

The impact of using GeoGebra interactive applets on conceptual and procedural knowledge

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Abstract

This work deals with the impact of using digital textbooks consisting of GeoGebra interactive applets on students' conceptual and procedural knowledge in mathematics. The authors propose the model of computer-guided discovery learning by using dynamic geometry software which includes three basic elements: learning objects, tasks for students and learning outcomes. The model is primarily based on the theory of constructivism, Pólya's heuristic strategy and Schoenfeld's problem-solving model.

The experimental research plan with experimental and control groups was used and there were 703 students in twelve elementary schools in Croatia participating in the research. The experimental group of students was taught by the model of computer-guided discovery learning and the control group of students was taught by teachers using traditional teaching methods. The covariance analysis determined the impact of the model of computer-guided discovery learning on students' conceptual and procedural knowledge and the results have shown statistically significant differences between the groups in favor of the experimental group.

The qualitative and quantitative analysis of students' and teachers' questionnaires and forum discussions was used to assess their attitudes towards the proposed learning model. The importance of teachers' experience in teaching using this model has also been noted. This learning model indicates the potential of computers and dynamic geometry software GeoGebra for scaffolding support in learning mathematics.

Keywords: *conceptual knowledge, digital textbook, discovery learning, dynamic geometry software, GeoGebra, interactive applet, learning model, procedural knowledge*

1. Introduction

Modern mathematics teaching is based on the theory of constructivism and focused on the student. Being an active participant in the teaching and learning process, he/she observes, investigates, asks questions, creates hypotheses, solves problems, cooperates with others, analyzes and evaluates his/her own work and the work of others. The student develops new knowledge on his/her own experiences and observations in physical and social environment, and, according to the radical constructivism learning theory, this knowledge is unique to each individual (von Glasersfeld, 1995). The teacher's role in such teaching is twofold: (1) organizing the teaching process and preparing appropriate activities for students and (2) guiding students in the learning process and encouraging them to think. Von Glasersfeld (1995) stands for scaffolding teacher support which means that the teacher initially stimulates and leads students to use their knowledge and skills and helps them learn some new strategies for solving the problem, but simultaneously lets them come to the solution on their own and be responsible for their progress (Dennen, 2004). Except for the teacher, this kind of support can be provided by computers. A computer can be a very useful tool for helping students in their active knowledge construction and especially for the implementation of student experiments and discovery learning.

2. Overview of previous research and the research question

The computer simulation is considered to be exceptionally suitable learning environment for discovery learning (Pinto, Couso and Hernández Rodrigues, 2010). De Jong and van Joolingen (1998) recognize some problems that students may encounter in discovery learning using computers, and emphasize the need for good scaffolding. They suggest three principles for successful discovery process: (1) displaying information and simulation simultaneously, (2) giving the students specific tasks and (3) structuring the learning environment in a few steps. Mayer (2004) shows that structured or guided discovery learning, with specific teaching goals and stimulating students' cognitive activities, best supports constructivist learning approach. Wangpipatwong and Papisratorn (2008) design the constructivist e-learning environment with a focus on research, cooperation and the construction of new knowledge and they conclude it gives better results than traditional learning environment.

In mathematics teaching, Glasnović Gracin (2008) recommends the use of dynamic geometry software because it supports multiple representations of mathematical objects and enables simple transferring from one mathematical view to another. As an example, she proposes GeoGebra since it supports graphic, symbolic and spreadsheet views. Furthermore, GeoGebra provides experimental learning that Schneider (2002) describes as active, independent and revealing learning primarily aimed at the development of intuitive perceptions and ideas, as well as at deeper and more comprehensive understanding of mathematical concepts. Bu, Spector and Haciomeroglu (2011) also emphasize the importance of multiple representations and students' interactive dealing with learning objects. They also suggest GeoGebra as an extremely useful mathematical tool which provides an intellectual bridge between certain domains of mathematics theory, school mathematics and instructional design frameworks. According to Iranzo and Fortuny (2011) the use of GeoGebra has a positive impact on the mathematical problem solving strategies since it helps students visualize the problem and avoid algebraic obstacles so students can focus on understanding geometry more. Shadaan and Leong (2013) find that GeoGebra is extremely useful as students' scaffolding support in understanding mathematical concepts and also as an excellent motivational tool for increasing self-confidence and fostering higher-level student thinking.

The previous researches show the great potential of GeoGebra as a tool for discovering and acquiring new mathematical knowledge which is especially related to conceptual knowledge. So the subject of this study is to examine the impact of GeoGebra and computer-guided discovery learning on conceptual and procedural knowledge of students. For this purpose GeoGebra is used to create various interactive applets and GeoGebraTube service is used to organize the applets in a digital textbook. In addition to these learning objects, it is necessary to plan the activities for students according to the desired learning outcomes. All three elements together form a learning model which is described in the next chapter.

