Decentralized Control Functions In Trajectory Guidance Of A Non-Holonomic AUV

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***Abstract*— This paper describes a modification of authors' previous work on virtual potential methods for planning and guidance of autonomous underwater vehicles (AUVs) along trajectories in $\mathbb{R}^2$ and $\mathbb{R}^3$. The modification replaces the algebraic sampling procedure of scalar potentials, performed in the previous algorithm in order to allow for the numerical approximation of the true local gradient of the potential, with the direct analytical solution for the local gradient. The modification also allows for a far more flexible integration of rotors, allowing fine-tuning by revealing analytical relationships between the stator and rotor components of calculated acceleration vectors. The modification is critical to the control of holonomically constrained AUVs with finite dynamics and significant lags in response to rudder and change of propeller rpms.**

I. INTRODUCTION

Algebraic methods, of which the decentralized control function control is a family, have long been a preferred tool of marine control system engineers, in solutions of optimization, trajectory planning, or direct guidance problems for various marine vessels [1 – 5]. As the technology, design and miniaturization of embedded control systems progresses, leveraging more and more processing power versus relatively low tact and mediocre memory capacity, this trend is not about to stall. Some notable advantages of algebraic methods is that they usually operate at a high level of abstraction. Although this facilitates solution to posed control problems with relatively few processor cycles and even less latency in memory access, the benefit is offset by a relatively large extent of theoretical research that needs to be undertaken in order to assure stability, well-posedness, polynomial complexity of algorithms involved and other critical considerations that need to be undertaken before committing production code to an actual vessel. In favor of algebraic methods, once they have been founded in sufficiently well understood theory and equipped with necessary proofs and caveats, they are extremely flexible and represent a good choice for rapid prototyping.

Due to the peculiar relationship between semantic and syntactic contents of data in algebraic frameworks (with relatively high semantic contents in relatively parsimonious syntactic containers), these methods are also attractive for use in coordinated and cooperative frameworks of multi-agent systems.

In line with all of the above, a distributed, cooperative framework for virtual potential-based guidance of unmanned vessels has been developed previously in [6 – 8] . The developed framework is scalable to 2D and 3D applications. The former is a guidance problem in a 2D plane with 3 degrees of freedom either at some constant depth or altitude off the bottom, or at the water surface. Alternatively, this algorithm can be used for the control of three out of the six degrees of freedom of an AUV cruising along an arbitrary 2D manifold which is a one-to-one map of a horizontal plane, for which z is being controlled (varied) by some outside algorithm, e.g. a a constant-altitude bottom-tracking. The latter is a guidance problem in a 3D water-space with 6 degrees of freedom.

However, this paper explores the outgrowth of the virtual potential-based algorithm, based on a scalar field of potentials "living" on the representation of the water-space of discourse, into the *decentralized control-function* algorithm that allows for analytical solution of the gradient of steepest descent.

In order to allow for a clearer understanding of some of the relationships involved, and implied connections between the virtual potentials and decentralized control functions, the paper focuses on the 2D guidance problem, with a note that the findings are applicable to a 3D problems, as mentioned in the Concluding remarks.

The paper proceeds with the preliminary definitions and overview of the theoretic framework in Section II. The main body of theory of the decentralized control functions is covered in Section III. Section IV short-lists the assumptions on the decoupled linear closed-loop dynamics of the controllable degrees of freedom of the AUV, based on the results of the authors' work on identification by use of self-oscillations in [9 – 11]. Results of simulations are presented in Section V. Section VI concludes the paper, pointing to future work and planned experimentation in the spring/summer of 2010.

