Maritime component reliability assessment and maintenance using bayesian framework and generic data

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ABSTRACT: The use of reliability models is often limited by a lack of satisfactory data for the estimates of system parameters. This paper discusses the problem of life cycle management with insufficient failure record of the equipment. This deficiency is addressed by using expert opinion and Bayesian modeling. The framework for Bayesian updating of the reliability models is presented and the different aspects of updating process are discussed. The use of expert opinion, generic data and the types of uncertainties associated is presented. Since the most of the generic data provides only constant failure rate, updating process for the exponential sampling distribution is described in detail.

1 INTRODUCTION

Through the use of reliability models, reliability and maintenance engineers use equipment life data to estimate its probability to perform the required functions and to support activities like: planning of the spare parts, inventory and maintenance activities, evaluating the personnel or choosing manufacturer or supplier, Murthy & Rausand & Osteras (2008) and Percy (2008). In growing complexity of systems and the need to achieve more with leaner budget, making the right decisions immediately after the problem arises is crucial.

Some facts should be mentioned. Failures encountered during the operation of the equipment are well founded on the fact that the data undoubtedly reflect all influential factors on its reliability and as a consequence the engineering reliability is failure oriented. The main goal of engineering maintenance, on the other hand, is to reduce the number of failures. Therefore, even if we are dealing with a component that is in service for some time, the use of reliability models is often limited by a lack of satisfactory data upon which it can be based. This stands for the so-called 'frequentists' approach to estimation of unknown model parameters where null hypotheses are possible if a sufficient data are observed. To solve this problem we may build reliability model on the data gathered with similar components. Considering the fact that operating conditions are not necessarily the same, one may say that we are often dealing with one-of-a-kind situations and the reliability estimates founded on previous experience alone are not always applicable.

Above mentioned clearly states that neither failure record or previous experience alone can solve reliability and maintenance problems and it makes a great deal of practical sense to use all the information available.

It should be pointed out that the non-safety equipment is expected to fail and the failure record will increase with time so the concept of learning from the data is applicable. Safety components that are impractical to be monitored and due to respective failure modes are expected to fail are usually in parallel or redundant configuration. Therefore the well-prepared maintenance actions significantly contribute to reliability of the system.

The Bayesian framework for model development offers a possibility of taking the previous knowledge and failure data into account. Therefore the model development is a learning process and knowledge is continually updated as more information becomes available. Such analyses are most credibly performed when subject matter experts are involved to play a key role.

Almost all commercially available reliability databases provide only constant failure rates, see Rausand & Høyland (2004) and OREDA (2002). This may cause some problems. First of all, the exponential model is not a reasonable choice for equipment that could endanger the people or environment. Secondly, many failure modes listed are prone to some kind of deterioration process, so the exponential model is not expected. A further fact should be mentioned in order to support the use of this paper. Although the non-aging property would seem to limit the usefulness of the constant failure rate (exponential model), it has continued to play a critical role in reliability calculations. Many probabilistic models, like reliability block diagrams or Markov models, are founded on exponential probabilistic model of failure and can be used without too much difficulty whenever the respective failure rates are given.

In this paper, first we present the interpretation of the constant failure rate in reliability modeling. Next we provide a brief overview of the key concepts of Bayesian modeling relevant to later discussion. Since the validity of the results depends on the validity of the assumptions required by the model, the derivation of exponential model is presented. Finally, through some examples we will explain how to incorporate prior knowledge such as generic data and expert opinion into the estimation process.

2 CONSTANT FAILURE RATE IN RELIABILITY MODELING

The example of generic data base is OREDA (2002). OREDA (Offshore Reliability Data) is the most known database whose main objectives are: "*collection and analysis of maintenance and operational data, establishment of a high quality reliability database, and exchange of reliability, availability, maintenance and safety technology among the participating companies*". Failure events are gathered from two or more installations, and reflect a weighted average of the experience. All the failure rates presented are based on the assumption that the failure rate is constant and independent of time. The data compiled in OREDA are directly relevant for offshore conditions. However, in some cases, considering the maritime operating conditions and particular failure mechanism we might justifiably expect similar frequencies.

For example the failures of valves are presented from population of 1170 items installed on 40 different offshore platforms. The accumulated (calendar) time in service is $36.67\cdot 10^6$ hours, and accumulated operational time is $31.62\cdot 10^6$ hours. During that period the 1017 failures were recorded. Failure rate is given as mean and its value is 31.05 per 10^6 and standard deviation is 18.89 per 10^6 hours for calendar time. The lower and upper bounds with 90% confidence intervals, and are respectively λ_L=7.74 and λ_U=67,14 failures per 10^6 hours.