3. The model of computer-guided discovery learning

Elliott, Sweeney and Irving (2009) develop the conceptual model of e-learning, which is based on the theory of constructivism, the discovery learning strategy and the problem-based learning. They explore its effectiveness in the natural sciences (Figure 1). The tasks for students are defined as a simple research cycle which is similar to the stages of

scientific research. The idea of solving problem in four steps is taken from Pólya (1945): (1) problem analysis, (2) inquiry planning, (3) research loop and (4) problem closure.

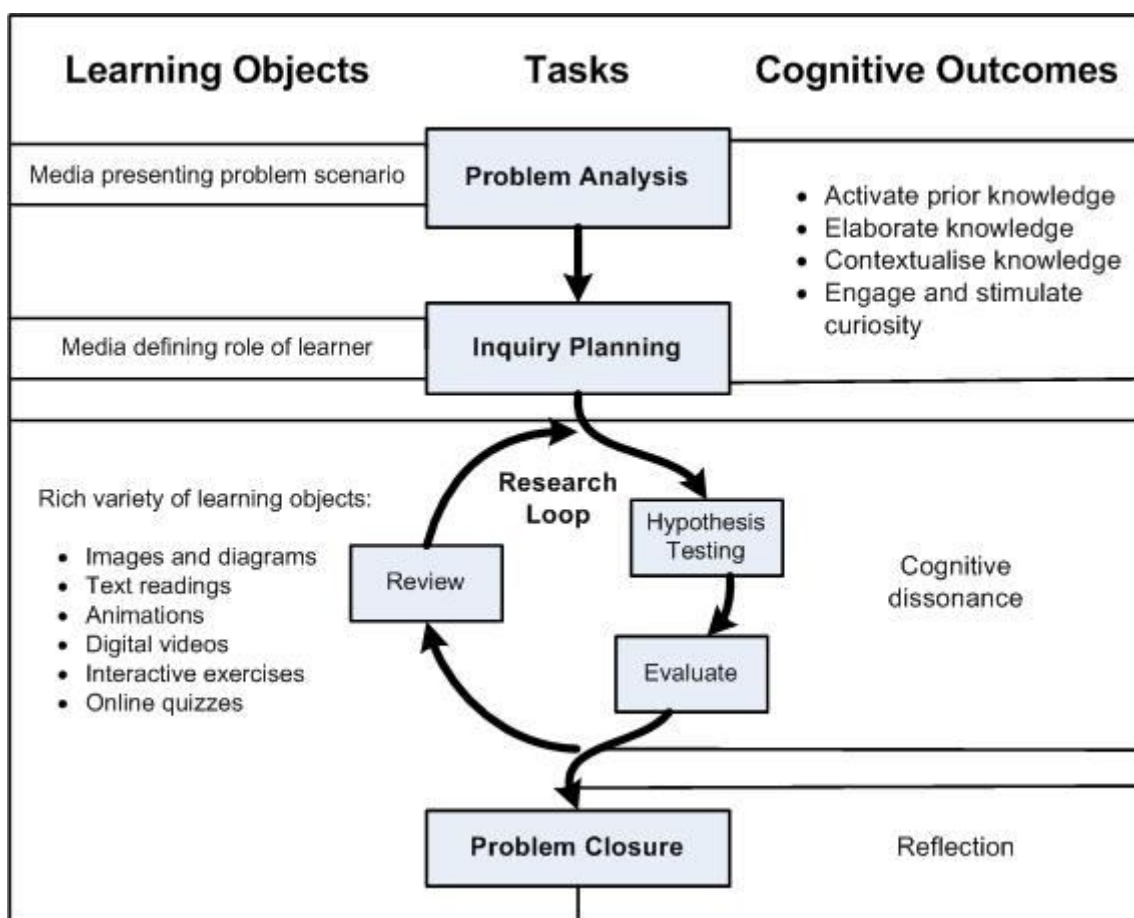


Figure 1. The conceptual model of e-learning (Elliott, Sweeney and Irving, 2009, p. 660)

On the basis of previous model, the model of computer guided discovery learning by using dynamic geometry software in mathematics teaching¹ is developed. Pólya's heuristic problem solving strategy is embedded in GeoGebra's dynamic learning environment, and according to Karadag and McDougall (2009) it can be used for explaining, exploring and modeling. If we want a student to successfully solve a given problem, with or without a computer, except possessing the basic mathematical knowledge (resources), the problem solving techniques (heuristics), proper selection of previous knowledge and techniques (control), it is necessary for the student to believe in his/her own abilities and have a positive attitude regarding mathematics (Schoenfeld, 1985). This Schoenfeld's frame of mathematical

¹ This learning model is developed as first author's doctoral thesis "Development of the model of computer guided discovery learning by using dynamic geometry software in teaching mathematics" under the supervision of second author and Mario Dumančić, PhD, and presented at The Faculty of Teacher Education, University of Zagreb on 16th February 2017.

problem solving model is complemented with socio-cultural factors related to the interaction of students with teachers, other students and the learning environment (Lester, Garofalo and Kroll, 1989), or, in this case, the dynamic discovery learning environment of GeoGebra.