II. THE PRELIMINARIES

Therefore, all $\vec{x}(k)$, $\tilde{\vec{x}}_i(k)$, $\vec{v}(k)$, $\vec{a}(k) \in \mathbb{R}^2$. Also, the time-index, k will be omitted whenever not critical to understanding a mathematical relationship, in order to lighten the notation. In such a setup, let the knowledge necessary for trajectory planning be encoded as a set $\mathbb{W}$ in (1).

$$\mathbb{W} = \{W_i\}\,, \quad i = 1 \ldots N \tag{1}$$

Where N is the total number of *features of the water-space* and W_i-s are the individual features, consisting of a single point, or a convex connected open set of points $\mathbb{A}_i \subset \mathbb{R}^2$ contained in an open ε-ball in $\mathbb{R}^2$. If a physical feature, such as an obstacle, part of geography or a pre-selected no-go-area is of non-convex shape, it can always be decomposed into open convex subsets. Such basic features are formally defined in (2).

$$W_i = (p_i(d), \vec{x}_i, S_i, d_i(\vec{x}, \vec{x}_i, S_i)) \tag{2}$$

In (2):
- $p_i(d) : \mathbb{R}_0^+ \rightarrow \mathbb{R}$, is the *isotropic potential distribution function* (pdf) varying only with distance from the i-th feature. The isotropic pdf maps the exterior of the feature (since d_i is only defined on the exterior of a feature) to a real;
- $\vec{x}_i$ is the *location of the* i*-th feature*, which needs to be a center of at least one open ε-ball in $\mathbb{R}^2$. This allows for a short-hand "W_i at $\vec{x}_i$". A good candidate for $\vec{x}_i$ is the center of the *minimum bounding sphere* (where the ε-ball is of minimum radius $\underline{\varepsilon}$). However, logical choice is such a $\vec{x}_i$ that allows for the most parsimonious notation of a set equations describing the geometry of $\mathbb{A}_i$ by defining piecewise or totally its convex hull;
- $S_i(t)$, the *full measurement of the* i*-th feature*, itself an N_S-tuple of reals $(s_1^{(i)}, s_2^{(i)}, \cdots, s_{Ns}^{(i)})$ of cardinality equal to the number of parameters necessary in addition to $\vec{x}_i$ to most sparsely populate the least number of equations defining the convex hull of $\mathbb{A}_i$. Naturally, S_i is a 0-tuple when $\mathbb{A}_i = \{x_i\}$, i.e. when the feature is point-wise;
- $d_i(\vec{x}, \vec{x}_i, S_i) : \mathbb{R}^2 \backslash \mathbb{A}_i \times \mathbb{R}^2 \rightarrow \mathbb{R}_0^+$ is a non-negative Euclidean 2D distance from W_i at $\vec{x}_i$, to $\vec{x} \in \mathbb{R}^2$, the position of the AUV:

$$d_i(\vec{x}, \vec{x}_i, S_i) = \min_{\vec{a} \in \mathbb{A}_i} \|\vec{a}(\vec{x}_i) - \vec{x}\| \tag{3}$$

Where $\mathbb{A}_i(\vec{x}_i) = \{\vec{a}(\vec{x}_i)\}$ is the convex hull of the obstacle centered on $\vec{x}_i \in \mathbb{R}^2$, and therefore all $\vec{a} \in \mathbb{A}_i(\vec{x}_i)$ are dependent on $\vec{x}_i$. Therefore the distance d_i can be conceived as taking any two pairs of vectors, with the first, $\vec{x} \in \mathbb{R}^2 \backslash \mathbb{A}_i$ describing any point of Euclidean 2-space external to the i-th obstacle, and the second being the center $\vec{x}_i \in \mathbb{R}^2$ of that obstacle. It is generally the minimum value in the set of distances of $\vec{x}$ to curves constituting the convex hull of $\mathbb{A}_i$ ("edges") and distances to intersections of the same curves ("vertices"). The determination of whether the closest feature of the convex hull of the obstacle centered at $\vec{x}_i$ to the point $\vec{x}$ is a vertex, an edge or a face is dependent on where $\vec{x}_i$ is in relation to $\vec{x}$.

A. Feature Classification

The features of interest in the trajectory-planning problem for an AUV are *obstacles*, such that the trajectory must not pass through them, and *way-points*, such that the trajectory of the AUV must pass as close as possible, ideally through, them.

