Active metamaterial cell using non-collocated velocity feedback

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Introduction

In the field of structural dynamics, the reciprocity principle is a useful property. It states that, if the points of excitation and measurement on a passive, linear time-invariant system are switched, the measured variable will remain unchanged. This allows for many practical applications, particularly in the field of modal analysis [1].

On the other hand, a system not complying with this principle may sometimes be preferable, if the goal is to allow for example vibration transmission in one direction, while hindering it in the other. Examples where such behaviour is desired include the development of invisible acoustic sensors [2], cloaking devices [3], and vibration isolation [4], to name a few.

The theoretical loss of reciprocity in linear vibration systems has been noted when they are activated by non-collocated velocity feedback [4]. While in general dual and collocated sensor-actuator pairs are preferred in vibration control of flexible structures, due to the fact that they result in unconditionally stable systems [5], such systems also exhibit reciprocal behaviour. On the other hand, in [4] it has been shown that a 2 dof vibration isolation system equipped with a non-collocated sensor-actuator feedback pair can in fact be unconditionally stable, if the uncoupled natural frequency of the receiving body is lower than the uncoupled natural frequency of the source body. This property of the passive mechanical system allows the alleviation of the destabilising effects of the reactive control force [6].

Recently, the theoretical considerations in [4] were also experimentally validated [7]. It was also shown that, even if the mechanical system is designed such that the uncoupled natural frequency of the receiving body is lower than the uncoupled natural frequency of the source body, which should make the system unconditionally stable, in practice the sensor-actuator dynamics prohibit unconditional stability. Still, the results implied that significant feedback gains could still be implemented, while retaining decent stability margins, and while accomplishing a considerable loss of reciprocity over a wide frequency band.

To further explore the applicability of non-collocated velocity feedback, in this paper a theoretical concept of an active metamaterial cell is presented. The system utilises two non-collocated sensor-actuator pairs in a decentralised control scheme in order to achieve non-reciprocal behaviour. To validate the theoretical findings based on such a conceptual model, a 3D printed experimental setup equipped with sensors and actuators is designed. Both the theoretical considerations and experimental results show that such a system exhibits significant loss of reciprocity in a broad frequency band. While not unconditionally stable, the system can still retain considerable gain stability margins when implementing large feedback gains. The control scheme presented here may find future practical use in the field of active acoustic metamaterial design.

The paper is structured as follows. In the second section the mechanical model of the active metamaterial cell is introduced. In addition, the experimental setup used for the validation of the conceptual model is described. The third section deals with the stability and performance of the activated system. Stability is assessed through the use of the generalised Nyquist stability criterion, while performance is considered by the difference in amplitudes between the characteristic transfer functions of the activated system, as the points of excitation and measurement are switched.

Active metamaterial cell concept and experimental setup

The conceptual lumped parameter model of the active metamaterial cell is given in Figure 1.

