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Mixed Logical Dynamical Modelling of a Stratified Storage with Phase-Change Material ⋆

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Abstract: Thermal storages that use phase-change materials are increasingly applied in heating systems with renewable energy sources. Their main advantage is storing energy of a high density while keeping the storage at lower temperatures. To exploit the full potential, optimal control is applied, which requires an appropriately designed model of the heating system. This paper proposes a model of the latent stratified storage in a mixed logical dynamical form. The proposed model is developed and verified on a real heating pilot system with a latent heat storage. Thermodynamic behaviour of this system is simulated, both by using mixed logical dynamical model and a non-linear thermodynamical model. Obtained results show high accuracy of the models.

Keywords: Heating system, stratified thermal energy storage, phase-change material, mixed logical dynamical modelling, mixed-integer linear program

1. INTRODUCTION

Global industrial direct CO_2 emissions due to energy consumption increased by 13% between 2008 and 2018 as given in REN21 (2021). Because of that, numerous policies and initiatives have been created with a goal to increase energy efficiency. One of them surely is EU 2030 Energy Strategy that aims to replace 20-20-20 targeted ratio of CO_2 reduction, renewables shares and energy savings into a 40-27-27 ratio by the year of 2030. All parts of this strategy rely on reducing the energy waste. This can be achieved by filling the gap between energy production and energy consumption by using Thermal Energy Storages (TES). They allow to store the energy generated in favourable conditions when there is no demand for it. Also, if an energy production becomes expensive or inefficient, the energy can be delivered to a system from a TES, which leads to better energy efficiency and economic savings. Furthermore, TES is an important factor in usage of Renewable Energy Sources (RES) because it can smooth out the intermittent production of RES. If a TES is connected to a renewable source and it is used together with another conventional energy source, then reduction of site peak demand is achievable as it is shown in OMNIBUS (2015).

Li et al. (2017) classified TES by material used in them into sensible, latent and thermochemical storages. Sensible storages are filled with a high thermal mass medium, usually water. Latent storages, besides water, also contain a phase-change material (PCM) such as paraffin, salt hydrates or thermoplastic polymers. The PCM enables energy transfer at a constant temperature and it manages to store a large amount of energy at lower temperatures in comparison to water. Thermochemical heat storages utilize reversible chemical processes to keep storage at a constant temperature while energy exchange occurs.

Recently, an emphasis is placed on latent TESs, which use PCM to store energy. Pielichowska et al. (2014) show that their great advantage is providing energy of high density while keeping the TES at lower temperatures. The choice of material that is used in this type of storage plays a major role in achieving high energy efficiency. Some of the material properties required are: good heat transfer, high density, small volume change, long-term chemical stability, no toxicity and no fire hazard (see Sharma et al. (2007)).

A generic method for obtaining a simple but effective model for latent storage is not created yet. Touretzky et al. (2015) based their model on temperature-enthalpy correlation. They used enthalpy as a function of time because it is continuous during the phase change process. This model contains a storage that is not stratified, which leads to lower physical accuracy caused by representing the whole mass of PCM with a single temperature variable. Such models are suitable for employing a mixed-integer approach in modelling. For more complex thermal systems this however becomes computationally very intensive and it cannot be done on-line because it depends on discretizing difference equations using finite difference approximations and dealing with a large number of IF-THEN statements. Finite-volume model of a PCM integrated in Building Air-Distribution Systems is presented by Jiang et al. (2020). The model is obtained by iden-

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tifying PCM enthalpy, and mixed-integer formulation of the enthalpy-temperature relationship is established by introducing auxiliary variables that decide phase of the material. Mohamed et al. (2017) made geometry model of latent storage using meshing methods. Obtained models are physically very precise, but they are suitable for simulation purposes only. Including them into a thermal system control would significantly extend the control algorithm computation time.

Heat sources of varying efficiency and availability, dynamically changing energy market conditions and ancillary services provision necessitate a highly dynamic thermal storage response to gain maximum economical benefit while ensuring heat supply to the consumption points connected to it. To control such a storage, suitable mathematical model is needed. In this paper, a procedure of modelling stratified storage with PCM is presented. By applying this procedure latent storage model is written in a Mixed Logical Dynamical (MLD) form using rules proposed by Bemporad et al. (1999). The main contribution of this procedure is creating a model that is suitable for Model Predictive Control (MPC), but is still physically very accurate. Moreover, this paper shows a comparison between simulations obtained by the obtained MLD model and the initial non-linear thermodynamical model.

