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Estimating parameters of groundwater recharge model in frequency domain

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Abstract This paper presents the results of the application of the spectral method for determining the values of parameters of the groundwater recharge model for the Jadro and Žrnovnica springs located in the Dinaric karst area of Croatia. By using this frequency domain approach, groundwater-balance calculations are avoided so that the method can be applied on unexplored karst aquifers where groundwater-balance cannot be achieved without extensive geological and hydrogeological investigations. Since the results of several tracer tests show that the Jadro and Žrnovnica springs mostly share the same catchment, the values of parameters of the groundwater recharge model are estimated separately for both springs and the obtained results are compared and discussed.

Key words karst; groundwater recharge; spectral method; karst spring; hydrological monitoring

INTRODUCTION

Groundwater recharge is defined as the portion of total rainfall falling into a drainage basin that ultimately reaches the water table in the phreatic zone of an aquifer. As groundwater recharge cannot be measured directly, it is estimated by using various methods. Depending on the available data, groundwater recharge can be calculated with a simple water balance model using precipitations and potential evapotranspiration, or with more complex and data demanding hydrological models (e.g. Rushton & Ward, 1979; Besbes & De Marsily, 1984; Korkmaz, 1990; Sophocleous, 1991; Wu *et al*., 1996, 1997; Avery *et al*., 1999; Ketchum *et al*., 2000). A fluid mass balance approach is a common characteristic of all these methods. Fluid mass balance is calculated for the entire aquifer or separately for the vadose zone (soil moisture balance) and the phreatic zone (groundwater balance). The soil moisture balance approach assumes that the initial precipitation left after evapotranspiration is taken up as soil moisture and when the soil is saturated (soil moisture holding capacity filled) water begins to form surface runoffs and percolation through the vadose zone. The percolation recharges the phreatic zone finally. The moisture lost in the process of evapotranspiration is measured directly or empirical equations are used for estimations. The groundwater balance approach is generally based on the mass conservation equation of the phreatic zone, i.e. the recharge for an analysed period is equal to the accumulated groundwater plus total discharge. Just as they are applied to granular media, these conceptual approaches are used also in karst. Soulios (1984) calculated the annual fluid mass balance for three karst springs in Greece by using Thornthwaite & Mather's (1955) soil moisture balance method. Bonacci (2001) used a hydrograph separation method for the estimation of monthly effective infiltration coefficients. Jocson *et al*. (2002) used a conceptual model for the estimation of karst island aquifer recharges. The parameters of the conceptual model were determined by equalizing the simulated recharges with aquifer discharges. Denić-Jukić & Jukić (2003) used Palmer's (1965) soil moisture balance method and composite transfer functions for the estimation of daily effective rainfall rates. The soil moisture holding capacities and the parameters of the composite transfer functions were determined simultaneously by minimizing the differences between observed and simulated discharges. In addition to these conceptual approaches, numerical models are used also for estimations of groundwater recharge in karst (Contractor & Jenson, 2000).

Hydrogeologic characteristics of karst are complex and significantly different from the characteristics of granular media. Karst aquifers are usually characterized with the time-variant boundaries that are dependent on fluctuations of groundwater levels. Only in exceptional cases do the surface and subsurface watershed lines coincide and only in those places where the boundaries between catchments are located in impermeable rocks (Bonacci, 1987). In addition, groundwater exchanges with adjacent aquifers through underground piracy routes or inflows from surface streams and accumulations are common in karst. The above-mentioned hydrogeologic characteristics cause increasing difficulties in identifying groundwater basins and writing of groundwater balance for karst aquifers. Practically, groundwater balance cannot be achieved without extensive geologic and hydrogeologic investigations, especially if a short period is analysed. Alternatively, the soil moisture balance approach requires measurements of groundwater levels for determining the values of various parameters characterizing the vadose zone.

This paper presents the results of application of the spectral method (Jukić & Denić-Jukić, 2004) for determining the values of parameters of the groundwater recharge model for the Jadro and Žrnovnica springs located in the Dinaric karst area in Croatia (Fig. 1). Investigations were performed in order to validate previous results, to test practical applicability of the spectral method, and to give a contribution to the existing knowledge about hydrological functioning of the Jadro and Žrnovnica Springs.