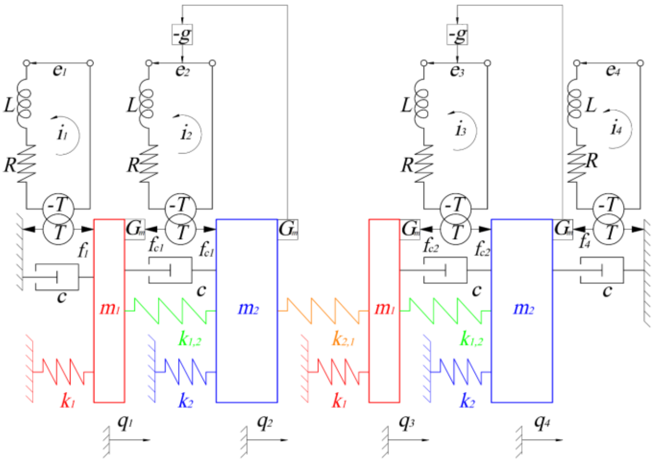

Figure 1: Lumped parameter model of the active metamaterial cell

It is essentially a 4 dof vibration system composed of two identical 2 dof vibration subsystems with masses m_1 and m_2, stifnesses $k_1, k_{1,2}, k_2$ and damping c, connected in series. The coupling stiffness $k_{2,1}$ ensures that vibrations may be transmitted between these two subsystems. Both excitation and control is accomplished by means of actuators modelled as first order electrical circuits with resistance R, inductance L and a back electromotive force constant T. On the other hand, the dynamic behaviour of the seismic sensors is well described by second order frequency response functions $G_m(\mathrm{j}\omega) = \omega_m^2/(\omega_m^2 + 2\mathrm{j}\zeta_m\omega_m\omega - \omega^2)$, where ω_m and ζ_m denote the sensor's natural frequency and damping ratio, while ω and j represent the angular frequency and imaginary unit respectively. Feedback is accomplished by controllers with gain g, generating an input voltage proportional to the measured velocity signals from masses m_2 at the control actuators, which in turn, impart control forces f_{c1} and f_{c2} to the mechanical system. As each of the control actuators uses a degree of freedom (mass m_1) as a base which it may react off, without accessing the velocity information of that particular degree of freedom, the sensor-actuator pairs are in a non-collocated configuration. Additionally, the control system as a whole is considered decentralised, since each of the controllers is fed only a part of the total state of the system, and there is no information exchange between them. For simplicity it is assumed that all sensors and actuators are identical and that the feedback gains of the two feedback loops are equal.

The experimental setup used to validate the theoretical model from Figure 1 may be observed in Figure 2. The setup is a 3D printed frame to which solid blocks of material are connected via thin members – this constitutes the mechanical subsystem. The lumped masses are realised as said solid blocks of material, having a much higher stiffness compared to the thin, leaf-like members, which have negligible mass, are much more flexible, and can therefore be considered the springs $k_1, k_2, k_{1,2}$ and $k_{2,1}$. Miniature voice-coil actuators are used as the primary and control actuators. These components also act as the dampers c from Figure 1, since the air-gap damping of the voice-coils is found to be much more significant when compared to the structural damping. The charge-output accelerometers nested within the blocks representing the lumped masses act as sensors. The output of the accelerometers is processed by a conditioner, which is equipped with an analogue integrator, allowing velocity signals to be obtained.

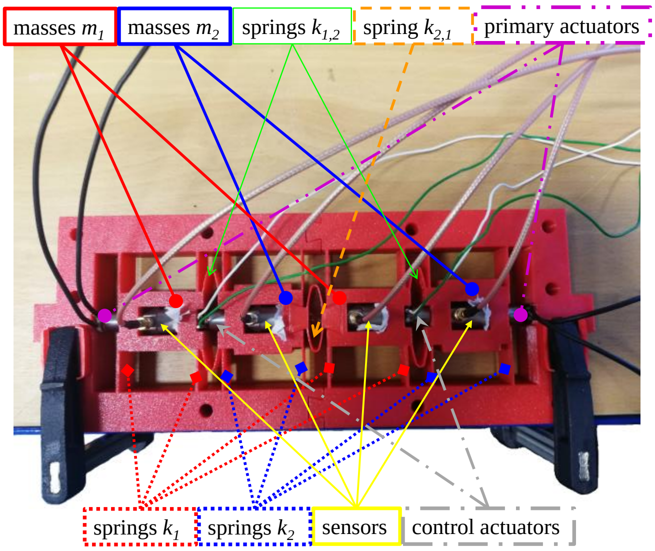

Figure 2: 3D printed experimental setup used to validate the theoretical concept of the active metamaterial cell

Since feedback loops are in a non-collocated configuration, in general, closed-loop stability is not guaranteed for all gains g [5]. However, results from [8] show that if this particular mechanical subsystem configuration is carefully designed, such that inequalities $m_2 > m_1$ and $\sqrt{k_2/m_2} < \sqrt{k_1/m_1}$ hold, the closed loop system can in fact be unconditionally stable with respect to the feedback gain g. This can be accomplished assuming that the sensors and actuators are ideal, i.e. that they do not possess any dynamic behaviour of their own. On the other hand, both the sensors and actuators are dynamical systems themselves, and as such contribute to the dynamic behaviour of the system as a whole. Due to this, in a setup where the sensors and actuators are not ideal, tuning the mechanical system parameters such that they satisfy the aforementioned inequalities does not result in an unconditionally stable system.

Stability and performance analysis

Since the 3D printed components of the experimental setup are custom-made, their properties (masses and stiffnesses) were determined experimentally, while the properties of both sensors and actuators were gathered from their respective datasheets. All of the relevant properties used in the mathematical model of the metamaterial cell are listed in Table 1.