Let both of these distinct classes of features, obstacles and way-point be described by a *potential distribution function*:

$$p(\vec{x}(k)) : \mathbb{R}^2 \backslash \mathbb{A}_i \rightarrow \mathbb{R}_0^+, \tag{4}$$

Mapping all points in the mission-space to the exclusion of the interior of the obstacle $\mathbb{A}_i$ to some real value of a potential. Let these pdf-s be uniquely decomposable into $p \equiv p_i \circ d_i$. Here, p_i is an *isotropic potential distribution function* that maps all points at the same distance from the feature, $\{\vec{x} \in \mathbb{R}^2, d(\vec{x}) = d\}$ to the same $p_i(d)$. $d_i(\vec{x})$ is the distance between an obstacle or a way-point and the location of the AUV $\vec{x}(k) \in \mathbb{R}^2$. This allows for the modeling of obstacles and way-points within the framework described by (1 – 3).

III. DECENTRALIZED CONTROL FUNCTIONS

The preceding observations lead to the definition of a *class* of p_i-s, introduced in (2), $\mathcal{P}$. To allow further simplification of the trajectory guidance mechanism, this class shall be explicitly defined by (5 – 7).

$$\mathcal{P} = \{p_{obs}(d), p_{wp}(d)\} \tag{5}$$

$$p_{obs}(d) = \exp\left(\frac{A^+}{d}\right) - 1, \quad A^+ > 0 \tag{6}$$

$$p_{wp}(d) = \begin{cases} d \leq r : & \frac{A^-}{2r} d^2 + \frac{A^-}{2} r - A^- d_0 \\ d > r : & A^-(d - d_0) \end{cases}$$
$$A^-, r > 0 \tag{7}$$

With:
- p_{obs}, given in (6) being the form of potential distribution function used for obstacles;
- p_{wp}, given in (7) being the form of potential distribution function used for way-points.

Previous considerations finally lead to the possibility of manipulating the fundamental equation of the virtual potential methods [12], as follows:

$$\begin{aligned} \vec{a} &= \nabla P_\Sigma(\vec{x}) = \nabla \sum_i p_i(\vec{x}) \\ &= \sum_i \nabla p_i(\vec{x}) = \sum_i \nabla p_i \circ d_i(\vec{x}, \vec{x}_i, S_i) \end{aligned} \tag{8}$$

Directionalizing the distance d_i by means of (9), (8) can be surmised into (10):

$$\begin{aligned} \forall d_i(\vec{x}) &= \min_{\vec{a} \in \mathbb{A}_i} \|\vec{a} - \vec{x}\| : \\ \exists \vec{d}_i &= \underline{\vec{a}} - \vec{x}, \quad \underline{\vec{a}} = \arg\min_{\vec{a} \in \mathbb{A}_i} \|\vec{a} - \vec{x}\| \end{aligned} \tag{9}$$

$$\vec{a} = \sum_i \frac{\partial p_i}{\partial d}\Big|_{d_i} \cdot \frac{\vec{d}_i}{d_i} \tag{10}$$

By intervening in the above to lighten the notation, and denoting $\partial p_i / \partial d$ with a_i, and $\vec{d}_i / d_i$ with $\hat{\vec{a}}_i$ (10) is reduced to (11) below.

$$\vec{a} = \sum_i a_i \hat{\vec{a}}_i \tag{11}$$

$$a_i = \begin{cases} p_i \sim p_{obs} : & -\frac{A^+}{d^2} \exp\left(\frac{A^+}{d}\right) \\ p_i \sim p_{wp}, \ d \leq r : & \frac{A^-}{r} d \\ p_i \sim p_{wp}, \ d > r : & A^- \end{cases} \tag{12}$$

With:
- $\vec{a}$ being the *irrotational acceleration along an ideal conservative trajectory*;
- $a_i = \partial p_i / \partial d_i$ is the *decentralized acceleration stator-control function of the* i*-th obstacle*;
- $\hat{\vec{a}}_i$ being the unit-vector denoting the direction of the gradient of p_i, being $(\vec{x}_i - \vec{x}(k))/\|\vec{x}_i - \vec{x}(k)\|$ for way-points, or $\hat{\vec{a}}_i \perp \mathbb{A}_i$, $\|\hat{\vec{a}}_i\| = 1$, a unit incident normal on the closest edge, vertex or submanifold of the convex hull of $\mathbb{A}_i$.