The computer-guided discovery learning model consists of three elements: (1) learning objects, (2) activities for students and (3) learning outcomes (Figure 2).

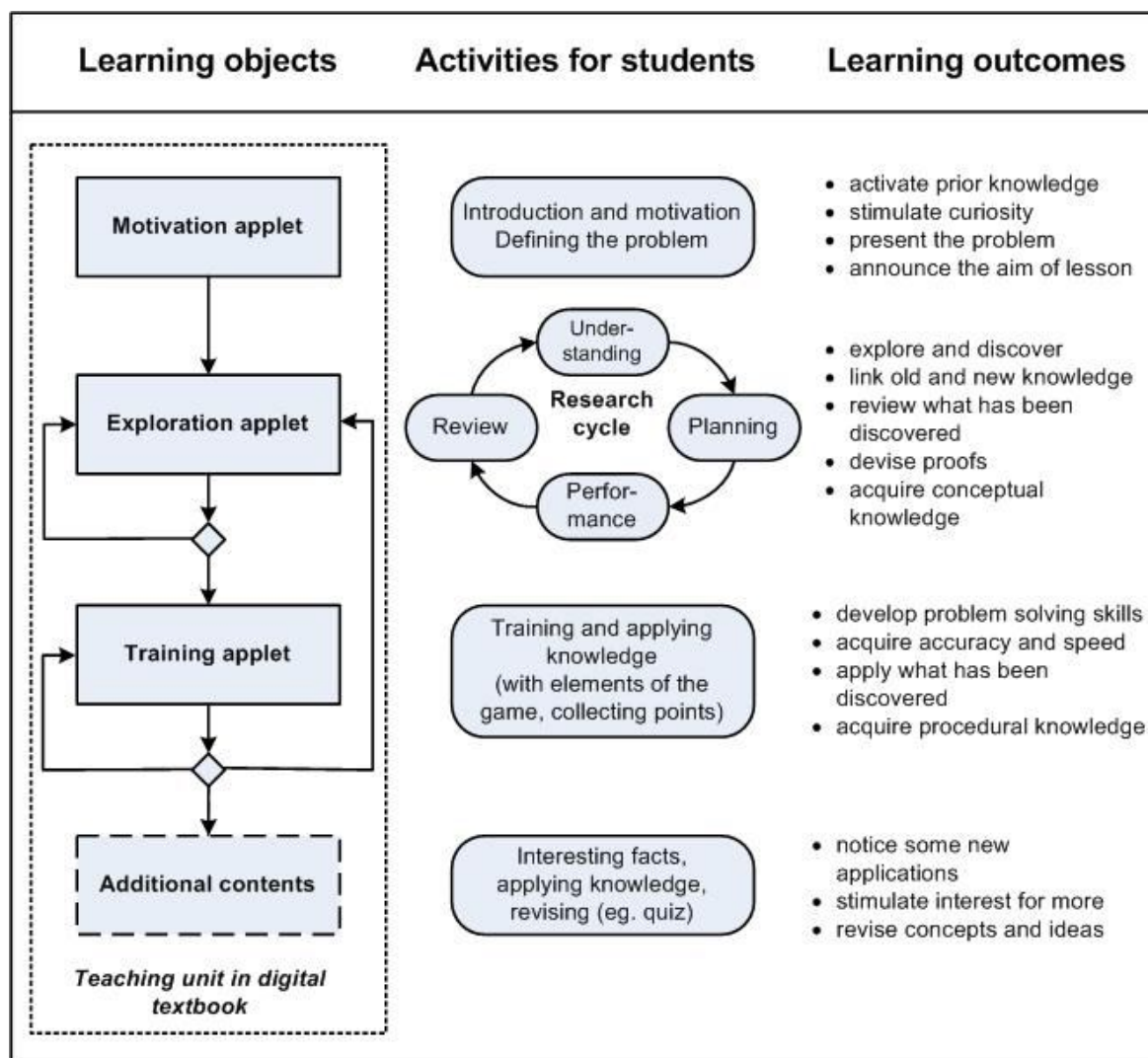


Figure 2. Scheme of the computer-guided discovery learning model (Dijanić, 2017, p. 152)

Learning objects are created like interactive applets or mathlets. According to Hohenwarter and Preiner (2007) mathlet is a small learning object dealing with specific mathematical topic or problem. It can be used for teacher demonstration to the whole class or for individual student's learning. The mathlet is made of a static and a dynamic part. Bjelanović Dijanić (2009) explains that the dynamic section includes mathematical objects (such as points, lines, geometric shapes, etc.) that can be moved so students could observe

their interactivity and perform experimental work. The static section of mathlet contains instructions, questions, tasks, suggestions or help, which allow guiding student along his/her way to conclusions with minimal risk of failure. According to Dijanić (2017) these interactive applets are organized in the digital textbook with chapters for each teaching unit and every applet has one of the following functions:

