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ROCKING OF SINGLE AND DUAL RIGID-BLOCK SYSTEMS SUBJECT TO GROUND EXCITATION: EXPERIMENTAL AND COMPUTATIONAL ANALYSIS OF OVERTURNING CONDITIONS

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ABSTRACT

A prismatic block is analysed for overturning when subject to a constant ground acceleration of prescribed magnitude and duration. The analytical overturning condition and a special numerical procedure are developed and assessed against experiments. Generalisation of the approach to rocking of a dual-block stack is outlined.

Key Words: *rocking; rigid block; constant ground acceleration; overturning*

1. Introduction

The esteemed member of the United Kingdom Association of Computational Mechanics, and Emeritus Professor at University of Glasgow, Nenad Bićanić, joined the Engineering Mechanics research group at the Faculty of Civil Enineering of University of Rijeka in his native Republic of Croatia upon his retirement from Glasgow in autumn 2010. Following his life-long interests in earthquake engineering and computational dynamics and recognising the need to characterise in more detail dynamic sensitivity of discontinuous systems, from dry walls in ancient and traditional construction to thermal insulation of graphite cores in nuclear power-plant reactors, he set up an experimental dynamics laboratory and started a number of new and exciting research topics in experimental and computational dynamics. Following his untimely demise in October 2016, we present to the Association a part of the research its valued member led so enthusiastically, related to rocking of blocky structures subject to ground motion.
For this occasion, we investigate the conditions under which a single slender prismatic rigid block overturns under influence of constant-acceleration ground motion of various duration analytically, numerically and experimentally. A closed-form solution clearly separating the regions of pure translation, stable rocking, and overturning, is derived. A non-linear numerical time-stepping scheme specifically designed to detect the precise time of contact and preserve the angular momentum balance at the time of contact is also presented and its accuracy assessed against the analytical solution. An experimental test rig is described next involving the set-up designed to eliminate slipping and prescribe various magnitudes of constant acceleration and their duration, and used to validate the analytical and numerical results.
The method may be generalised to the dual-block stack, where a situation in which the ground is set in motion by an impulse through a relatively short time period, left in uniform motion during which the system rocks in a stable fashion, and then subjected to a counter-impulse, is of critical importance. Such acceleration history often makes the upper block overturn and was a subject of Nenad's special interest, and we conclude this contribution by presenting a test-rig designed by Nenad himself, capable of performing such a counter-impulse in a controlled manner.

2. Single block: analytical, numerical and experimental analysis

A rigid prismatic block of mass m, a rectangular base of unit thickness, width b and height h (or the half-diagonal R of its frontal side and the angle of slenderness $\alpha = \tan^{-1}\frac{b}{h}$) lies on a rigid ground platform subject to a constant acceleration a_0 during the prescribed time-segment $t \in [0, t_a]$ which drops

to zero for $t > t_a$. We address the case whereby sliding between the block and the ground is prevented and investigate whether for a prescribed set of input parameters R, α, a_0, t_a the block will translate with the ground, rock in a stable fashion or overturn. It is assumed that contact between the block and the ground is maintained throughout the motion.
For a relatively small a_0, the block moves along with the ground without rocking. The block shall tilt around one of its corners when the moment of its weight with respect to that corner becomes equal to an infinitesimal time change of the corresponding angular momentum. In particular, for $a_0 > g\tan\alpha$ the block ceases to move translationally and starts rotating by θ around one of its corners according to

$$\ddot{\theta} + p^2[\sin(\alpha - \theta) - a\cos(\alpha - \theta)/g] = 0, \tag{1}$$

where $p = \frac{1}{2}\sqrt{\frac{3g}{R}}$ is the so-called *frequency parameter*. For small α and θ, this equation becomes linear with a constant coefficient of the form $\ddot{\theta} - p^2\theta = p^2\,(a/g - \alpha)$, having the solution $\theta = (\frac{a_0}{g} - \alpha)(\cosh pt - 1)$ for $t \leq t_a$ and $\theta = \alpha(1 - \cosh pt) + \frac{a_0}{g}\,[(1 - \cosh pt_a)\cosh pt + \sinh pt_a \sinh pt]$ for $t > t_a$. The least critical situation in which the block will overturn takes place when $\theta = \alpha$ and $\dot{\theta} > 0$ (i.e. when $\theta > \alpha$) for $t > t_a$. This gives the following condition for overturning

$$\frac{\alpha - \frac{a_0}{g}(1 - \cosh pt_a)}{\frac{a_0}{g}\sinh pt_a} < \tanh pt \leq 1 \quad \Longrightarrow \quad pt_a > -\ln\frac{\frac{a_0}{\alpha g} - 1}{\frac{a_0}{\alpha g}}. \tag{2}$$

