Prediction of alpha-acids accumulation in hop cones according to empiric mathematical model, general linear regression, and Cobb-Douglas production model-function

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Abstract

As the most important quality parameters of the hops the development of mathematical models for predicting of alpha-acids accumulation in hop cones is very important for planning of hop stocks in order to avoid the market stress. The aim of this study is to verify the accuracy of the empiric mathematical model for predicting the content of alpha-acids in hop cones of cv. Aurora using the linear regression, general linear regression and Cobb-Douglas production model-function. The values of standard deviations (σ) of differences between predicted and analyzed content of alpha-acids for used methods are much lower in the comparison with the empiric mathematical model of alpha-acids prediction in the case of non-compliance with the strictly set condition for reliability of the empirical mathematical model, according to which the daily reference crop evapotranspiration (ET_0) in July ≤ 4.5.

Key words. hop, alpha acids prediction, empiric mathematical model, general linear regression, Cobb-Douglas prediction model-function

Introduction

It is well-known that the alpha-acids are the most important quality parameters of hops on the world market. The world hop industry depends on the alpha acids balance or disbalance caused by their offer and demand in every year and consequently the hop stocks are carefully planned according to available quantities of alpha acids. However, it is not easy to predict the alpha-acids content in hop cones technological maturity because of interaction between genotype of hop cultivars and environmental conditions during the growing season (PAVLOVIČ 2014). According to the results of all previous studies (SREČEC et al. 2004; SREČEC et al. 2008; KUČERA & KROFTA 2009; MOZNY et al. 2009; PAVLOVIČ et al. 2013) the quantity and distribution of rainfalls during the hop vegetation, stays in positive correlation with the accumulation of alpha acids during the technological maturity of hop cones. On the other hand, higher temperatures in July and August have a strong negative impact on alpha-acids accumulation in hop cones (SREČEC et al. 2008; KUČERA & KROFTA 2009; PAVLOVIČ et al. 2013).

Development of mathematical models for prediction of alpha-acids content in relation of weather conditions during the hop vegetation was and still is in focus of hop research, particularly because of the global heating and consequently global weather extremes, because the overall hop supplies depends not just of hop acreage, but also because of the weather attributes. Nevertheless, there are different approaches in mathematical modeling of alpha-acids prediction in hop cultivars in order to predict the hop cones yield as well as yield of alpha-acids (KUČERA & KROFTA 2009; MOZNY et al. 2009; SREČEC et al. 2013; PAVLOVIČ et al. 2013).

The aim of this study is to verify the accuracy of the empiric mathematical model for predicting the content of alpha-acids in hop cones of cv. Aurora.

Material and methods

For verification of the accuracy of the empiric mathematical model for predicting the content of alpha-acids in hop cones of cv. Aurora, the empiric mathematical model reached by Eureqa Nutonian Inc. mathematical software (SREČEC et al. 2013) was used (1).

$$y = \frac{5.38w - 453 - 1.33w^2}{x} : (-10) \qquad (1)$$

Where:

y – alpha-acids content in dry matter (%)

x – sum of effective temperatures (°C) from second germination after the spring pruning until technological maturity

w – sum of total rainfalls (mm) for the same period

It is also important to point out that the authors set up a condition for accuracy of equation (1), according to which the daily reference crop evapotranspiration (ET_0) in July ≤ 4.5 (SREČEC et al. 2008; SREČEC et al. 2013).

The accuracy of the empiric mathematical model (1) was estimated by using the linear regression (FREEDMAN 2005), general linear regression (ELLENBERG 1972), and Cobb-Douglas production model-function (Cobb & Douglas 1928).

All evaluations were done on the basis of our own *Mathematica*-modules, and were performed on the computer with a 2.90 GHz Intel(R) Core(TM)i7-75000 CPU with 16GB of RAM.

Results

First, the functional dependence of the proportion of α-acids in dependence on the amount of rainfalls w (in mm) and the sum of effective temperatures x (°C) was determined using ordinary linear regression (2) and the parameters are determined by the principle of least squares by minimizing the function (3)

$$y = a + bw + cx \qquad (2)$$

$$F(a, b, c) = \sum_{i=1}^{27} [a + bw_i + cx_i - y_i]^2 \qquad (3)$$

and we get the following expression (4):

$$y = 15.117 + 0.0135w - 0.00515x \qquad (4)$$

and $\sigma = 1.934$ of differences between predicted and analyzed content of alpha-acids.

Second, instead of equation (1) functional dependence between the content of alpha-acids and x and w, was also determined using the regression with general parameters a, b, c (5):

$$y = a\frac{1}{x} + b\frac{w}{x} + c\frac{w^2}{x} \qquad (5)$$

That means the model formally written in equation (1) is accepted, but instead of parameters

(53.8, -453, -1.33)/(-10)

we searched the new parameters (a, b, c) and the parameters are determined by the principle of least squares by minimizing the function (6)

$$F(a, b, c) = \sum_{i=1}^{27} \left[a\frac{1}{x_i} + b\frac{w_i}{x_i} + c\frac{w_i^2}{x_i} - y_i\right]^2 \qquad (6)$$

and we get the following expression (7):

$$y = -67.8938\frac{1}{x} + 99.0955\frac{w}{x} - 0.12397\frac{w^2}{x} \qquad (7)$$

and $\sigma = 1.931$ of differences between predicted and analyzed content of alpha-acids.