- (1) Motivation applet, at the beginning of each teaching unit. Its main role is to present a problem or topic to the student and to motivate him/her for further work.
- (2) Exploration applet; its role is to enable performing mathematical experiments to the student and help him/her discover new mathematical knowledge. There are one or more research cycles in each applet which stages are taken from Pólya's problem solving strategy: understanding the problem, devising a plan, carrying out the plan and looking back. At this learning step the basic outcomes are to acquire conceptual knowledge and understand the basic ideas in mathematics.
- (3) Training applet; its role is to enable students solve some math problems related to the topic. For additional students' motivation these applets include the elements of game and students' progress is encouraged by scoring points. The learning outcomes are to apply discovered skills in solving problems and acquire procedural knowledge.
- (4) Additional contents; like interesting facts from the history of mathematics, some specific knowledge application in everyday life, extended mathematical contents for gifted pupils, additional teaching activities for students with less developed mathematical abilities, quizzes and the like.

For the purpose of the research which included the development of computer guided discovery learning model and the verification of this learning model's efficiency (Dijanić, 2017), four teachers and the first author of this paper collaboratively created three digital textbooks:

6th grade: *Angle and triangle* (<http://tube.geogebra.org/student/b272303>)

(Dijanić, Belavić, Grgić, Pein i Vuković, 2014),

7th grade: *The similarity of triangles and polygons*

(<http://tube.geogebra.org/student/b364307>)

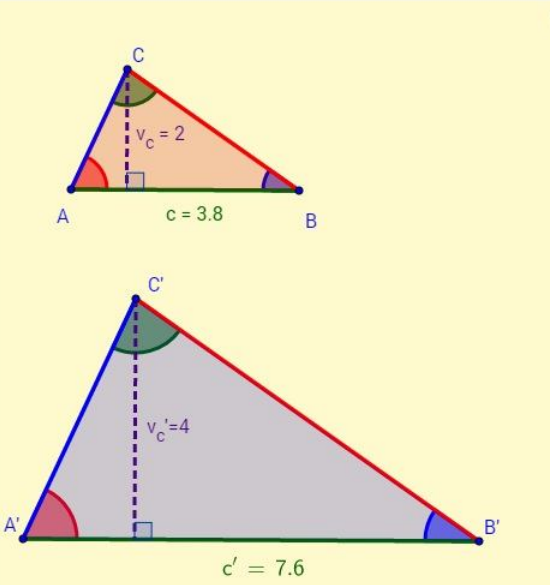
(Dijanić, Grgić, Pein i Vuković, 2014),

8th grade: *Pythagorean theorem* (<http://tube.geogebra.org/student/b297339>)

(Dijanić, Belavić, Grgić i Pein, 2014).

In these digital textbooks the emphasis is on the applets that enable discovering new mathematical knowledge with the scaffolding support of GeoGebra. For example, Figure 3 shows the exploration mathlet from digital textbook *The similarity of triangles and polygons*, teaching unit *The perimeter and area of similar triangles*. The idea is to enable students observe several cases of similar triangles, write down their observations in the table and discover the relationship between the areas of similar triangles by inductive reasoning. The mathematical proof of discovered assertion is interactively demonstrated to students in the next applet of this textbook.

Omjer površina sličnih trokuta - istraživanje



Na slici lijevo su dva slična trokuta. Koeficijent sličnosti
Nacrtaj u bilježnicu ovakvu tablicu.

	k	$P = \frac{c \cdot v_c}{2}$	$P' = \frac{c' \cdot v_c'}{2}$	$\frac{P'}{P}$
1.				
2.				
3.				
4.				

1. Izračunaj površinu oba trokuta.
Popuni 1. redak tablice podacima sa slike.
Izračunaj omjer površina trokuta A'B'C' i ABC (zaokruži na jednu decimalu). Što uočavaš?
2. Promijeni veličinu trokuta ABC pomicanjem vrhova A, B i C.
Izračunaj površine pa popuni 2. redak tablice.
3. Promijeni veličinu trokuta A'B'C' pomicanjem vrha B' tako da je
 - a) trokut A'B'C' veći od trokuta ABC (za 3. redak tablice),
 - b) trokut A'B'C' manji od trokuta ABC (za 4. redak tablice).
4. Što zaključuješ?
Kako se odnose površine sličnih trokuta? Provjeri
Zapiši to matematičkom formulom!

Figure 3. Exploration applet: The ratio of the areas of two similar triangles

Some other types of interactive applets that are used in the proposed learning model with a detailed description of activities and tasks for the students can be found in Dijanić (2015; 2017).