The problem may be also analysed numerically. The motion is described via

$$\begin{aligned} \ddot{\theta} + p^2[\sin(\alpha - \theta) - a\cos(\alpha - \theta)/g] &= 0, \quad \theta \geq 0 \\ \ddot{\theta} - p^2[\sin(\alpha - \theta) + a\cos(\alpha - \theta)/g] &= 0, \quad \theta \leq 0. \end{aligned} \tag{3}$$

These equations will be numerically solved using Newmark's trapezoidal time-stepping rule [1] at discrete time instants separated by a time step Δt, along with the Newton–Raphson iterative solution procedure. To make the transition from one of the equations of motion to the other without any constraint violation it becomes important to detect the time of contact precisely. We propose a technique in which the rotation at the end of a time step is monitored throughout the analysis for the change of sign. When such change is detected, say at a time t_{n+1}, dynamic equilibrium over the time step is repeated for an *unknown time-step length* $\Delta t'$ under the condition that $\theta_{n+1} := 0$. After the impact, the original time-step length is restored and the time-stepping procedure switches to the other equation of motion.
When the exact time of the impact is detected, angular velocity $\dot{\theta}^+$ at the start of the first post-impact step needs to be reduced with respect to that at the end of the last pre-impact step ($\dot{\theta}^-$) following the angular-momentum balance via $\dot{\theta}^+ = \left(1 - 1.5\sin^2\alpha\right)\dot{\theta}^-$ [2], while angular acceleration $\ddot{\theta}^+$ follows from the corresponding post-impact equation of motion, in effect giving $\ddot{\theta}^+ = -\ddot{\theta}^-$. The detail is given in [3].
To validate the theory and assess the numerical procedure, a test rig is set up and a range of measurements conducted on a near frictionless air track shown in Fig. 1 enabling a constant acceleration of a chosen magnitude to be applied to a slider (blue) for a prescribed duration. The near-absence of friction is obtained by means of an air cushion between the air track and the slider. This is achieved by pumping air, which escapes the air track through a large set of tiny holes drilled on its top surface.
Before the analysis, the block of mass m is placed on the slider of mass M and the system is set floating by supplying sufficient air pressure to the air track. The slider–block system is kept in equilibrium through a force in the string attached to the slider and running over a pulley on the right-hand side of the air track and supporting a hanging mass $\bar{m}$, which is counter-balanced by the force in the second string securing the slider in a fixed position by connecting it to the left-hand side of the air track.
The contact conditions needed (continuous contact between the block and the slider without slipping) are provided by attaching the block to the slider via adhesive tapes as also shown in Fig. 1 enabling free rotation of the block about both corners without sliding and vertical detachment from the slider.

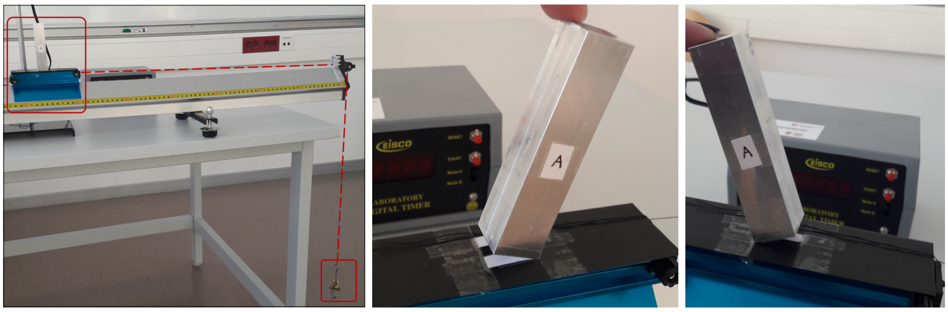

Figure 1: Air-track, mechanical system at start of analysis, provision for non-sliding non-bouncing contact

The slider–block system is set in motion by cutting the left-hand string, thus subjecting the system to a constant acceleration of magnitude $a_0 = \frac{\bar{m}g}{M+m+\bar{m}}$. For the given slider–block system, therefore, the constant acceleration a_0 is completely defined by the hanging mass $\bar{m}$ which may be freely varied.
The exposure t_a of the system to such acceleration is defined by this mass *and* the initial distance from the bottom of the hanging mass to the floor y as $t_a = \sqrt{\frac{2y}{g}\frac{M+m+\bar{m}}{\bar{m}}}$. In other words, for the chosen $\bar{m}$ defining a_0, varying this distance provides different exposures t_a of the system to such acceleration. When the ground acceleration drops to zero, the slider remains free to move uniformly without any horizontal disturbances thus completely reproducing the problem stated.
We analyse the block with the half-diagonal length $R = \frac{1}{2}\sqrt{85}$cm and angle of slederness $\alpha \equiv \tan^{-1}(2/9)$ and conduct the time stepping for the range of non-dimensional constant accelerations $\frac{a_0}{\alpha g} \in \langle 0, 3]$ and non-dimensional exposures to such accelerations $pt_a \in \langle 0, 5]$ using the time-step length $\Delta t = 0.001$ s and the rotation norm in the Newton–Raphson procedure set to 10^{-10}. The results are shown in Fig. 2, from where it is obvious that they agree with the analytical overturning predictions very closely, the difference in the results between the two analyses being completely attributed to the fact that the numerical analyses has been conducted for the non-linear case.
A range of measurements is then made for given mass of the slider $M = 120$ g and the block $m = 95.5$ g and different values of the input data $(\bar{m}, y) \Leftrightarrow (a_0, t_a)$. The hanging masses are chosen from within the range $\bar{m} \in [15\,g, 115\,g]$ and the vertical distances from within the range $y \in [0.2\,\text{cm}, 25.3\,\text{cm}]$. The results of the experiment are shown in Fig. 2 and compared to the analytical and the numerical results.