From the foregoing discussion, the analyst must have an opinion about parameters that must be expressed through a probability distribution. It is desirable that the distribution parameters have proper operational interpretation. First question immediately rises: why the constant failure rate is given even for the failures of mechanical components that are clearly caused by fatigue or wearout. Therefore the exponential approximation of increasing failure rate should be considered. The data available for the analysis is usually the number n of failures during a observation time in service $\Delta t = t_2 - t_1$. The failure rate estimated by n/t will thus be an Average Failure Rate (AFR)

$$AFR(t_1,t_2) = \frac{1}{t_1 - t_2}\int_{t_1}^{t_2} \lambda(u)\,\mathrm{d}u = \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1} \tag{1}$$

where the $R(t)$ is general reliability function and $\lambda(t)$ is the respective failure rate function. For the Weibull model, the failure rate and reliability functions are

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \tag{2}$$

$$R(t) = exp\left[-\left(t/\theta\right)^{\beta}\right] \tag{3}$$

The Weibull probability density is

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} exp\left[-\left(t/\theta\right)^{\beta}\right] \tag{4}$$

where β is the shape parameter and θ is the scale paramenter. Last three equations are different representations of the same model. If the 'real' life distribution is Weibull distribution with an increasing failure rate function, $\beta > 1$, and we use a constant failure rate estimate, we overestimate the failure rate in the initial phase and underestimate the failure rate in the last part of the observation or prediction interval. This is illustrated in Figure.1.

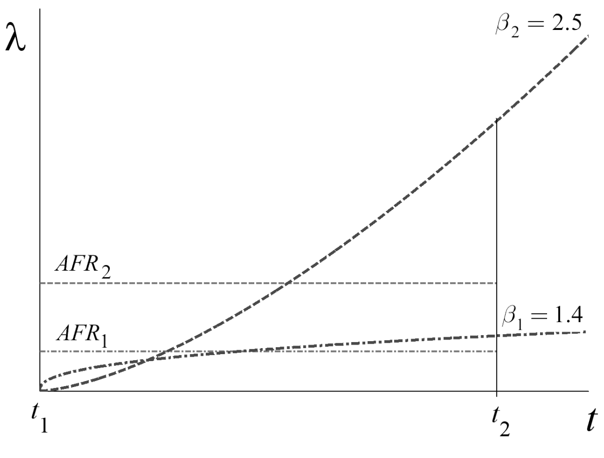

Figure 1. AFR of the Weibull models with β parameter set to 1.4 and 2.5.

This general conclusion rejects all information regarding most of the mechanical failures with increasing failure rate, expressed in the form of exponential model. To deal with the above shortcoming we should consider the exponential model when 'real' Weibull model estimates β is in the range 1÷2.

Generally, in that range of β, it in unlikely that the any kind of periodic maintenance is practically feasible. In such cases, even if the failure rate is increasing, reliability model can be reasonably well developed on exponential basis. Validity of this approximation rises when observation window Δt decreases.

In the case of the Weibull distribution, the shape parameter β i usually related to particular failure mode and the quality measures of the component in hand and therefore has no operational meaning. Therefore the parameter β can be assumed as failure specific, and constant across the range of stress levels. This cannot be assumed for the scale parameter θ because it generally changes with the stress applied. Consequently θ can be assumed as usage or operational specific parameter.

3 BAYESIAN INFERENCE

This section describes the concept of subjective or personal probability. In Bayesian reliability analysis, the statistical model consists of two parts: the likelihood function, $f(x \mid \varphi)$ and the prior distribution, $\pi_0(\varphi)$. The distribution that represents our knowledge about these parameters is the prior distribution, $\pi_0(\varphi)$. The likelihood function is typically constructed from the sampling distribution of the data and is considered as fixed. The sampling distribution contains the vector of unknown parameters φ. Once we acquire the failure data, we regard the sampling distribution as a function of the unknown parameters. In Bayesian analysis, the likelihood function and the prior distribution are the basis for parameter estimation and inference. Mathematically we can combine prior knowledge with current data through equation:

$$\pi_1(\varphi \mid x) = \frac{f(x,\varphi)}{f(x)} = \frac{f(x \mid \varphi)\,\pi_0(\varphi)}{\int_\Omega f(x \mid \varphi)\,\pi_0(\varphi)\,\mathrm{d}\varphi} \tag{5}$$