4. Research methodology

The aim of the research is to investigate the impact of using proposed computer-guided discovery learning model on students' conceptual and procedural knowledge in mathematics.

There were 15 mathematics teachers and 703 of their students from the sixth, seventh and eighth grades from twelve Croatian school participating in the research. The study was conducted during mathematics classes during school year 2014/2015. Before the experiment started, the students had taken the exams testing their prior knowledge in mathematics. The experimental research plan with experimental and control groups was used. Each teacher participated with two parallel classes, one being experimental and the other control group. The experimental group of students was taught by the model of computer-guided discovery learning (as the intervention) working individually or in pairs in IT classroom and the control group of students was taught by teachers using traditional teaching methods. The research process included five lessons of learning new geometry content in the sixth, seventh and eighth grades for both groups of students. All the teachers used the same lesson plans for teaching students in the control group and used digital textbooks mentioned in the previous section for teaching students in the experimental group. The final examinations of acquired knowledge were conducted in the same way as the initial testing.

Besides the quantitative analysis of students' results at testing conceptual and procedural knowledge, the qualitative analysis of the teachers' discussions in the forum of learning management system Moodle was conducted, as well as the analysis of the semi-structured interview of the teachers. The results indicate multiple advantages of the proposed learning model.

5. Results

The covariance analysis has been used to determine the impact of the model of computer-guided discovery learning on students' conceptual and procedural knowledge. The students' pretest results were used as a covariate in the analysis to control the potential impact of the difference in the initial testing results of students on the differences between the groups in the final testing results.

The following tables show the differences between groups (control and experimental) and statistical significance of these differences for two dependent variables: conceptual knowledge (Table 1) and procedural knowledge (Table 2).

Table 1. The results: conceptual knowledge

	Control group	Experimental group		
Grade	<i>M (SD)</i>	<i>M (SD)</i>	<i>F (df)</i>	<i>p</i>
6 th	7.283 (2.940)	7.996 (3.068)	5.608 (1/238)	<i>p</i><0.05
7 th	8.308 (2.694)	8.516 (3.310)	0.346 (1/177)	<i>p</i> >0.05
8 th	8.780 (2.595)	9.459 (2.774)	6.524 (1/279)	<i>p</i><0.05
all together (6 th - 8 th)	8.166 (2.802)	8.699 (3.098)	8.296 (1/700)	<i>p</i><0.01

Table 2. The results: procedural knowledge

	Control group	Experimental group		
Grade	<i>M (SD)</i>	<i>M (SD)</i>	<i>F (df)</i>	<i>p</i>
6 th	3.734 (2.627)	4.060 (2.900)	1.443 (1/238)	<i>p</i> >0.05
7 th	3.485 (2.837)	3.370 (2.806)	0.173 (1/177)	<i>p</i> >0.05
8 th	4.254 (2.831)	5.300 (3.167)	16.413 (1/279)	<i>p</i><0.01
all together (6 th - 8 th)	3.888 (2.776)	4.369 (3.095)	9.173 (1/700)	<i>p</i><0.01

If we look at the whole sample of students together, the covariance analysis gives the following results. The students' average scores at the conceptual knowledge variable are statistically significantly different between the intervention and the comparable group ($F=8.296$, $df=1/700$; $p<0.01$). Concurrently, the students taught using the computer-guided discovery learning model show higher conceptual knowledge ($M=8.699$) than the students taught traditionally ($M=8.166$). The students' average scores at the procedural knowledge variable are statistically significantly different between the intervention and the comparable group ($F=9.173$, $df=1/700$; $p<0.01$). Concurrently, the students taught using the computer-guided discovery learning model show higher procedural knowledge ($M=4.369$) than the students taught traditionally ($M=3.888$).

If we look at the grades individually, the students' average scores at the conceptual knowledge variable are statistically significantly different between the intervention and the

comparable group only at the sixth-grade ($F=5.608$; $df=1/238$; $p<0.05$) and eighth-grade students ($F=6.524$; $df=1/279$; $p<0.05$). Concurrently, the students taught using the computer-guided discovery learning model show higher conceptual knowledge ($M_6=7.996$; $M_8=9.459$) than the students taught traditionally ($M_6=7.283$; $M_8=8.780$). The students' average scores at the procedural knowledge variable are statistically significantly different between the intervention and the comparable group only at the eighth-grade students ($F=16.413$; $df=1/279$; $p<0.01$). Concurrently, the students taught using the computer-guided discovery learning model show higher conceptual knowledge ($M_8=5.300$) than the students taught traditionally ($M_8=4.254$). There is no statistically significant difference between the groups of seventh-grade students at both conceptual and procedural knowledge variable.

The average results comparison of initial (pre-test) and final (post-test) assessment of students' knowledge is given in the Figure 4 and Figure 5 separately for the sixth, seventh and eighth-grade students.