A. Rotor Modification

The previously described irrotational acceleration, if reproducible exactly by AUV's actuators, will guide an AUV along a trajectory with two basic unintended features:

1) A (non-decaying) limit cycle or a strange attractor centered on a local minimum of the total potential P_Σ,
2) Existence of local minima other than at the location of the way-point, $\vec{x}_{wp} \in \mathbb{R}^2$.

To take care of the former, a breaking of symmetry of a vector field $\vec{a}(\vec{x})$ implicitly constructed by (11) is required [citeBar:Rotors. One of the ways of doing so, covered extensively in [6] is the introduction of *rotors* (or *curls*, in American literature), by design. The introduction demonstrated in [6], suffers of a certain level of involvement, due to the algebraic approach of sampling in the potential field, needing to formulate “slanted tableaus” (pitched surfaces of potential) rotating or sliding around the edges of $\mathbb{A}_i$-s of obstacles.

In the proposed decentralized function format, the *rotational component* of the along-trajectory acceleration contributed by the i-th feature can be represented as:

$$\vec{a}_i^{rot} = a_i^{rot} \cdot \hat{\vec{a}}_i^{rot} \tag{13}$$

Where:
- a_i^{rot} is the *decentralized acceleration rotor-control function of the* i*-th feature*, defined by feature class explicitly, for sake of simplicity by taking $a_i^{rot} \equiv a_i$, i.e. a_i^{rot}-s being also of the form (12);
- $\hat{\vec{a}}_i^{rot}$ being the unit vector giving the direction of the rotational component, defined explicitly by feature class below.

$$\hat{\vec{a}}_i^{rot} = \begin{cases} p_i \sim p_{obs}, \sin(\rho) \leq 0: & [\,\hat{\vec{a}}_i^{\mathrm{T}} \,|\, 0\,]^{\mathrm{T}} \times [\,0\;0\;1\,]^{\mathrm{T}} \\ p_i \sim p_{obs}, \sin(\rho) > 0: & [\,0\;0\;1\,]^{\mathrm{T}} \times [\,\hat{\vec{a}}_i^{\mathrm{T}} \,|\, 0\,]^{\mathrm{T}} \\ p_i \sim p_{wp}: & \vec{0} \end{cases} \tag{14}$$

Where:
- ρ is $\angle(\vec{x}_{wp} - \vec{x}_i, \vec{x}_{wp} - \vec{x})$, given in (15), the azimuth of the AUV w.r.t. the line through the waypoint at $\vec{x}_{wp}$ and obstacle W_i at $\vec{x}_i$.

$$\rho = \mathrm{atan}_2 \frac{(\vec{x} - \vec{x}_{wp})\hat{\vec{\jmath}}}{(\vec{x} - \vec{x}_{wp})\hat{\vec{\imath}}} - \mathrm{atan}_2 \frac{(\vec{x}_i - \vec{x}_{wp})\hat{\vec{\jmath}}}{(\vec{x}_i - \vec{x}_{wp})\hat{\vec{\imath}}} \tag{15}$$

Finally, the *acceleration along an ideal conservative trajectory* can, in terms of the newly developed *decentralized control functions* be expressed as:

$$\vec{a}_\Sigma = \sum_{i=1}^{N} a_i \cdot \hat{\vec{a}}_i + a_i \cdot \hat{\vec{a}}_i^{rot} = \sum_{i=1}^{N} a_i (\hat{\vec{a}}_i + \hat{\vec{a}}_i^{rot}) \tag{16}$$

IV. DYNAMICS

The first of the unintended features mentioned w.r.t. to the irrotational acceleration along an ideally conservative trajectory – the appearance of non-attenuated limit cycles is resolved due to the fact that real AUVs themselves do not have energetically conservative dynamics. To name a few, added mass effects, Coloumb and viscous friction, Coriolis forces, vortex shedding and other compound effects dissipate energy from an AUV undergoing motion in a waterspace.