Table 1: Active metamaterial cell parameters

parameter	value	units
m_1	0.045	kg
m_2	0.06075	kg
k_1	55400	Nm^{-1}
k_2	9100	Nm^{-1}
$k_{1,2}$	18150	Nm^{-1}
$k_{2,1}$	17850	Nm^{-1}
c	0.8	Nsm^{-1}
T	0.45	$\mathrm{NA}^{-1}, \mathrm{Vsm}^{-1}$
L	63×10^{-6}	H
R	1.5	Ω
ω_m	$2\pi\times42\times10^{3}$	rads^{-1}
ζ_m	0.00158	(dimensionless)
g	300	Vsm^{-1}

The inputs to the system are considered to be the voltages at the primary actuators e_1 and e_4 (see Figure 1 and Figure 2), while the outputs are the velocities of each of the degrees of freedom. The velocities $\mathrm{d}q_1/\mathrm{d}t$ and $\mathrm{d}q_4/\mathrm{d}t$ (see Figure 1) are of particular interest, since they represent the vibration response at the boundaries of the system. An input-output relationship is therefore established between these quantities in the form of a frequency response function matrix $\mathbf{G}(j\omega)$ with dimensions 4×2 (since there are four measured outputs and two inputs), where matrix entries $G_{1,2}(\mathrm{j}\omega)$ and $G_{4,1}(\mathrm{j}\omega)$ are used as functions characterising the propagation of vibration from right to left and vice-versa (see Figure 1 and Figure 2). On the other hand, the closed-loop stability properties of a system with multiple inputs and outputs may be analysed utilising various methods [9]. Here, the generalised Nyquist stability criterion is used, where an open-loop sensor-actuator frequency response function is derived and its eigenvalues are analysed using the single-input single-output Nyquist criterion. In this case, since there are two sensor-actuator pairs, the open-loop sensor-actuator frequency response function has two eigenvalues.

A detailed development of the mathematical model representing the active metamaterial cell, utilising the numerical values from Table 1 may be found in [8].

The closed-loop stability of the experimental setup is evaluated by forcing the system using a white noise input voltage at either of the control actuators, measuring the response of sensors mounted to masses m_2 and assembling the sensor-actuator open-loop frequency response function matrix. Then, one may extract its eigenvalues and analyse them using the ordinary, SISO, Nyquist stability criterion. After the suitable feedback gain g is found, which ensures a ~6 dB gain margin (given in Table 1), the performance of the system with the feedback loops closed may be evaluated. This is done by applying white noise excitation voltage at the terminals of the primary actuators and measuring the velocity response of each of the masses. This is done both with and without feedback active, in order to compare the responses of the passive and activated system.

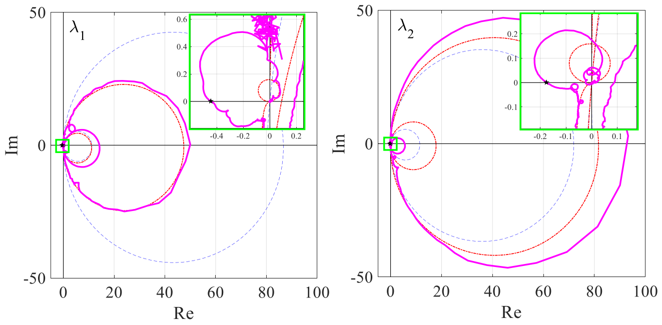

Figure 3: Nyquist contours of the eigenvalues λ_1 and λ_2 of the sensor-actuator open-loop frequency response function. Faint dashed blue line – model without sensor-actuator dynamics, solid dash-dotted red line – model with sensor-actuator dynamics, thick magenta line – experimental data. The star represents the point at which the experimentally curated contours cross the negative real axis

Figure 3 shows the Nyquist contours of eigenvalues of the sensor-actuator open-loop frequency response function for both the mathematical models and the experimental setup. The model including sensor-actuator dynamics in particular predicts the behaviour of the experimental system quite well. In order to better grasp the stability properties of the system, the amplitude and phase plots of the eigenvalues are given in Figure 4 and Figure 5. From these plots it is evident that the model which considers ideal transducers remains stable, since the phase never reaches -180°. The model where the transducer dynamics are taken into account however, predicts that the phase will cross -180° at a frequency just below the natural frequency of the sensor, but even so the amplitude remains very small (-36.22 dB), and therefore the model predicts a very high gain margin. On the other hand, while the behaviour of the experimental setup is modelled well up to about 1kHz, at higher frequencies the model and data start to diverge. In particular, the experimental setup is noted to have a lower gain margin (7 dB) than the mathematical model, and the phase crosses -180° at a lower frequency. The deviation between the responses of the model and experiment are explained by higher order vibrational modes present in the flexible structure, which are not modelled. This is also a likely cause of the much smaller gain stability margin in the experimental setup, as the finite dimensional controller interacts with a system having infinite vibration modes [10].