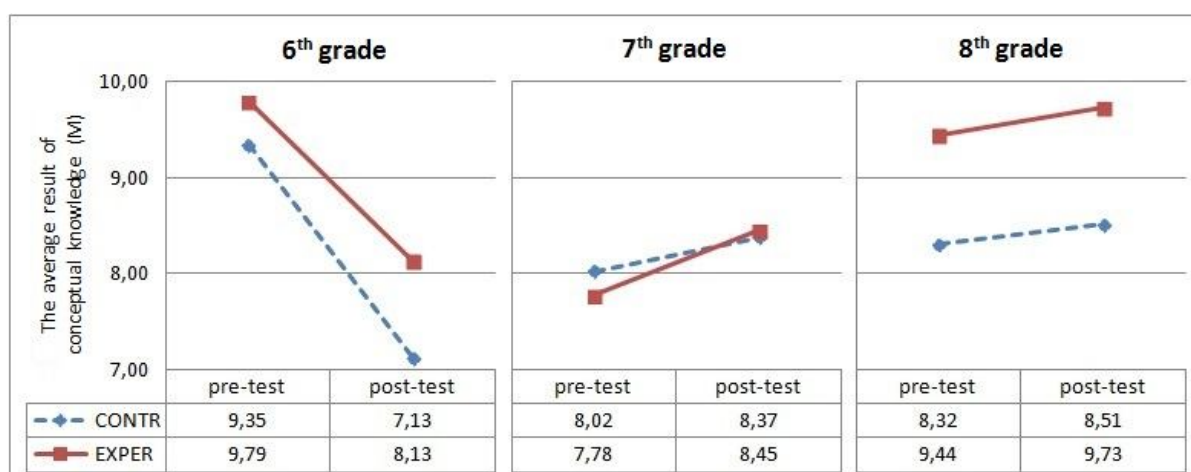


Figure 4. The average pre-test and post-test results of conceptual knowledge



Figure 5. The average pre-test and post-test results of procedural knowledge

The difference in achievement at sixth, seventh and eighth-grade students can be observed. Eighth-grade students achieved the best results using computer-guided discovery learning model, while such intervention in mathematics teaching and learning had no impact on knowledge of seventh-grade students. We presume there are some other factors that influence achievement in mathematics, like the complexity of studied mathematical content, the puberty stages of students, teacher's experience in teaching using this learning model, students' experience in discovery learning and in learning individually using ICT.

The qualitative analysis of teachers' questionnaires and forum discussions shows their attitudes towards the proposed learning model. The results indicate multiple advantages: an individualized approach to each student, active work of all the students at their own pace, the possibility of repeating the lesson several times or returning to the previous lesson, the exploration and discovery of new knowledge for all the students, the visualization of mathematical contents and reasoning based on the visualization, the interactivity and dynamism of digital learning materials that enables experimental work, immediate feedback, easier and more interesting learning and solving problems through playing a game, additional contents for gifted pupils and teachers also noted that students with learning difficulties can get more attention and help from them (whereas others work independently).

Some of the perceived deficiencies of the computer-guided discovery learning are: disorientation of some students in the new learning environment, the lack of understanding the operating instructions and reading without understanding, some students rush through the learning material without proper reasoning, some students have the feeling they are left to themselves, the lack of feedback to the teacher about the level of acquired mathematics knowledge for each student as well as the inadequacy of this learning model for lessons in which the drawing and the geometric constructions are the key features. The importance of the teachers' experience in teaching by this model has also been noted.

6. Conclusion

The results show that computer-guided discovery learning model by using dynamic geometry software and GeoGebra interactive applets in mathematics teaching has certain potential and can provide better results in acquiring both, conceptual and procedural

knowledge than traditional teaching methods can. Similar results were obtained in some other experimental research dealing with conceptual knowledge (Svedružić, 2006; Shadaan and Leong, 2013), procedural knowledge (Nguyen and Kulm, 2005) or both (Güzeller and Akın, 2012; Jupri, Drijvers and van den Heuvel-Panhuizen, 2015). According to Cheung and Slavin (2013) the use of ICT in mathematics teaching shows positive but small effects on mathematical performance. Therefore, they highlight the need for new and better digital tools to take advantage of the possibilities of ICT to improve mathematics achievement of all students. We believe that proposed learning model, which uses digital textbooks consisting of GeoGebra interactive applets with carefully planned activities for students, can make some scientific contribution in that direction. This learning model also indicates the potential of computers and dynamic geometry software GeoGebra for scaffolding support in learning mathematics.

However, there were differences in acquiring conceptual and procedural knowledge of students considering their age and also related to the specific learning content (different grades students used different digital textbooks). Therefore, the influence of students' age (but also gender which plays an important role during teenage years) requires additional examination, as well as the complexity of mathematical content to be studied.