Now, the failure model parameters are random variables Φ in space Ω, it values φ (for example Weibull parameters β and θ), are random variables which behave according to distribution $\pi_0(\varphi)$. Hence, before any failures being observed, a priori estimate about the expected value of φ from Φ is $E[\pi_0(\varphi)]$. Furthermore

$$f(x,\varphi) = f(x \mid \varphi)\,\pi_0(\varphi) \tag{6}$$

constitutes the joint density and expression in denominator of Equation 5

$$f(x) = \int_\Omega f(x \mid \varphi)\,\pi_0(\varphi)\,\mathrm{d}\varphi \tag{7}$$

is called the marginal density and can be interpreted as normalizing constant, i.e. a constant whose role is to ensure that posterior $\pi_1(\varphi \mid x)$ is a proper density function. Hence, posterior distribution is always proportional to joint density:

$$\pi_1(\varphi \mid x) \propto f(x \mid \vartheta)\,\pi_0(\varphi) \tag{8}$$

This process must be repeated as another failure is acquired and the posterior distribution of Φ becomes the prior. The available failure data is denoted in chronological order by $\boldsymbol{T}=\{T_1, T_2, ..., T_n\}$. After the n-th failure is observed, the prior distribution of Φ , $\pi_{n-1}(\varphi)$, is updated to the posterior distribution of Φ, $\pi_n(\varphi \mid \boldsymbol{T})$.

From computational viewpoint regarding the foregoing scheme the first problem arises in solving the integral in denominator of Equation 5. It is clear that for the different choices of the prior distribution $\pi_0(\varphi)$ the joint density, Equation 6, may take one algebraic form or another. For certain choices of the prior, the posterior has the same algebraic form as the prior. Such a choice is a conjugate prior. Conjugate priors are shown in Table 1.

Table 1. Conjugate priors

Sampling Distribution	Conjugate Prior
Binomial (π)	Beta
Exponential (λ)	Gamma
Gamma (κ)	Gamma
Multinomial (π)	Dirichlet
Multivariate Normal (μ, Σ)	Normal Inverse Wishart
Negative Binomial (π)	Beta
Normal (μ, σ^2 known)	Normal
Normal (σ^2, μ known)	Inverse Gamma
Normal (μ, σ^2)	Normal Inverse Gamma
Pareto (β)	Gamma
Poisson (λ)	Gamma
Uniform(0, β)	Pareto

There is no one 'correct' way of inputting prior information and different approaches can give different results. From practical viewpoint, prior distribution should reflect the best available knowledge or information about unknown parameters and should not be specified simply for computational convenience. If the conjugate prior distribution that provides an adequate representation of information cannot be found, numerical technique, such as MCMC should be used, Hamada et al. (2008) and Singpurwalla (2006).

The first order, or so called 'classical', approach to inference on future failures replaces the unknown parameters with respective mean value approximation $E[\varphi]$

$$f(x \mid \varphi) \approx f(x \mid \hat{\varphi}) \tag{9}$$

which generally yields good approximation in presence of vast amount of data. Within the Bayesian framework it is correct to predict the future failures with prior-predictive distribution

$$f(x) = \int_{\Omega} f(x \mid \varphi)\, \pi_0(\varphi)\, \mathrm{d}\varphi \tag{10}$$

or with posterior-predictive distribution

$$f(x \mid \boldsymbol{T}) = \int_{\Omega} f(x \mid \varphi)\, \pi_1(\varphi \mid \boldsymbol{T})\, \mathrm{d}\varphi \tag{11}$$

Since only difference between the two prediction models is in chronological order, the distribution that contains more data (the latest) is used for prediction.

Typically physical and statistical arguments regarding the prior distribution model should be given by the informed opinion of the analyst and/or any chosen subject matter specialists. Arguments must be supported on: physics of failure theory and computational analysis, prototype testing, generic reliability data and past experience with similar devices. The foregoing Bayesian updating framework is a general one. Now, its application will be illustrated by considering revision of an initial estimate of the failure rate in the gamma-exponential conjugate model.