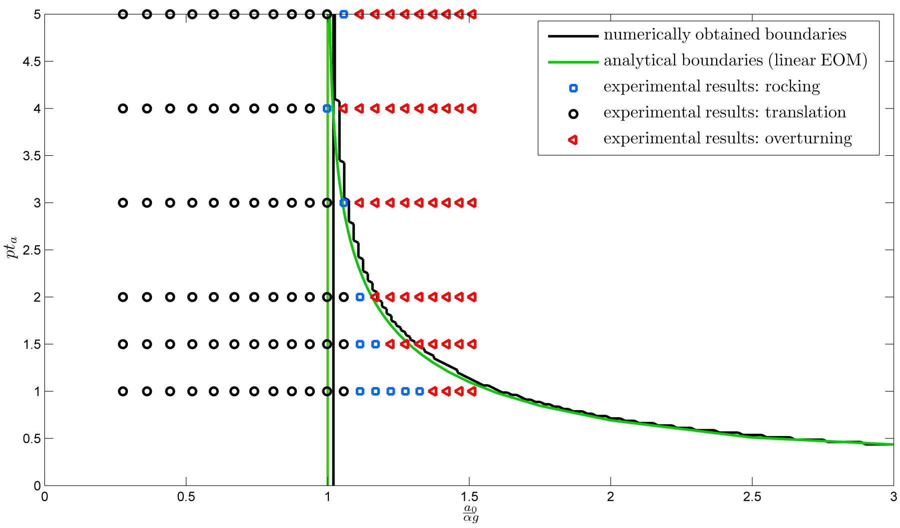

Figure 2: Single block: analytical, numerical and experimental results

Clearly, the results overally agree quite well and the small differences between the experimental and the numerical results may well be caused by imperfect manual input of the problem parameters.

3. Dual-block stack: problem description, partial overturning and future work

The above analysis may be applied to motion of a non-bouncing dual-block stack subject to ground acceleration a and in our future work we plan to address this problem in more detail. Such motion can be described by four different motion patterns as shown in Fig. 3.

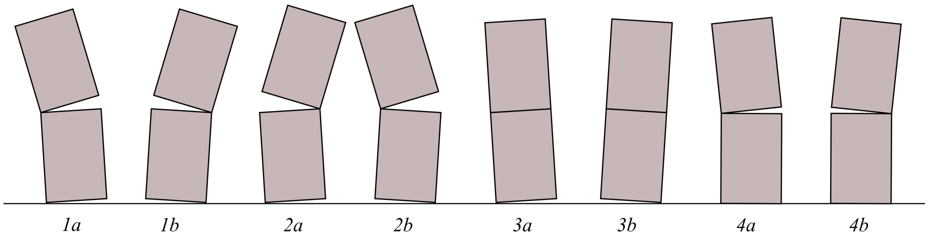

Figure 3: Dual-block stack: patterns of rocking motion

For such a problem, the so-called partial overturning (overturning of the upper block) may take place for an alternative acceleration history, in which at a time $t_b > t_a$ a constant acceleration $a_1 < 0$ takes place and lasts until $t_c > t_b$, after which it again drops to zero.

To analyse this problem experimentally, a bespoke pendulum rig has been designed inspired by the real-size tests conducted at the Roorke University in India [4]. The rig is shown in Fig. 4 and, in contrast to the air track, (i) it sets the slider–block system in motion by providing an input moment impulse via a pendulum arm and (ii) provides a *counter-impulse* when the slider hits the stopper thus reversing the direction of motion of the slider and potentially causing overturning of the upper block.

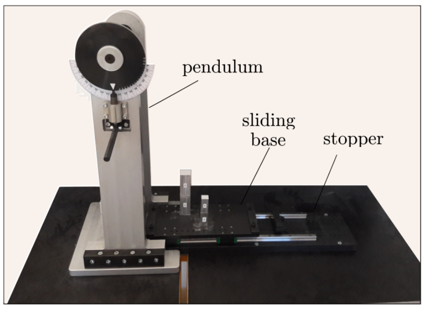

Figure 4: Pendulum test rig

Our future work will address this phenomenon both numerically and experimentally.

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