Likewise, there have been noted some limitations of this research. Firstly, using the proposed learning model, students have acquired new knowledge in geometry since the visualization has been highlighted as one of the most significant potential for using GeoGebra. So, there is the question to what extent would this learning model influence the discovery of knowledge from other mathematical areas (such as arithmetic, algebra, analysis, statistics). Secondly, the research has been conducted in Croatian schools during regular mathematics classes which are mostly traditional (and the results have confirmed some deficiencies of this traditional classroom environment). There is the question how this learning model would influence the conceptual and procedural knowledge of students in some other educational systems (e.g. where problem solving, communication and reasoning are emphasized). Furthermore, besides the problem with technology equipment in Croatian schools, there is also the problem related to in-service teacher training on using technology, as well as their readiness for applying modern teaching and learning models like the one described in this issue.

References

- [1] Bjelanović Dijanić, Ž. (2009). Mathlet – interaktivni digitalni materijal namijenjen samostalnom učenju. *Pogled kroz prozor*, 6.
<http://pogledkrozprozor.wordpress.com/2009/03/31/mathlet-interaktivni-digitalni-materijal-namijenjen-samostalnom-ucenju/> [Accessed 10/01/2017]
- [2] Bu, L., Spector, J. M., & Haciomeroglu, E. S. (2011). Toward model-centered mathematics learning and instruction using GeoGebra: A theoretical framework for learning mathematics with understanding. In: L. Bu, & R. Schoen (Eds.), *Model-Centered Learning: Pathways to Mathematical Understanding Using GeoGebra* (pp. 13-40). Rotterdam: Sense Publishers.
- [3] Cheung, A. C., & Slavin, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review*, 9, 88-113.
- [4] De Jong, T., & van Joolingen, W. R. (1998). Scientific discovery learning with computer simulations of conceptual domains. *Review of educational research*, 68(2), 179-201.
- [5] Dennen, V. P. (2004). Cognitive apprenticeship in educational practice: Research on scaffolding, modeling, mentoring, and coaching as instructional strategies. In D. H. Jonassen (Ed.), *Handbook of Research on Educational Communications and Technology* (2nd ed., pp. 813-828). Mahwah, NJ: Lawrence Erlbaum Associates.
- [6] Dijanić, Ž., Belavić, D., Grgić, N., Pein, B., & Vuković, A. M. (2014). *E-udžbenik: Kut i trokut* [Elektronički udžbenik].
<http://tube.geogebra.org/student/b272303> [Accessed 10/01/2017]
- [7] Dijanić, Ž., Grgić, N., Pein, B., & Vuković, A. M. (2014). *E-udžbenik: Sličnost i mnogokuti* [Elektronički udžbenik].
<http://tube.geogebra.org/student/b364307> [Accessed 10/01/2017]
- [8] Dijanić, Ž., Belavić, D., Grgić, N., & Pein, B. (2014). *E-udžbenik: Pitagorin poučak* [Elektronički udžbenik].
<http://tube.geogebra.org/student/b297339> [Accessed 10/01/2017]
- [9] Dijanić, Ž., (2015). Zadaci u kontekstu e-udžbenika. U: R. Gortan (Ur.), *Zbornik radova Devetog stručno-metodičkog skupa Metodika nastave matematike u osnovnoj i srednjoj školi: Zadaci u nastavi matematike* (str. 56-71). Pula: Matematičko društvo Istra.
- [10] Dijanić, Ž., (2017). *Development of the model of computer guided discovery learning by using dynamic geometry software in teaching mathematics* (Unpublished doctoral dissertation). Faculty of Teacher Education, University of Zagreb.
- [11] Elliott K., Sweeney K. & Irving H. (2009). A learning design to teach scientific inquiry. In: L. Lockyer, S. Bennett, S. Agostinho, & B. Harper (Eds.), *Handbook of Research on Learning Design and Learning Objects: Issues, Applications, and Technologies* (pp. 652-675). Hershey & London: IGI Global.
- [12] Glasnović Gracin, D. (2008). Računalo u nastavi matematike: Teorijska podloga i metodičke smjernice. *Matematika i škola*, 46, 10-15.