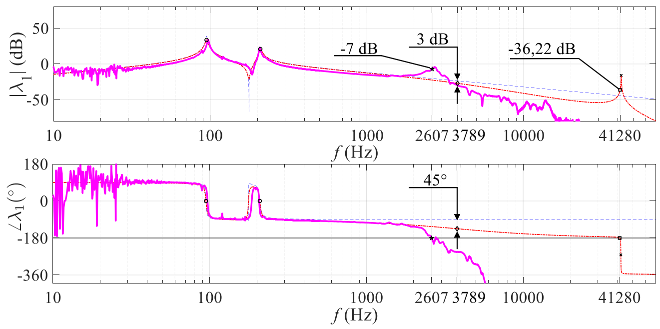

Figure 4: Bode plots of the first eigenvalue of the sensor-actuator open-loop frequency response function. Faint dashed blue line – model without sensor-actuator dynamics, solid dash-dotted red line – model with sensor-actuator dynamics, thick magenta line – experimentally curated data. Circles indicate the natural frequencies of the mechanical system, "×" indicates the natural frequency of the sensor, the diamond indicates the cut-off frequency of the actuator, while the square and star indicate the frequencies at which the phase crosses -180° for the model and experimental data respectively

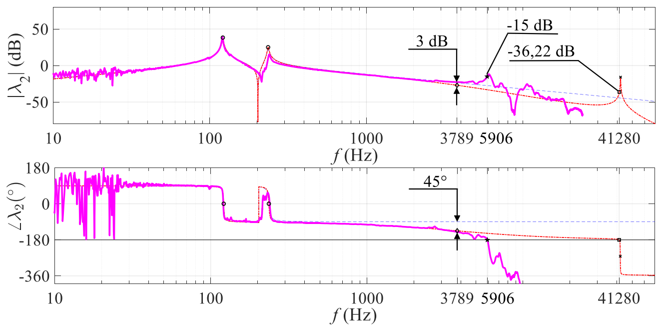

Figure 5: Bode plots of the second eigenvalue of the sensor-actuator open-loop frequency response function. Faint dashed blue line – model without sensor-actuator dynamics, solid dash-dotted red line – model with sensor-actuator dynamics, thick magenta line – experimentally curated data. Circles indicate the natural frequencies of the mechanical system, "×" indicates the natural frequency of the sensor, the diamond indicates the cut-off frequency of the actuator, while the square and star indicate the frequencies at which the phase crosses -180° for the model and experimental data respectively

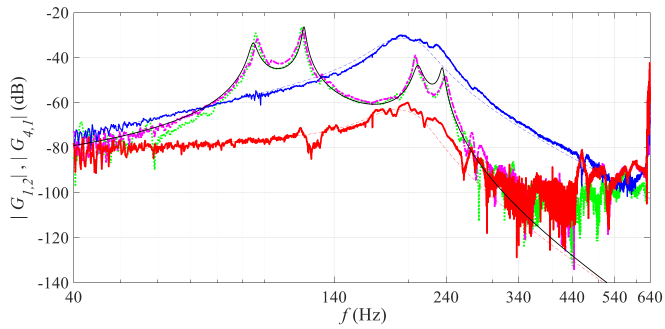

Figure 6: Comparison between the characteristic frequency response functions of the active metamaterial cell: solid black line – model, passive ($G_{1,2} = G_{4,1}$), green dotted line – experimental, passive ($G_{1,2}$), magenta dash-dotted line – experimental, passive ($G_{4,1}$), thick red line –experimental, active ($G_{4,1}$), thick blue line – experimental, active ($G_{1,2}$), faint dashed red line – model, active ($G_{4,1}$), faint dashed blue line – model, active ($G_{1,2}$)

From Figure 6 it is clear that the passive system exhibits reciprocal behaviour, as the two characteristic frequency response functions coincide at all frequencies. However, after the feedback loop is closed, the responses change drastically, as the difference in their amplitudes reaches 30 dB in the resonance controlled range, and increases further at higher frequencies (the mass controlled range). The behaviour is predicted quite well by the mathematical model, however at higher frequencies the measured amplitudes become quite small, and therefore reach the noise-floor, preventing the extraction of any significant information.

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Conclusion

Utilising two non-collocated decentralised velocity feedback loops, an active metamaterial cell, which does not exhibit reciprocal behaviour is developed and experimentally validated. Even though the dynamic behaviour of the transducers limits the stability properties of the closed-loop system, careful design of the mechanical subsystem allows for relatively large feedback gains to be implemented, while still retaining acceptable stability margins. The loss of reciprocity in the structure is evident by large differences in vibration transmission in the two characteristic directions of the system. These differences are present in a wide frequency band and reach 30 dB at frequencies in the resonance controlled range. The results from this research show the possibility of applying such control schemes in the design of active acoustic metamaterials. Such systems could then be tuned both in terms of their mechanical subsystem parameters (in order to bolster their closed-loop stability), as well as in terms of the gain of the controller in order to achieve non-reciprocal sound transmission.

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