The exponential distribution in terms of the failures is

$$f(x) = \Lambda\, exp(-\Lambda x) \tag{12}$$

with Λ, $x > 0$. The constant failure rate Λ should be treated as a random variable, and it is necessary to specify the form of the distribution that it follows. The model assumed is again exponential

$$\pi_0(\lambda) = \nu\, exp(-\nu\lambda) \tag{13}$$

with v, $\lambda > 0$. In Equation 13, v is the parameter of this distribution. The dimension of v is lifetime, being the inverse of those of λ. If λ designates an estimate of Λ then Equation 12 becomes

$$f(x \mid \lambda) = \lambda exp(-\lambda x) \tag{14}$$

with λ, $x > 0$. And after the occurrence of the first failure T_1, Equation 5 results in:

$$\pi_1(\lambda \mid T_1) = \frac{f(T_1 \mid \lambda)\, \pi_0(\lambda)}{\int_0^{\infty} f(T_1 \mid \lambda)\, \pi_0(\lambda)\, \mathrm{d}\lambda} \tag{15}$$

In this equation, λ is the unknown parameter of interest distributed with posterior $\pi_1(\lambda \mid T_1)$ and $\pi_0(\lambda)$, given in Equation. 13, is the prior distribution of λ. Subsequently $f(T_1 \mid \lambda)$, given in Equation 14, is the likelihood function that updates a prior distribution. Equation 13 is computationally convenient but seriously limited in expressing prior knowledge because it has only one parameter. Within the Bayesian framework we are free to choose other forms of priors. Equation 13, can be assumed in gamma form

$$\pi_0(\lambda) = \frac{{\kappa_0}^{\alpha_0}}{(\alpha_0 - 1)!}\, \lambda^{\alpha_0 - 1}\, exp(-\kappa_0 \lambda) \tag{16}$$

with κ, α, λ >0. Expected value and variance of the gamma distribution are

$$\hat{\lambda}_0 = E\left[f(\lambda_0)\right] = \frac{\alpha_0}{\kappa_0} \quad ; \quad Var(\lambda_0) = \frac{\alpha_0}{{\kappa_0}^2} \tag{17}$$

Gamma prior, Equation 16, is equal to exponential prior, Equation 13, when α_0=1 and κ_0=v. Gamma distribution can generate a wide variety of shapes for the prior distribution by simply modifying the numerical values assigned to parameters α_0 and κ_0 without impacting mathematical simplicity.

The posterior mean (expected value) is the most frequently used Bayesian parameter estimator. The posterior mode (maximum belief) and median (central value) are less commonly used alternative estimators. Prior to first failure all inference should be founded on the α_0, κ_0. The occurrence of the first failure T_1 gives the evidence that must be considered in order to validate the assumptions stated by the prior distribution. The marginal density becomes

$$f(T_1) = \int_0^{\infty} \left(\lambda e^{-\lambda T_1}\right)\left(\frac{{\kappa_0}^{\alpha_0}}{(\alpha_0 - 1)!}\, \lambda^{\alpha_0 - 1}\, exp(-\lambda \kappa_0)\right) \mathrm{d}\lambda$$
$$= \frac{\alpha_0 {\kappa_0}^{\alpha_0}}{(T_1 + \kappa_0)^{\alpha_0 + 1}} \tag{18}$$

and by applying Equation 15 the posterior is

$$\pi_1(\lambda \mid T_1) = \frac{\left(\lambda e^{-\lambda T_1}\right)\left(\frac{{\kappa_0}^{\alpha_0}}{(\alpha_0 - 1)!}\, \lambda^{\alpha_0 - 1} e^{-\lambda \kappa_0}\right)}{\frac{\alpha_0 {\kappa_0}^{\alpha_0}}{(\kappa_0 + T_1)^{\alpha_0 + 1}}}$$
$$= \frac{(\kappa_0 + T_1)^{\alpha_0 + 1}}{\alpha_0 !}\, \lambda^{\alpha_0} e^{-(\kappa_0 + T_1)\lambda} \tag{19}$$

That is according to Equation 16 also the gamma distribution with parameters $\alpha_1 = \alpha_0 + 1$, $\kappa_1 = \kappa_0 + T_1$ and expected value

$$\hat{\lambda}_1 = E\left[f(\lambda \mid T_1)\right] = \frac{\alpha_0 + 1}{\kappa_0 + T_1} \tag{20}$$

After the second failure T_2 the calculation is repeated starting from Equation 18. If the parameters α_0 and κ_0 are replaced with α_1 and κ_1, and time to failure T_1 with T_2 we obtain:

$$f(\lambda \mid T_1, T_2) = \frac{(\kappa_1 + T_2)^{\alpha_1 + 1}}{\alpha_1 !}\, \lambda^{\alpha_1} e^{-(\kappa_1 + T_2)\lambda} \tag{21}$$

which is gamma distributions with parameters $\alpha_2 = \alpha_0 + 2$ and $\kappa_1 = \kappa_0 + T_1 + T_2$. Consequently, for the failures $\boldsymbol{T} = \{T_1, T_2, ..., T_n\}$ we get gamma distribution with parameters

$$\alpha_n = \alpha_0 + n \quad , \quad \kappa_n = \kappa_0 + \sum_{i=1}^{n} T_i \tag{22}$$

