THE RELEVANCE OF TURBULENT MIXING IN ESTUARINE NUMERICAL MODELS FOR TWO-LAYER SHALLOW WATER FLOW

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Shallow water equations (SWE) are derived by vertically integrating the Navier-Stokes equations. These hyperbolic partial differential equations (PDE) are a good approximation of the fluid flow when the horizontal length scale is much greater than the vertical length scale. Shallow water equations are also applied for stratified flows, in which gradients of salinity and temperature form layers of different densities. For example, highly stratified estuaries are characterized by an upper layer of freshwater flowing towards the river mouth, over a denser layer of salt-water advancing upstream. A similar configuration is also found in sea straits connecting two oceans or seas of different densities. Dynamic flows in such environments are readily modelled by coupled systems of two-layer SWE [1, 2, 3].

Two-layer models usually include friction effects; however, turbulent mixing is rarely accounted for. The reason for this may be found in the fact that stratification suppresses the intensity of vertical mixing between the layers. Even for highly dynamics flow conditions, when shear stress is known to locally generate interfacial instabilities and intensify turbulent mixing, the resulting processes occur on a much smaller scale in comparison to the thickness of the upper or lower layer [4]. In highly stratified estuaries, however, even a week entrainment may change the dynamics of a nearly-stagnant salt-water layer over longer reaches. Furthermore, entrainment may also help to maintain a locally compromised hyperbolic character of the governing system of equations when interfacial instabilities appear. Therefore, at least some aspects of the turbulent mixing should be introduced in two-layer SWE.

The following PDE system is derived for a one-dimensional two-layer shallow water flow in prismatic channels including friction and entrainment:

$$\frac{\partial \mathbf{w}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{w})}{\partial x} = \mathbf{B}(\mathbf{w})\frac{\partial \mathbf{w}}{\partial x} + \mathbf{g}(\mathbf{w}) + \mathbf{s}_{\mathrm{F}}(\mathbf{w}) + \mathbf{s}_{\mathrm{E}}(\mathbf{w}), \tag{1}$$

the vector of conserved quantities $\mathbf{w}$ and the flux vector $\mathbf{f(w)}$ are respectively defined as follows:

$$\mathbf{w} = \left\{h_1 \quad q_1 \quad h_2 \quad q_2\right\}^{\mathrm{T}}, \quad \text{and} \quad \mathbf{f}(\mathbf{w}) = \left\{q_1 \quad \tfrac{q_1^2}{h_1} + \tfrac{g}{2}h_1^2 \quad q_2 \quad \tfrac{q_2^2}{h_2} + \tfrac{g}{2}h_2^2\right\}^{\mathrm{T}}, \tag{2}$$

where h_i is the layer thickness, $q_i = h_i u_i$ is the layer flow rate per unit width, u_i is the layer horizontal velocity, g is acceleration of gravity, and index $i = 1,2$ denotes the respective upper and lower layer. The first source term appears as a result of coupling the two-layer system, in which $\mathbf{B(w)}$ is defined as

$$\mathbf{B}(\mathbf{w}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -gh_1 & 0 \\ 0 & 0 & 0 & 0 \\ -grh_2 & 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

and $r = \rho_1/\rho_2$ is the ratio between the upper layer density ρ_1 and the lower layer density ρ_2. The remaining three source terms account for the channel bed slope, friction and entrainment, as follows:

$$\mathbf{g}(\mathbf{w}) = \begin{Bmatrix} 0 \\ -gh_1\frac{\mathrm{d}b}{\mathrm{d}x} \\ 0 \\ -gh_2\frac{\mathrm{d}b}{\mathrm{d}x} \end{Bmatrix}, \quad \mathbf{s}_{\mathrm{F}}(\mathbf{w}) = \begin{Bmatrix} 0 \\ -\frac{\tau_i}{\rho_1} \\ 0 \\ -\frac{\tau_b}{\rho_2} + \frac{\tau_i}{\rho_2} \end{Bmatrix}, \quad \mathbf{s}_{\mathrm{E}}(\mathbf{w}) = \begin{Bmatrix} \frac{1}{r}E|\Delta u| \\ u_1 E|\Delta u| \\ -E|\Delta u| \\ -u_2 E|\Delta u| \end{Bmatrix}, \tag{4}$$

where b is the bed elevation, τ_b and τ_i are the respective bed and interfacial shear stress, $E = \frac{w_e}{|\Delta u|}$ is the entrainment rate, w_e is the entrainment velocity, and $\Delta u = u_1 - u_2$. The shear stresses are defined by the quadratic friction law and the respective bed and interfacial friction coefficients, λ_b and λ_i.