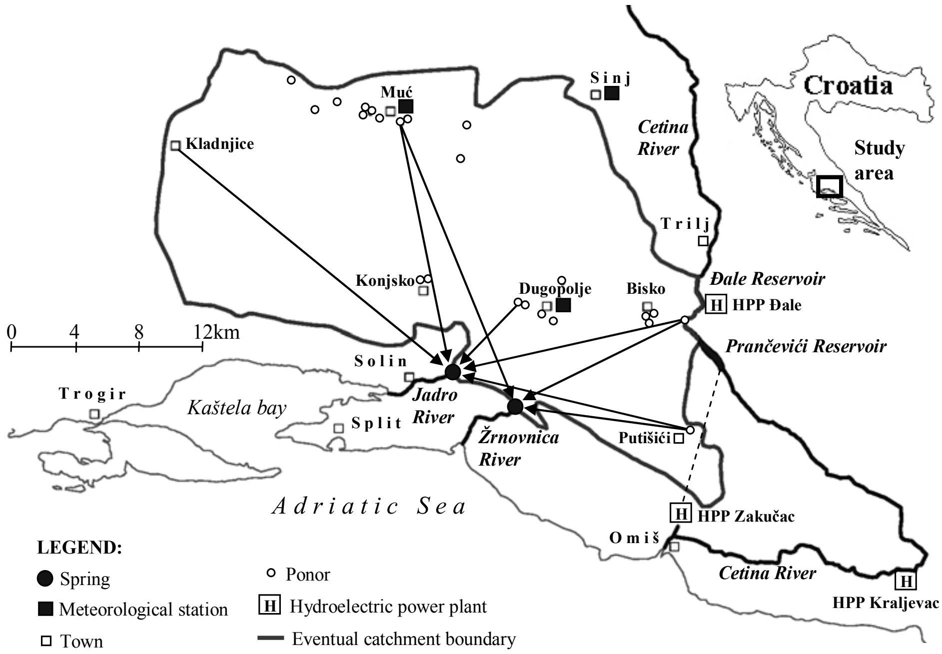

Fig. 1 Map of the Jadro and Žrnovnica Springs catchment with the results of tracer tests.

The Jadro and Žrnovnica springs are situated in the vicinity of the city of Split, Adriatic Sea coast, Croatia. The Jadro spring is located at an elevation of 35 m a.s.l. whereas the water from the Žrnovnica spring emerges at different elevations, from 78 to 90 m a.s.l., depending upon groundwater levels in the hinterland of the karst massive. The catchment is located in the bare Dinaric karst, mainly formed of carbonate rocks and partly of impermeable flysch. There is no reliable hydrological delineation of the Jadro and Žrnovnica springs catchment area so Fig. 1 shows only eventual hypothetical boundaries. The assumed eastern limit is approx. 15 km from the springs in the vicinity of the perennial Cetina River flowing through a carbonate formation at an elevation of 300 m a.s.l. Tracer tests of the ponor Grabov Mlin located near the Cetina River riverbed that were performed in 1963 revealed the direct connection between the Cetina River and the Jadro and Žrnovnica springs. About 96% of the injected tracer appeared at the springs 23 days after the tracer had been injected into the ponor. Construction of the hydroelectric power plants along the Cetina River (reservoirs Đale and Prančevići) had a strong influence on the springs discharges. The capacity of the Jadro Spring has been increased (Bonacci, 1987) and the Žrnovnica Spring has never dried up since then. Denić-Jukić (2002) concluded that the inflows from the Cetina River bed and catchment to the Jadro Spring are permanent, significant, time variant and dependent on groundwater levels. The results of several tracer test carried out later to improve understanding of the catchment area functioning are also presented on Fig. 1, including the tracer tests of the ponors Jablan and Ponikve located near the town Muć, which revealed fast connection between this area and the springs.

Available data are daily rainfall rates from the meteorological stations Dugopolje and Muć, daily temperatures and relative humidity from the nearest available station Sinj, and daily discharges from the Jadro and Žrnovnica Springs. The period from 2001 to 2005 is analysed in this study. The general statistics for total precipitations and the spring discharges are presented in Table 1.

Table 1 General statistics for the annual precipitations and the daily discharges.

Period	Daily discharge ($m^3 s^{-1}$)						Annual precipitation (mm)					
	Jadro Spring			Žrnovnica Spring			Station Dugopolje			Station Muć		
	Min	Max	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max	Mean
2001–2005	3.87	70.1	9.97	0.37	16.7	1.90	963	1647	1264	1061	1577	1301

METHODOLOGY

In this study, groundwater recharge rates are calculated by the mathematical model based on Palmer's (1965) soil-moisture balance method. The groundwater recharge in this context represents the effective rainfall, i.e. the portion of total rainfall that instantaneously recharges the phreatic zone. The parameters are the soil-moisture holding capacities of the upper layer of soil s_1 and the lower layer s_2. From the upper layer, which implicitly includes land cover, the moisture is lost freely in the processes of evaporation and transpiration, while the moisture lost from the lower layer depends on the saturation of the upper layer. The moisture lost in the process of evapotranspiration is calculated by using the expression of Eagleman (1967). Although Palmer's (1965) soil-moisture balance method and the expression of Eagleman (1967) were not developed originally for the karst areas, which are practically devoid of vegetation and soil cover, results of several applications in the Dinaric karst have shown that they can be successfully applied in preliminary analyses (e.g. Denić-Jukić, 2002; Denić-Jukić & Jukić, 2003; Jukić & Denić-Jukić, 2004; Jukić, 2005).