In previous work, notably [9 – 11], it is shown how using a technique of identification by induced self-oscillations in a closed loop, a non-linear coupled dynamic model of an AUV can be found. Subsequently an I-PD controller can be designed [9] to assure robust linear uncoupled behavior of the two most important AUV's degrees of freedom controlled in a closed loop. It can be shown that for most torpedo-types AUVs, I-PD controllers plus inverse non-linearities can be designed that allow for the dynamics of *surge speed*, $v(k)$ and heading rate of change, $\omega(k)$, to be (17) and (18), respectively.

$$\begin{aligned} G_v(s) &= \frac{1}{T_v^2 s^2 + 2\zeta_v T_v s + 1} && (17) \\ G_\omega(s) &= \frac{1}{T_\omega^2 s^2 + 2\zeta_\omega T_\omega s + 1} && (18) \\ G_{v,\omega}(z) &= \tfrac{z-1}{z} \mathcal{Z}\left\{ \tfrac{1}{s} G_{v,\omega}(s) \right\} \end{aligned}$$

Additionally, both dynamics are allowed to saturate in level and rate, i.e., there exist:

$$\begin{aligned} \overline{v} &= \rceil \mathcal{Z}^{-1} \left\{ G_v(z) \cdot V_c(z) \right\} \lceil \\ \overline{a} &= \rceil \mathcal{Z}^{-1} \left\{ \tfrac{z}{z-1} G_v(z) \cdot V_c(z) \right\} \lceil \\ \overline{\omega} &= \rceil \mathcal{Z}^{-1} \left\{ G_\omega(z) \cdot \Omega_c(z) \right\} \lceil \\ \overline{\alpha} &= \rceil \mathcal{Z}^{-1} \left\{ \tfrac{z}{z-1} G_\omega(z) \cdot \Omega_c(z) \right\} \lceil \end{aligned}$$

To drive these control loops, the signals $v_c(k)$ and $\omega_c(k)$ proceed from (16) as follows:

$$\vec{v}_c(k) = \frac{T}{2} \left(\vec{a}_\Sigma(k) + \vec{a}(k-1) \right) + \vec{v}(k-1) \tag{19}$$

$$\vec{v}_c(k) \equiv v_c \angle \varphi_c(k):$$

$$\varphi_c(k) = \mathrm{atan}_2 \frac{(\vec{a}_\Sigma(k) + \vec{a}(k-1) + \frac{2}{T}\vec{v}(k-1))\hat{\vec{\jmath}}}{(\vec{a}_\Sigma(k) + \vec{a}(k-1) + \frac{2}{T}\vec{v}(k-1))\hat{\vec{\imath}}} \tag{20}$$

$$\omega_c(k) = \frac{2}{T} \left(\varphi_c(k) - \varphi(k-1) \right) - \omega(k-1) \tag{21}$$

Where φ, ω, $\vec{a}$ and $\vec{v}$ are respectively the measured or estimated heading, yaw rate, acceleration and velocity of the AUV, and the quantities subscribed by c are the controller commands or variables otherwise used in the process of calculating the commands.

V. SIMULATION RESULTS

To demonstrate the direct applicability of the *decentralized control functions* to guidance of AUVs whose dynamics can be modeled by (17,18), a simulation was performed. Actual AUV operations are constrained by weather, time of year, logistics and safety concerns. Simulations alleviate the concerns (primarily w.r.t. collision avoidance and existence of non-attenuated, non-convergent behaviors) and guarantee that the code executing the theoretical consideration in the previous section, can be bundled into the MOOSDB embedded control architecture of the AUV [14,15].

The simulations assume a finite ideal sensor range of 10m, such that as soon as any obstacle is within 10m of an agent, the agent is aware of the obstacle and performs necessary pdf calculations. However, the implementation of the simulated ranging sensor is such that obstacle masking, i.e. non-transparency is assured. An agent approaching an obstacle isn't able to sense through it. Therefore, if another farther obstacle exists behind a closer obstacle whose extent subtends a range of angles wider than the one subtended by the farther obstacle, the former will not be included in the calculation of the decentralized control function.