This form is applicable only if the parameters are integers. Further generalization can be presented in the general form of gamma distribution

$$f_{\Lambda|\boldsymbol{T}}\left(\lambda \mid \boldsymbol{T}\right) = \frac{\kappa_n^{\alpha_n}}{\Gamma(\alpha_n)} \lambda^{\alpha_n - 1} exp\left(-\kappa_n \lambda\right) \tag{23}$$

and any real and positive values can be used for expressing the prior distribution. The expected value of the failure rate is

$$\hat{\lambda}_n = E\left[f\left(\lambda \mid \boldsymbol{T}\right)\right] = \frac{\alpha_0 + n}{\kappa_0 + \sum_{i=1}^{n} T_i} \tag{24}$$

and variance.

$$Var\left(\lambda_n\right) = \frac{\alpha_0 + n}{\left(\kappa_0 + \sum_{i=1}^{n} T_i\right)^2} \tag{25}$$

Some comments are in order now. From Equation 24 and 25, it is clear that the weight given to the prior decreases as the sample size increases. In other words: evidence from the data has higher weight than the prior information. By employing Equations 24 and 25 with previously presented example of generic data regarding the valve one may calculate parameters α_0 and κ_0.

Considering the maintenance reality much of the failure data will be subjected to censoring. That is dealt with ease by employing the likelihood principle inherent to foregoing scheme

$$f\left(\boldsymbol{T} \mid \varphi\right) \propto \prod_{i=1}^{n} p_i\left(T_i \mid \varphi\right) \tag{26}$$

where

$$p_i\left(T_i \mid \varphi\right) = \begin{cases} f\left(T_i \mid \varphi\right); & T_i \text{ observed} \\ R\left(T_i \mid \varphi\right); & T_i \text{ right censored} \\ F\left(T_i \mid \varphi\right); & T_i \text{ left censored} \end{cases} \tag{27}$$

with F=1-R. Posterior-predictive distribution of the gamma-exponential model is

$$f\left(x \mid \boldsymbol{T}\right) = \int_{\Omega} \lambda\, exp\left(-\lambda x\right) \frac{\kappa_n^{\alpha_n}}{\Gamma(\alpha_n)} \lambda^{\alpha_n - 1} exp\left(-\lambda \kappa_n\right) d\lambda = \frac{\alpha_n \kappa_n^{\alpha_n}}{\left(x + \kappa_n\right)^{\alpha_n + 1}} \tag{28}$$

4 ILLUSTRATIVE EXAMPLES

The key to the Bayesian modeling, is the specification of a prior probability distribution on φ, before the data analysis. After describing how to calculate Bayesian model, we are presenting some examples in order to illustrate some of the main aspects related to prior specification. The data chosen to demonstrate different aspects of updating in chronological order is: 16, 4, 55, 10, 8, 7, 19, 22, 13 and 11. We assume that failures occur independently according to the exponential distribution. As stated before we are about to estimate the constant failure rate after each of the failures. Therefore we initially carry out a qualitative check to see how the all of data fits to exponential distribution, Figure 2. The graph appears to indicate a trend in the range 0÷20 but the lack of more data and the absence of additional information clouds this conclusion.

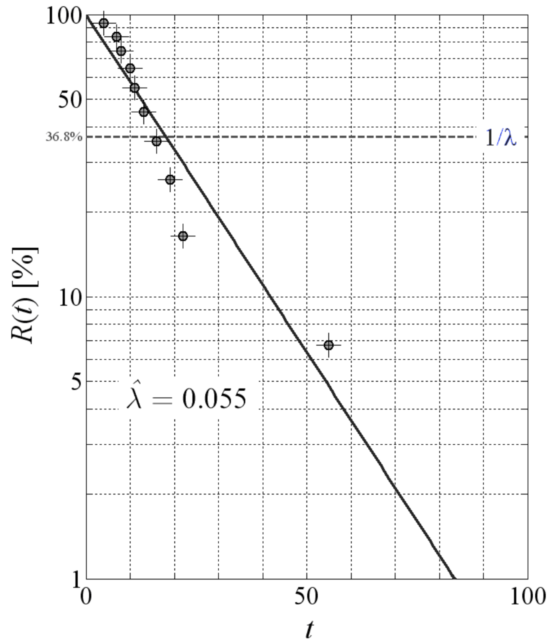

Figure 2. Exponential probability plot of complete data set.

It is well known that the maximum likelihood estimate is strongly biased for the small data sets, but just, for comparison purposes the same data were used in maximum likelihood estimate (MLE) of the reliability model, Figure 3.