The values of the model parameters are estimated in the frequency domain by the spectral method (Jukić & Denić-Jukić, 2004). The karst aquifer is considered as a lumped system that contains a phreatic and vadose zone. It is implicitly assumed that the vadose zone encloses the epikarst zone and the soil cover receiving rainfall rates as the input to the system. Groundwater recharges are the output from the vadose zone and the input to the phreatic zone, which is considered as the linear and time-invariant filter. The phreatic zone finally produces spring discharges as the output from the system. The additional assumption about exponential forms of autocorrelation functions of input and output time series is introduced in order to obtain a mathematical relationship between the Transfer Function of Total Rainfalls (TFTR) and the transfer function of groundwater recharges in frequency domain. However, the results of practical applications of the spectral method on several karst springs in Croatia (Jukić & Denić-Jukić, 2004; Jukić, 2005) have shown that this assumption cannot always be successfully applied to the time series of rainfall rates and spring discharges, especially if a short time period is analysed such as a year. For this reason, the scale factor of TFTR is considered as a model parameter ξ. Consequently, the optimal values of the parameters $S \equiv (s_1, s_2)$ and ξ for intervals α are determined by using approximate Whittle log likelihood function (Shumway & Stoffer, 2000) as the criterion:

$$\ln[L_\alpha(S,\xi)] \approx -\sum_{0<\nu_k<1/2}\left\{\ln[g_\alpha(S,\xi,\nu_k)] + \frac{I_{y,\alpha}(\nu_k)}{g_\alpha(S,\xi,\nu_k)}\right\} \qquad (1)$$

where ν_k is Fourier frequency and $I_{y,\alpha}(\nu_k)$ is periodogram of spring discharges for the interval α. The parametric periodogram in equation (1) is calculated by expressions:

$$g_\alpha(S,\xi,\nu_k) = \xi\left|H_T(\nu_k)\right|^2 I^R_{x,\alpha}(S,\nu_k) \qquad (2)$$

$$\left|H_T(\nu_k)\right|^2 = \frac{f_y(\nu_k)}{f_x(\nu_k)} \qquad (3)$$

where $I^R_{x,\alpha}(S,\nu_k)$ is periodogram of groundwater recharge rates for the interval α, $\left|H_T(\nu_k)\right|^2$ is TFTR in frequency domain, $f_x(\nu_k)$ and $f_y(\nu_k)$ are spectral density functions of rainfall rates and spring discharges, respectively.

RESULTS OF APPLICATION

The optimal values of the parameters are determined by maximizing the approximate Whittle log likelihood function (equation (1)) by using an automatic trial-and-error procedure based on systematic alterations in the acceptable values of the model parameters. During the search only the parameter set associated with the current maximum of the log likelihood function is retained, which, at the end of the search, is regarded as the optimal parameter set. In order to achieve a minimal and finite number of alterations, the maximum value of the log likelihood function was searched for the integer values of parameters s_1 and s_2 in the intervals $0 \le s_1 \le 20$ mm and $10 \le s_2 \le 120$ mm where the acceptable results for the Dinaric karst aquifers are located. This procedure could be applied because the number of the parameters is not significant and they have relatively short intervals of possible values. The parametric periodogram (equation (2)) was calculated by using the values of the polynomial forms of TFTR (Fig. 2). These forms were applied to eliminate the variances associated with the periodogram estimates of the spectral density functions in equation (3). Figure 2 shows that both aquifers act as low-pass filters and that they have similar system response. Although the differences between TFTR are very small, it can be noted that TFTR for the Žrnovnica Spring attenuates the amplitude of the input signal less intensively at high frequencies.

The optimal values of parameters and the resulting Effective Infiltration Coefficients (EIC) are presented in Table 2. Following the results of previous investigations of the Jadro Spring (Jukić & Denić-Jukić, 2004), parameter s_1 is equal to zero for each analysed period, which is the