Two batches of simulations were performed, to ascertain the qualitative ramifications of varying the stator versus the rotor part of the decentralized control functions of the obstacles. In both cases, the A^+ independent parameter in (12), but used to form either a_i in (11) or a_i^{rot} in (13), was varied in logarithmic increments between the values $(0.04 - -20.00)$, through values in table II.

The AUV's dynamics were modeled after (17, 18), and the coefficients used in both simulation batches are listed in table I.

TABLE I

PARAMETER VALUES, LINEAR DECOUPLED DYNAMICS OF SURGE SPEED AND HEADING RATE OF CHANGE

T_v	ζ_v	T_ω	ζ_ω
0.288	0.622	0.341	0.470

A. Variation Of the Stator Repulsive Factor

A set of simulations was performed to ascertain the criticality of setting the *stator repulsive factor*, A_s^+, i.e. A^+ in (12) when applied to (11), on the clearing of the trajectory with all obstacles. With the value of A_s^+ varied, all other parameters were kept to nominal levels specified in table III.

During the first batch of simulations, A_r^+ (being the A^+ parameter, but for a_i^{rot} in (13)) was kept constant at 2.1389 while A_s^+ was varied according to table II. The resultant trajectories are displayed in figure 1.

B. Variation Of the Rotor Repulsive Factor

Another set of simulations was performed to inspect the influence of the *rotor repulsive factor*, A_r^+, i.e. A^+ in (13). As well as in the first experiment, the first priority was the clearing of the trajectory with all obstacles. With the value of A_s^+ varied, all other parameters were kept to nominal levels specified in table IV.

During the second batch of simulations, A_s^+ was kept constant at 2.1389 while A_r^+ was varied according to table

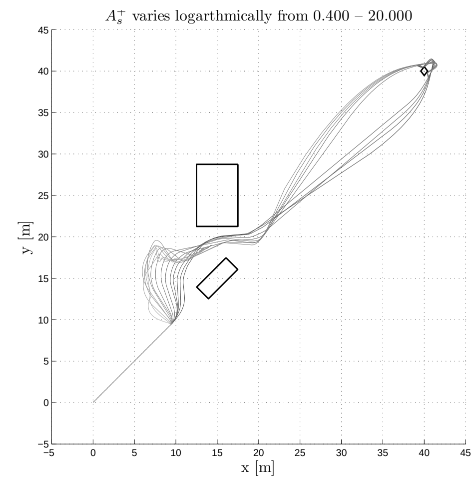

Fig. 1. The plot of paths resulting from the variation of A_r^+

TABLE II

VARYING VALUES OF PARAMETERS A_s^+ AND A_r^+ RESPECTIVELY

Simulation 1 $A_s^+(i)$	Simulation 2 $A_r^+(i)$
0.40000	0.40000
0.52895	0.52895
0.69947	0.69947
0.92496	0.92496
1.2232	1.2232
1.6175	1.6175
2.1389	2.1389
2.8284	2.8284
3.7402	3.7402
4.946	4.946
6.5405	6.5405
8.649	8.649
11.437	11.437
15.124	15.124
20.000	20.000

TABLE III

PARAMETER VALUES, STATOR REPULSIVE FACTOR SIMULATION BATCH

A_r^+	$\overline{v}$	$\overline{a}$	$\overline{\omega}$	$\overline{\alpha}$
2.1389	3.0 kts	1.5 kts/s	38°/s	16°/s^2

TABLE IV

PARAMETER VALUES, ROTOR REPULSIVE FACTOR SIMULATION BATCH

A_s^+	$\overline{v}$	$\overline{a}$	$\overline{\omega}$	$\overline{\alpha}$
2.1389	3.0 kts	1.5 kts/s	38°/s	16°/s^2