Notice that the turbulent mixing is included through the source term $\mathbf{s}_{\mathrm{E}}(\mathbf{w})$ and entrainment rate E (Eq. 4), which adds an additional vertical mass and momentum exchange between the layers. The assumption of constant density $\rho_i(x,t) = \rho_i$ in each layer is a first step approximation of the turbulent mixing process necessary to derive a governing system in a conservative form. The downside of such an approximation is that densities in both layers are unaffected by the mixing processes, and in practice only the volume of fluid is conserved inside the domain, instead of the actual mass. However, laboratory and field measurements indicate that in highly stratified environments, the entrainment effects are confined to the interfacial layer [4]. Therefore, if the thickness of the interfacial layer is much smaller than the thickness of the upper and lower layer, this approximation is justified.

System of Eqs. (1) is solved by an approximate Roe solver, which is based on a Finite Volume Method. In this case, a modified Q-scheme was applied [1], in which all source terms are upwinded [3]. This method is explicit in time and second-order accurate for steady solutions. The details of the numerical solver are documented in [2].

Two numerical tests are presented to illustrate the relevance of the turbulent mixing in a two-layer shallow water flow. First, a steady-state flow was computed to show the impact of E on the overall interface profile and lower layer dynamics in a highly stratified estuary. A 10 km long prismatic and horizontal channel was considered. The spatial step was set to $\Delta x = 10$ m, time step was chosen to satisfy the stability condition CFL = 0.9, also $g = 9.81$ ms^{-2} and $r = 0.975$. The downstream boundary condition was forced by a constant total depth $H = 1.5$ m, and h_1 was computed from the internal critical flow condition [2]. The upstream boundary condition was forced by constant flow rates, $Q = 1.5$ m^{3}s^{-1} and 2.5 m^{3}s^{-1}. The bed friction factor was computed from the Yen's explicit equation [5], the interfacial friction factor was set to $\lambda_i = 10^{-3}$, and E was computed from an empirical entrainment equation [6]. The simulation ran until quasi-steady flow conditions were established. The numerical solutions - interface profiles and flow rates in both layers - are shown in Fig. 1. The results suggest that the salt-water intrusion length is shortened when entrainment is considered, especially over longer reaches. This differences is mainly the result of lower layer dynamics, *i.e.*, an increased flow difference due to entrainment of fluid from the lower to the upper layer (Fig. 1B).

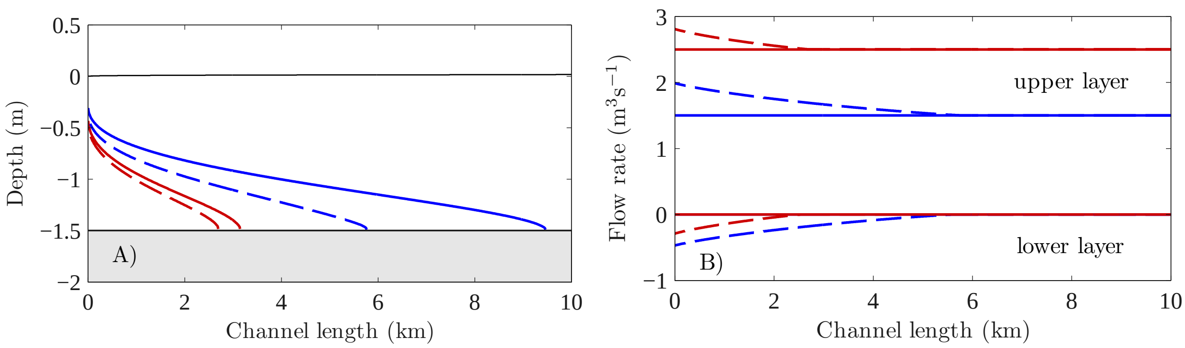

Figure 1: Numerical results for a steady-state flow in a horizontal prismatic channel: A) interface profiles and B) layer flow rates. Results from the model without entrainment are shown as a solid blue line for $Q = 1.5$ m^{3}s^{-1}, and as a solid red line for $Q = 2.5$ m^{3}s^{-1}, while the results from the model with entrainment are shown in the same colour dashed lines.