Third, instead of equation (1) and (5) functional dependence between the content of alpha-acids and x and w, was determined using the general linear regression (8).

$$y = a_0 + b\frac{1}{x} + b\frac{w}{x} + c\frac{w^2}{x} \qquad (8)$$

The parameters were also determined and the parameters are determined by the principle of least squares by minimizing the function (9)

$$F(a_0, a, b, c) = \sum_{i=1}^{27}\left[a_0 + a\frac{1}{x_i} + b\frac{w_i}{x_i} + c\frac{w_i^2}{x_i} - y_i\right]^2 \qquad (9)$$

and we get the following expression (10):

$$y = 1.99 - 6438.14\frac{1}{x} + 120.19\frac{w}{x} - 0.16\frac{w^2}{x} \qquad (10)$$

and $\sigma = 1.921$ of differences between predicted and analyzed content of alpha-acids.

Fourth, functional dependence between the content of alpha-acids and x and w, was determined using the Cobb-Douglas production model-function (11).

$$y = Aw^{\beta}x^{\gamma}, \quad \beta, \gamma > 0. \qquad (11)$$

The function (11) might be written in following form (12)

$$\ln y = \ln A + \beta \ln w + \gamma \ln x \qquad (12)$$

Parameters A, β, γ were also determined and the parameters are determined by the principle of least squares by minimizing the function (13).

$$F(a, \beta, \gamma) = \sum_{i=1}^{27}[a + \beta \ln w_i + \gamma \ln x_i - \ln y_i]^2, \text{ where } a = \ln A. \qquad (13)$$

and we get the following expression (14):

$$y = 787.738w^{0.518758}x^{-0.98109} \qquad (14)$$

and $\sigma = 1.978$ of differences between predicted and analyzed content of alpha-acids.

Discussion

Considering the values of standard deviations (σ) of differences between predicted and analyzed content of alpha-acids, it is obvious that in all four cases the σ values are much lower in the comparison with the empiric mathematical model of alpha-acids prediction (SREČEC et al. 2013), because the standard deviation of differences between predicted and analyzed content of alpha-acids for equation (1) is $\sigma = 3.383$, for equation (4) $\sigma = 1.934$, for equation (7) $\sigma = 1.931$, for equation (10) $\sigma = 1.921$, and for equation (14) $\sigma = 1.978$, respectively. Nevertheless, this is only at the first view. Namely, the authors of empiric mathematical model of alpha-acids prediction (1) set up a strict condition for reliability of the empirical mathematical model, according to which the daily reference crop evapotranspiration (ET_0) in July ≤ 4.5 (SREČEC et al. 2008; SREČEC et al. 2013). That means the extreme high values of the sum of effective temperatures and, consequently, the values of total rainfalls during the hop vegetation are excluded, because they are treated as out layers (Figure 1).

On the other hand, in this study all the values for all independent and dependent variables are included (i.e., every x, w, y data). Moreover, the accuracy of the equation (1) is confirmed in completely different environmental conditions of Savinja Valley, but only for the same hop cultivar, Aurora (SREČEC et al. 2013). That corresponds with the conclusions of previous authors that crop mathematical models are useful tools for assessing the vulnerability and response of crops to climate change if they are adequately tested against observed data during the validation process (MOZNY et al. 2009).

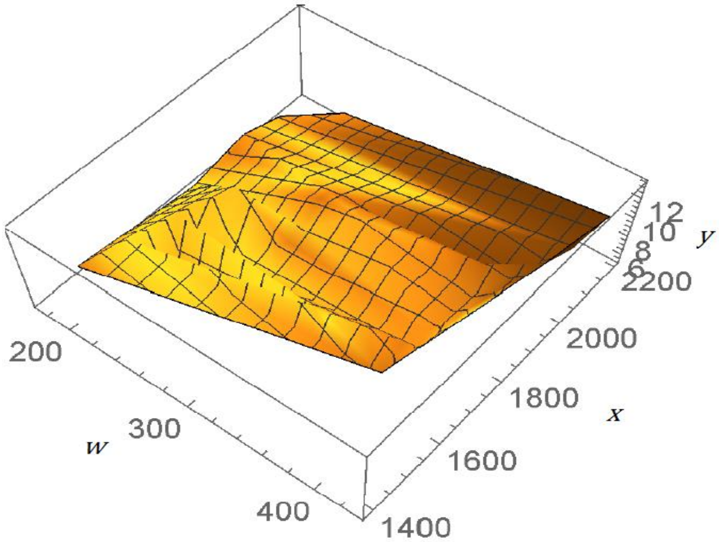

Figure 1. Interaction between accumulation of alpha-acids (*y*) regarding the total rainfalls (*w*) and the sum of effective temperatures (*x*)

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