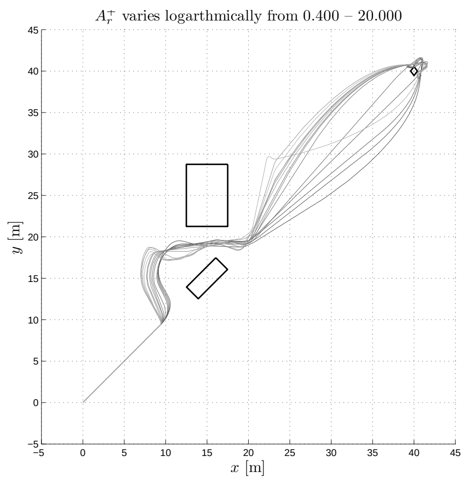

Fig. 2. The plot of paths resulting from the variation of A_r^+

II. The resultant trajectories are displayed in figure 2.

C. Preliminary Study Of Multi-Agent Guidance

Modeling other agents as circular obstacles of unit radius, and assuming perfect inter-agent awareness, a preliminary study of a multi-AUV mission was carried out. In the study, no other intervention in the decentralized control functions between i-th and j-th agents except the "circular obstacle" nature of one w.r.t. the other were undertaken (although results for the numerically sampled virtual potentials were obtained in [7]). The paths of the trajectories along which the AUVs were guided are displayed in Figure 3. The "creeping" effect already discussed in [7] is visible. However, the "creep" is less dynamic, due to lags incurred by dynamic modeling of AUVs involved.

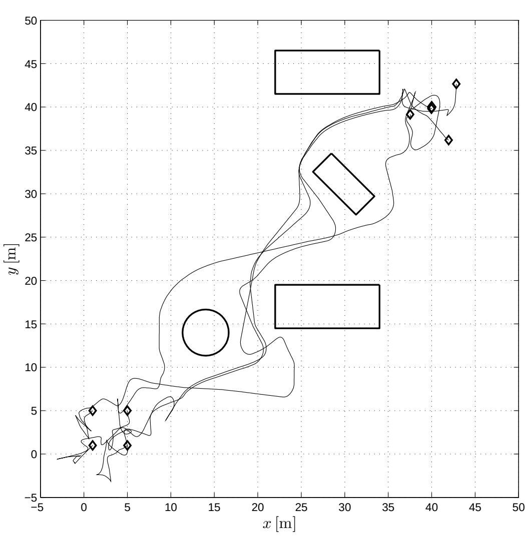

Fig. 3. The paths planned for 4 non-cooperating AUVs simultaneously traversing a cluttered water-space.

VI. Conclusions and Future Works

A. Conclusions

The results presented represent a significant advance in the state of the art of the virtual potential-based methods for trajectory planning previously developed through authors' research of virtual potentials. The proposed modification of the earlier method is demonstrably able to cope with holonomic constraints and non-ideal finite dynamics with lags in response to rudder and change of main propeller rpm.

B. Future Works

In view of the availability of actual experimental equipment [13], and based on the embedded control system architecture developed in [14, 15], a series of live water exercises shall be performed. The planned platform is the Iver 2 AUV, and the planned time-window is the spring/summer season of 2010. The goal of the exercises shall be to research all implications of operating in the actual water column, with influences of currents, wind and waves, and compare the performance of the algorithm when subjected to measurement noise and stochastic disturbances.

Important additions to the algorithm before committing to live water exercises include:

1) A method for non-linear anti-windup, having in mind the relationships between (a_i, a_i^{rot}) in (11,13), and (v_c, ω_c) in (19–21) assuring that the closed loops of control of surge speed and heading rate change remain responsive.
2) A modification of (13) such that there is no rotor action once a midpoint of a feature is passed, since rotors compromise the stability of way-points occluded by the obstacles, i.e. those near obstacles, and on the opposing side of the obstacle w.r.t. the initial position of the AUV(s),
3) A provision for backing up. The current framework assumes that the *heading* is equivalent to the *bearing*.

VII. Acknowledgments

This research was made possible through the following contributions:

1) *Professional and technological equipment* — An FP7 Capacities Project in Response to the REGPOT-2008-1 Call, ***"Strengthening the Croatian Underwater Robotics Research Potential – CURE"***.
2) *Scientific and professional secondment / internship grant* — "Unity Through Knowledge Fund", MOSES [1], Croatia.

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