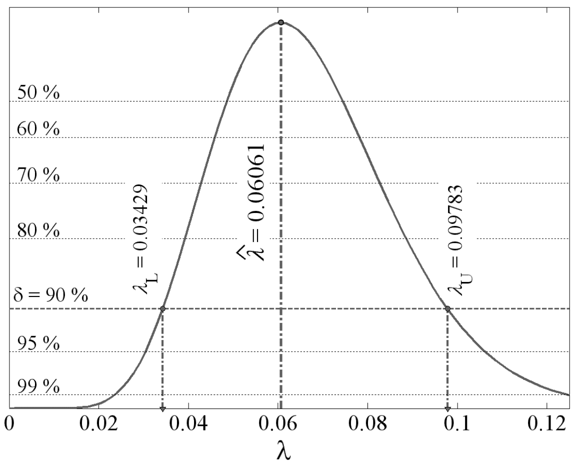

Figure 3. MLE estimate for the complete data set with exponential model (90% confidence interval).

The failure rate λ=0.055 will be used as the reference value in the future discussion. This should not be considered as the correct reliability model, but just as the result that will be reached MLE in case all of the data is available.

The gamma distribution is flexible and therefore capable to express a prior knowledge. By varying the parameters α_0 and κ_0 of gamma prior distribution Equation 16, we can find a distribution that approximately represents our prior belief about λ, and a standard Bayesian analysis can be carried out. When there is very little prior knowledge about the model parameters an non-informative or diffuse prior distribution preferred. In this case, we would typically specify a prior distribution that is at least approximately uniform over the range of indifference, Figure 4.

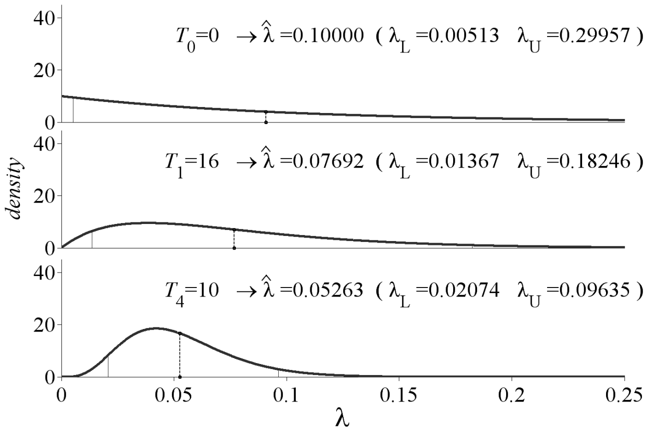

Figure 4. Bayesian updating process with diffuse prior distribution.

The first distribution (T_0=0) shows our prior belief about λ, and since the α_0=1 the prior is exponential. Note that the mass of this prior distribution is spread over a wide range. The lower and upper credibility bounds of our estimate are denoted by λ_L and λ_U respectively. In the two subsequent graphs are the posterior distributions. In the middle is a posterior distribution after the first failure T_1=16 and in the bottom is a posterior distribution after the fourth failure T_4=10. It is evident that following the information gained from the data, updating process relatively quickly adjust our estimate of the first stage prior. Also, with more data the estimate bounds narrows. In Bayesian updating scheme we are generally interested on estimates after the first few failures. After the 10th failure, Figure 5 the Bayesian results resemble to MLE shown on Figure 3. This illustrates a very general property of Bayesian statistical procedures. In plain terms, the data easily swamp the information in the prior and diffuse priors allow us to compare the results with the MLE. In large samples, they give answers that are similar to the answers provided by MLE.

Bayesian interval on Figure 5 agrees with the MLE confidence interval on Figure.3, though their probabilistic interpretations are different. The MLE is a frequency statement about the likelihood that numbers calculated from a sample capture the true parameter and provides us a confidence interval for an unknown parameter. On the Bayesian side the parameter estimates, along with credibility intervals, are calculated directly from the posterior distribution. Credibility intervals are legitimate probability statements about the unknown parameters, since these parameters now are considered random, not fixed. The credibility level for Bayesian model is also set to 90%.