- [13] Güzeller, C. O., & Akın, A. (2012). The effect of web-based mathematics instruction on mathematics achievement, attitudes, anxiety and self-efficacy of 6th grade students. *International Journal of Academic Research in Progressive Education and Development*, 1(2), 42-54.
- [14] Hohenwarter, M., & Preiner, J. (2007). Creating mathlets with open source tools. *Journal of Online Mathematics and its Applications*, 7, ID 1574.
http://www.maa.org/external_archive/joma/Volume7/Hohenwarter2/index.html
[Accessed 15/04/2015]
- [15] Iranzo, N., & Fortuny, J. M. (2011). Influence of GeoGebra on problem solving strategies. In: L. Bu, & R. Schoen (Eds.), *Model-Centered Learning: Pathways to Mathematical Understanding Using GeoGebra* (pp. 91-103). Rotterdam: Sense Publishers.
- [16] Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2015). Improving grade 7 students' achievement in initial algebra through a technology-based intervention. *Digital Experiences in Mathematics Education*, 1(1), 28-58.
- [17] Karadag, Z., & McDougall, D. (2009). Dynamic worksheets: visual learning with guidance of Pólya. *MSOR Connections*, 9(2), 13-16.
- [18] Lester, F. K., Garofalo, J., & Kroll, D. (1989). *The role of metacognition in mathematical problem solving: A study of two grade seven classes* (Final report NSF MDR 85-50346). Bloomington: Indiana university, Mathematics Education Development Center.
<http://files.eric.ed.gov/fulltext/ED314255.pdf> ERIC ED314255 [Accessed 18/06/2015]
- [19] Mayer, R.E. (2004). Should there be a three-strikes rule against pure discovery learning? The case for guided methods of instruction. *American Psychologist*, 59(1), 14-19.
- [20] Nguyen, D. M., & Kulm, G. (2005). Using web-based practice to enhance mathematics learning and achievement. *Journal of interactive online learning*, 3(3), 1-16.
- [21] Pintó, R., Couso, D., & Hernández Rodríguez, M. I. (2010). An inquiry-oriented approach for making the best use of ICT in the classroom. *eLearning Papers*, 20, 1-14.
<http://www.uh.cu/static/documents/RDA/An%20inquiry-oriented%20approach.pdf>
[Accessed 08/08/2016]
- [22] Pólya, G. (1945). *How to solve it*. Princeton, New Jersey: Princeton University Press.
- [23] Schneider, E. (2002). *Computeralgebrasysteme in einem allgemeinbildenden Mathematikunterricht: didaktische Orientierungen - praktische Erfahrungen*. München-Wien: Profil Verlag.
- [24] Schoenfeld, A. (1985). *Mathematical problem solving*. Orlando, Florida: Academic Press.
- [25] Shadaan, P., & Leong, K. E. (2013). Effectiveness of Using GeoGebra on Students' Understanding in Learning Circles. *Malaysian Online Journal of Educational Technology*, 1(4), 1-11.
- [26] Svedružić, A. (2006). Vrednovanje učinkovitosti učenja fizike putem računala za učenike osnovne škole. *Odgojne znanosti*, 8(2), 535-549.
- [27] Von Glasersfeld, E. (1995). *Radical Constructivism: A Way of Knowing and Learning*. London: Falmer Press.

- [28] Wangpipatwong, T., & Papasratorn, B. (2008). The influence of constructivist e-learning system on student learning outcomes. In: C. van Slyke (Ed.), *Information Communication Technologies: Concepts, Methodologies, Tools, and Applications* (Vol. 1, pp. 1109-1121). Hershey & London: IGI Global.

Utjecaj uporabe interaktivnih apleta kreiranih u GeoGebri na konceptualno i proceduralno znanje učenika

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Sažetak

Ovaj se rad bavi utjecajem uporabe digitalnih udžbenika koji sadrže interaktivne aplete kreirane u GeoGebri na konceptualno i proceduralno znanje učenika. Autori predlažu model računalno vođenoga učenja otkrivanjem korištenjem programa dinamične geometrije koji uključuje tri osnovna elementa: objekte učenja, aktivnosti za učenike i ishode učenja. Model se temelji na teoriji konstruktivizma, Pólyinoj heurističkoj strategiji i Schoenfeldovom modelu rješavanja problema.

U istraživanju je sudjelovalo 703 učenika iz dvanaest osnovnih škola u Hrvatskoj te je korišten eksperimentalni nacrt s eksperimentalnom i kontrolnom skupinom. Eksperimentalna skupina učenika učila je po modelu računalno vođenoga učenja otkrivanjem, a kontrolna skupina učenika u tradicionalno organiziranoj nastavi. Analizom kovarijance utvrđen je utjecaj modela računalno vođenoga učenja otkrivanjem na konceptualno i proceduralno znanje učenika, a rezultati pokazuju statistički značajne razlike između skupina u korist učenika eksperimentalne skupine.

Provedena je kvalitativna i kvantitativna analiza upitnika za učenike i učitelje te analiza rasprave učitelja na forumu kako bi se ustanovili stavovi učenika i učitelja prema predloženoj modelu učenja. Uočena je važnost iskustva učitelja u primjeni ovoga modela učenja. Predloženi model učenja ukazuje na potencijal računala i programa dinamične geometrije *GeoGebra* kao *scaffolding* podrške u učenju matematike.

Ključne riječi: *digitalni udžbenik, GeoGebra, interaktivni aplet, konceptualno znanje, model učenja, proceduralno znanje, program dinamične geometrije, učenje otkrivanjem*