The second example shows the importance of entrainment terms when dealing with a possible loss of the hyperbolicity. In two-layer flows, the loss of hyperbolicity occurs when Δu exceeds a certain value and the eigenvalues of the Jacobian flux become complex [7]. This problem is related to the occurrence of the interfacial shear instabilities, such as the Kelvin-Helmholtz waves [7]. The same model set-up was used as previously. The only difference is that the Rječina Estuary channel geometry was considered, λ_i was calibrated to field measurements [8], and the upstream boundary condition was forced by a variable flow rate, *i.e.*, an increase from $Q = 4.3$ m^{3}s^{-1} at $t = 0$ min to $Q = 9.7$ m^{3}s^{-1} at $t = 15$ min. The comparison of numerical results with and without entrainment is shown in Fig. 2. The results show how the model without the entrainment produced non-physical oscillations during the river flow increase (Fig. 2B). Whereas, the model with entrainment terms remained stable and reached steady-state results (Fig. 2C,D).

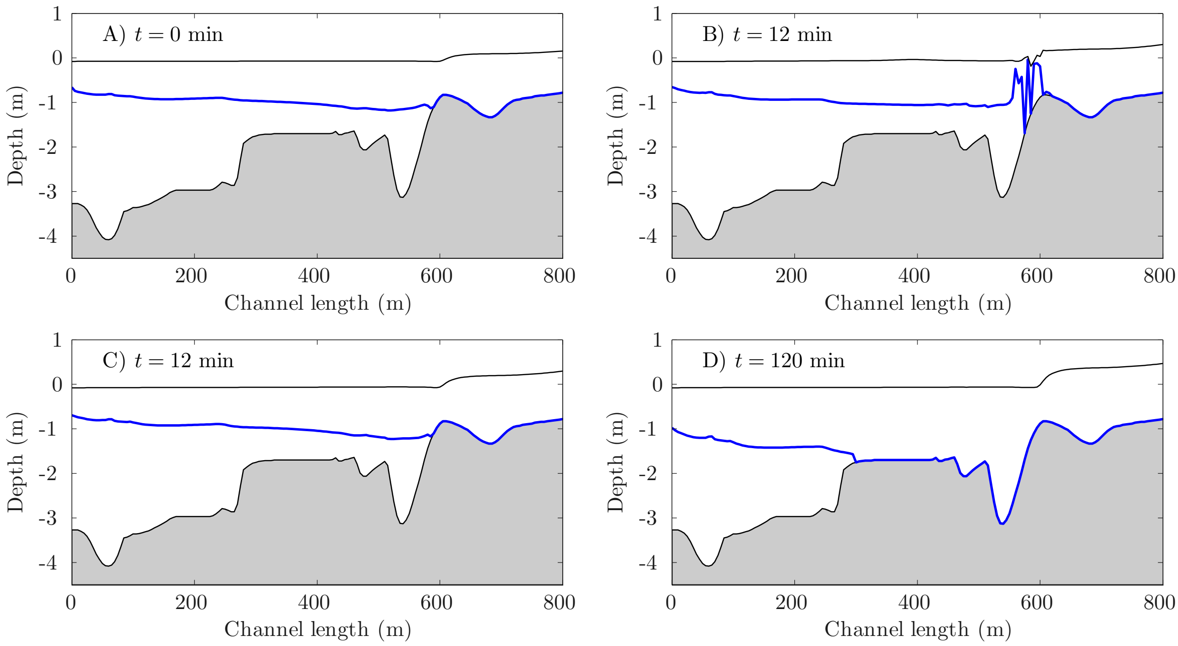

Figure 2: Numerical results for the variable two-layer flow in the Rječina Estuary: A) initial conditions at $t = 0$ min, B) instabilities at the interface at $t = 12$ min (without entrainment), C) interface profile at $t = 12$ min (with entrainment), D) steady-state solution at $t = 120$ min (with entrainment).

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