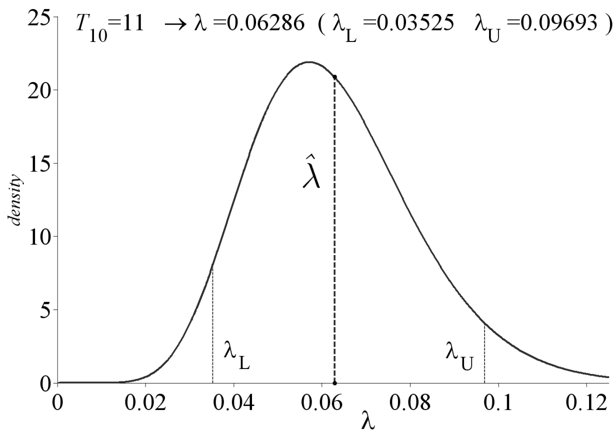

Figure 5. Bayesian updating process with diffuse prior distribution (90% credibility interval).

Although in large samples broad prior distribution give answers that are very similar to the answers provided by classical statistics, in small samples results depend on the chosen prior distribution. If we choose prior distributions that assign non-negligible mass to the region surrounding the 'true' value of a parameter, then the posterior distribution will slowly converge, Figure 6. Such a scenario is likely to occur if low failure rate, given by a manufacturer is adopted for analysis. Because of the attitude of people involved in estimate we call this prior to be 'optimistic'.

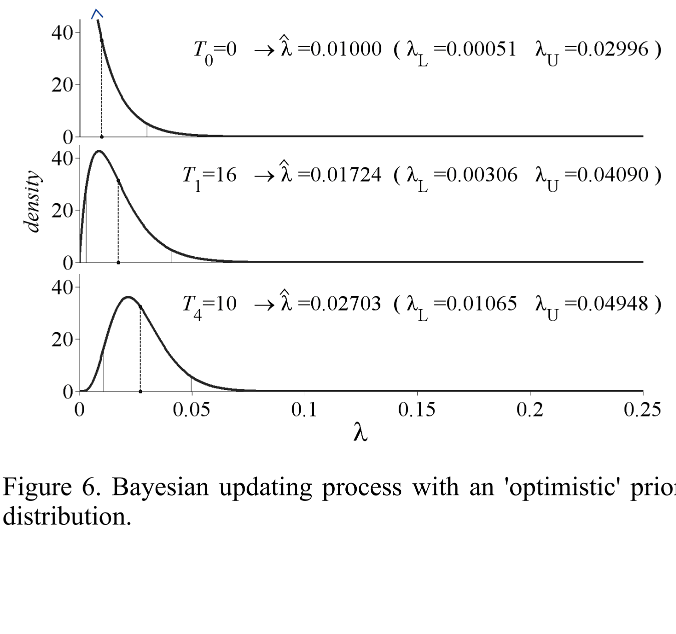

Figure 6. Bayesian updating process with an 'optimistic' prior distribution.

This demonstrates how the Bayesian solution depends on the prior adopted. It can be viewed as either advantage or disadvantage, depending on how you regard the prior density; as representative one to real prior information, and the aim of the investigation.

The informative prior distributions assign most of the prior weight around the 'true' value of the parameter estimated. They are appropriate only if our belief is founded on a quality information about the parameters. Therefore the prior model must be strongly supported by experts. If not, our updating process will converge more slowly even than the MLE and the evidence provided by early failures will not provide any significant impact on posterior distribution. This is shown on Figures 7 and 8. On the Figure 7 we use a wide, but still informative prior, and the 'true' estimate is well covered. The expected value of our estimate quickly converges to the estimate shown in Figure 2. In this case the prior is gamma with α_0=8.5 and κ_0=100.

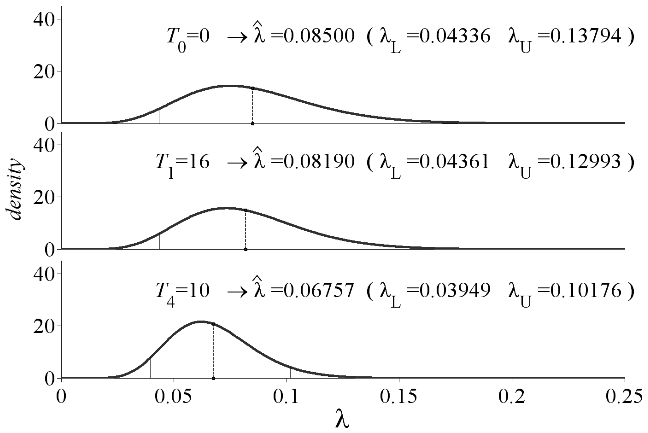

Figure 7. Bayesian updating process with an 'broad' gamma prior distribution.

The results for gamma prior with α_0=12.5 and κ_0=350 are shown on Figure 8.

