = 0.457876$
	$J(T_{14}) = 0.411357$

Table 1: Summary of the results

Acknowledgement

This work has been supported in *part* by *grant No.* 6524 of Croatian Science Foundation. Principal investigator: Dražen Čular, PhD.

References

Appleton, D. R. (1995). May the best man win? Statistician, 44, 529–538.
Chung F. R. K. and Hwang, F. K. (1978). Do stronger players win more knockout tournaments? J Am Statist Ass, 73, 593-596.
David, H. A. (1959). Tournaments and paired comparisons. Biometrika, 46, 139-149.
Glenn, W. A. (1960). A comparison of effectiveness of tournaments. Biometrika, 47, 253-262.
Horen J. and Riezman, R. (1985). Comparing draws for Single elimination tournaments. Oper Res, 3(2), 249-262.
Hwang, F. K. (1982). New concepts in seeding knockout tournaments. Am Math Mon, 89, 235–239.
Israel, R. B. (1981). Stronger players need not win more knockout tournaments. J Am Statist Ass, 76, 950–951.
Klaassen, F. and Magnus, J. (2003). Forecasting the Winner of a Tennis Match. Eur J Oper Res, 148, 257-267.
Marchand, E. (2002). On the comparison between standard and random knockout tournaments. Statistician, 51, 169-178.
McGarry, T. and Schutz RW (1997). Efficacy of traditional sports tournament structures. J Oper Res Soc, 48, 65–74.

Rosen, S. (1986). Prizes and Incentives in Elimination Tournaments. Am Econ Rev, 74, 701-715.
Scarf, P. A. and Shi, X. (2008). Measuring the importance of a match in a tournament. Comput Oper Res, 35, 2406–2418.
Scarf, P., Yusof. M. M. and Bilbao, M. (2009). A numerical study of designs for sporting contests. Eur J Oper Res, 198, 190–198.
Schwenk, A. (2000). What is the Correct Way to Seed a Knockout Tournament. Am Math Mon, 107, 140-150.
Searls, D. T. (1963). On the Probability of winning with different tournament structures. J Am Statist Ass, 58, 1064-1081.

O PROBLEMU REPASAŽA NA TURNIRIMA SA 16 NATJECATELJA

Sažetak
Radom je predloženo nekoliko novih modela koji su primjenjivi na eliminacijskom sustavu natjecanja sa repasažom u kojem dolazi do direktnog sučeljavnja natjecatelja u žrijebu do 16 sudionika s dodjelom jedne ili dvije brončane medalje. Analizirana je pravednost distribucije medalja svakog od predloženih modela ovisno o posignutim rezultatima i načinu provedbe repasaža (prije ili nakon završnog finalnog sučeljavanja dvaju naboljih natjecatelja). Ponuđena novi modeli povećali su razinu pravednosti podjele medalja sustava sa jednom brončanom medaljom sa : $J(T_6)$ =1,03 na $J(T_{17})$ = 0,42, a u sustava podjele dvije brončane medalje sa $J(T_1)$ =1,11 na $J(T_{14})$ = 0,41

Ključne riječi: *nokaut system natjecanja, repasaž, pravednost, simulacija, taekwondo*

Corresponding information:

Received: 03 September 2017
Accepted: 09 November 2017
Correspondence to: Dražen Čular
University:University of Split
Faculty: Faculty of kinesiology
Phone: +385 992509452
E-mail: dcular@kifst.hr