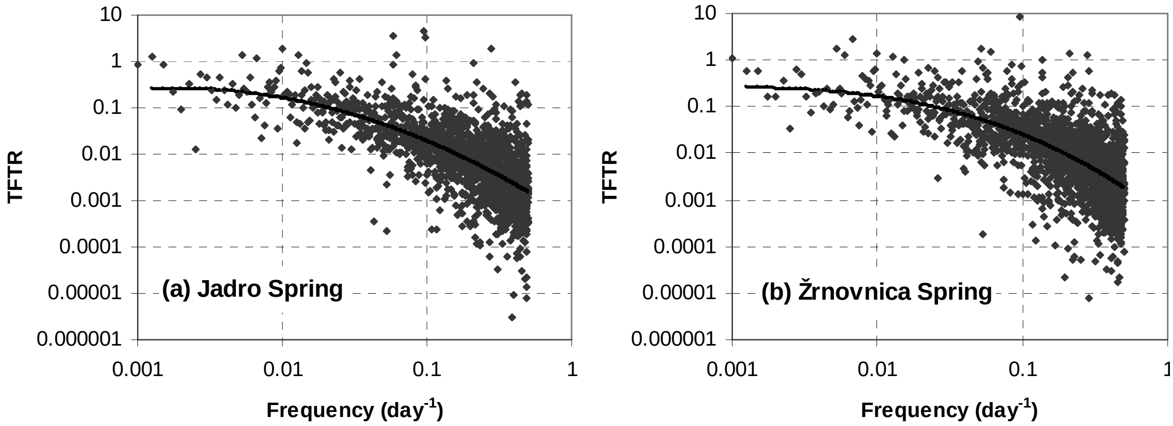

Fig. 2 Transfer functions of total rainfall (TFTR) for the Jadro Spring (a) and the Žrnovnica Spring (b).

Table 2 Values of parameters (ξ, s_1 and s_2) obtained by using the spectral method and the values of the effective infiltration coefficients (EIC) for the analysed periods.

Period	Jadro Spring				Žrnovnica Spring			
	ξ	s_1 (mm)	s_2 (mm)	EIC	ξ	s_1 (mm)	s_2 (mm)	EIC
2001	1.080	0	90	0.605	0.921	0	90	0.605
2002	1.258	0	90	0.502	1.572	0	40	0.562
2003	0.741	0	70	0.572	1.013	0	60	0.586
2004	1.325	0	20	0.777	1.134	0	20	0.777
2005	0.760	0	20	0.716	0.946	0	20	0.716
Mean	1.033	0	58	0.634	1.117	0	46	0.649

expected value for the karst areas practically devoid of soil cover. The values of parameter s_2 for both springs range widely from 20 to 90 mm, which shows that the soil moisture holding capacity cannot be considered as a time-invariant characteristic of these karst aquifers. This result can be explained with the simplifications that are made by considering the phreatic zone as linear and time-invariant filter and by applying the groundwater recharge model that only takes internal runoff practically into account. However, the mean value of s_2 for the Jadro Spring is in accordance with the previous result obtained by Jukić & Denić-Jukić (2004) for the period 1995–2000 (58 *vs* 43 mm), whereas the mean value of EIC is practically identical to the previous (0.634 for the period 2001–2005 *vs* 0.633 for the period 1995–2000). The results for the Žrnovnica Spring are very similar to the results of the Jadro Spring, especially concerning the mean EIC which is only 2% higher. The same soil moisture capacities are obtained for the three periods: 2001, 2004 and 2005.

CONCLUSION

The values of parameters of a groundwater recharge model are determined by comparing the parametric periodograms with the periodogram of observed discharges, where the approximate Whittle log likelihood function is used as the criterion. This log likelihood function summarizes discrete values in the Fourier frequencies so the emphasis in fitting is placed more on the short time system behaviour. For this reason, the spectral method is designed in the first place for the fast response karst aquifers.

By using this frequency domain approach, groundwater-balance calculations are avoided so that the method can be applied on unexplored karst aquifers where groundwater-balance cannot be achieved without extensive geologic and hydrogeologic investigations. The spectral method has a background in correlation. The sine and cosine coefficients of periodograms can be interpreted as regression coefficients, i.e. they tell us the degree to which the respective sine and cosine functions are correlated with the data at the respective frequencies. Thus, permanent inflows from surface streams, random changes of the catchment boundaries or random groundwater exchanges with adjacent aquifers through underground piracy routes have minor impact on the accuracy of the results obtained by the spectral method, while they can change the groundwater-balance completely. The term random includes in this context the processes affecting the output signal, but they are not correlated with the input signal. Consequently, the spectral method is convenient for applications on unexplored karst aquifers when groundwater balance is unknown. However, it should be emphasized that the spectral method is only a tool for determining the optimal values of parameters, but the accuracy of the obtained groundwater recharge rates is mostly related to the applied model.

The obtained values of the parameters of the groundwater recharge model and EIC are in accordance with the results of tracer tests which show that the Jadro and Žrnovnica Springs share mostly the same aquifer. However, the forms of TFTR reveal that these two adjacent springs do not have completely the same hydrological characteristics. The identified TFTR have different shapes for the quick component which shows that the relative importance of this component in total discharges is larger for the Žrnovnica Spring.

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