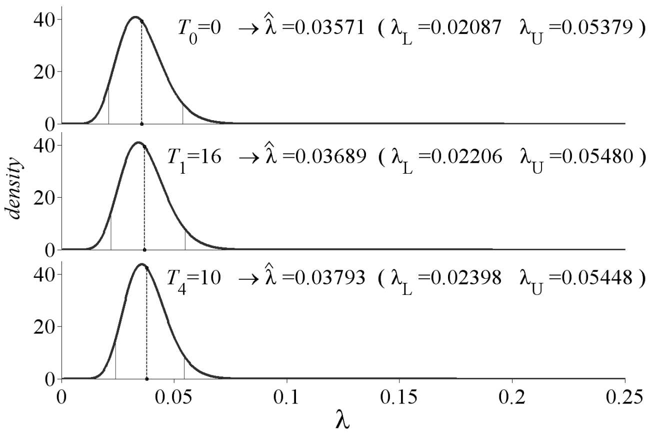

Figure 8. Bayesian updating process with a 'narrow' gamma prior distribution.

In this case the mean of the prior is closer to the 'true' value than in Figure 8, but the 'true' value is less covered. This result suggests that small intervals should be adopted only when there is plenty of relevant information, and larger intervals when there is a lack of information. Note that the standard deviation in OREDA (2002) is relatively high comparing to mean values of failure rate.

The main valid assumption if a successful reliability model is to be established is the specification of the sampling distribution. These arguments should be founded from the physics-of-failure on the device in question. Weibull distribution is frequently used in reliability analysis to fit failure data, because it is capable to handle decreasing, constant and increasing failure rates. Also the Weibull distribution is very flexible and it is not uncommon to fit a small set of failure data equally well as the 'real' failure distribution. Since the Weibull distribution is not a member of the exponential family Bayesian models are not amenable to simple analyses with conjugate priors. Weibull distribution has two parameters that must be considered.

We may suppose that components in this multi-parameter problem are independent so that their joint prior density is the product of corresponding univariate marginal priors. From the previous discussion it seems reasonable to simplify the analysis by assuming constant parameter β =3.5. We assumed the uniform prior on θ over the range 1÷10, and through the numerical methods the posterior is calculated for one failure T_1=4.25·10^3, Figure 9

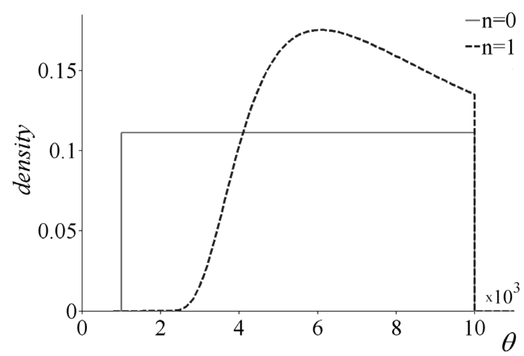

Figure 9. Prior and posterior distributions for Weibull example.

The respective posterior predictive distribution is calculated numerically and is presented in Figure 10.

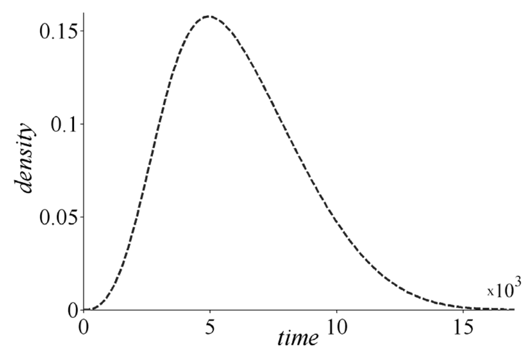

Figure 10. Posterior predictive distribution for Weibull example after the first failure.

5 CONCLUSION

When dealing with small data sets Bayesian inference represents excellent alternative to frequentist approach. Is is founded on the previous knowledge and on the concept of subjective probability that is non existent in the 'objective' concepts of frequentists approach.

Since it is unlikely in most applications that data will ever exist to objectively validate the reliability model our estimate must relay on the other sources of information. Since the most of the generic data provides only constant failure rate, updating process for the exponential sampling distribution is described in detail. In this paper the probabilistic concept for such analysis is discussed and, considering the closed form easily applicable solution is given.

The accompanying discussion describes the special care that should be given to the prior model selection. Narrow intervals should be adopted only if there is plenty of relevant information and larger intervals when there is a lack of information. In other words the analyst must subjectively find balance between optimistic and indifferent attitudes. The first results in narrow priors, and it is unlikely that data will lead the model to better estimates. The second attitude produces broad priors, the results are similar to classic inference and the advantage of Bayesian method is not fully utilized.

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