The work is organized as follows. In Section II, stratification of storage is explained and thermodynamic processes are examined. Section III presents the procedure of modelling PCM and partial system submodels. In section IV, a pilot example of a thermal energy system with two different types of heat sources and a latent TES is described. Also, simulations of the obtained model are shown. Section V concludes the paper.

2. THERMAL ENERGY STORAGE

A TES is usually cylindrically shaped tank filled with water, often made of steel and well insulated from the outside air. Energy is transferred to the storage from a heat source that is either supplied from utility grids or from locally available primary energy sources. The TES supplies thermal loads at needed medium flow and supply temperature. The time change in thermal energy of the storage is equal to the difference between inlet and outlet energies.

2.1 Stratification of TES

Storage stratification is used to achieve better accuracy of the mathematical model. It is done by dividing a single tank into several layers by height. Usually, more layers lead to a better model accuracy. The number of layers is not limited, but using an inappropriately large number of layers results in a more complex model and prolongs the calculation time.

2.2 Mathematical model of a latent TES

Mathematical model of a latent TES describes various heat transfers that occur in a tank. It can be applied on a stratified storage that has at least two layers. Every layer is expressed by three differential equations. One

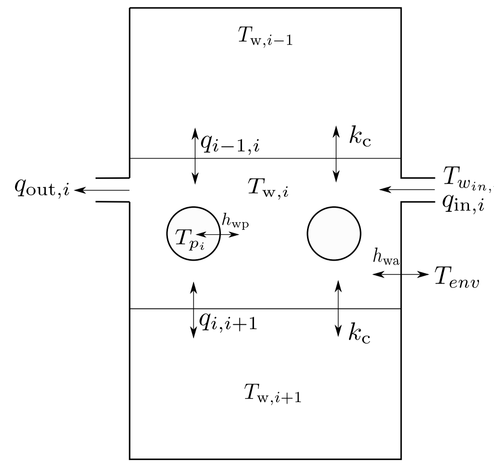

Fig. 1. Stratification of latent TES

of them represents temperature of the water, while the remaining ones stand for PCM's stored latent energy and temperature. It is sometimes the case in the latent TES that some of the layers do not contain PCM due to the tank shape. Also, the reason could be a barrier that separates the PCM layers from the non-PCM layers. So, the number of difference equations for the physical model given in this paper is:

$$\text{number of eq.} = 3 \text{ x PCM layers} + \text{non-PCM layers}. \quad (1)$$

Equations (2)-(4) represent the i-th layer of a tank that includes a PCM, such as the one from Fig. 1. The model physical parameters are described in Table 1. To make the model more accessible to use, all heat transfers are one-dimensional. The overall change of volume due to the phase transition is neglected as it is proposed by Sharma et al. (2007). All thermophysical properties of PCMs are constant and each PCM is isotropic and homogenous. Furthermore, PCMs densities and conductivities are equal for all phases as it is stated by Wang et al. (1999).

$$\begin{aligned} m_{w_i} c_w \frac{dT_{w_i}}{dt} &= q_{in,i} c_w T_{w_{in,i}} - q_{out,i} c_w T_{w_i} \\ &+ \delta_{i-1,i} q_{i-1,i} c_w T_{w_{i-1}} \\ &+ (1 - \delta_{i-1,i}) q_{i-1,i} c_w T_{w_i} \\ &- \delta_{i,i+1} q_{i,i+1} c_w T_{w_i} \\ &- (1 - \delta_{i,i+1}) q_{i,i+1} c_w T_{w_{i+1}} \\ &- k_c \frac{A_h}{\Delta x_{i,i-1}} (T_{w_i} - T_{w_{i-1}}) \\ &- k_c \frac{A_h}{\Delta x_{i,i+1}} (T_{w_i} - T_{w_{i+1}}) \\ &- h_{wa} A_{wa_i} (T_{w_i} - T_{env}) \\ &- h_{wp} A_{wp_i} (T_{w_i} - T_{p_i}), \end{aligned} \quad (2)$$

$$\frac{dE_{lat_i}}{dt} = \delta_i h_{wp} A_{wp_i} (T_{w_i} - T_{p_i}), \quad (3)$$

$$m_{p_i} c_p \frac{dT_{p_i}}{dt} = (1 - \delta_i) h_{wp} A_{wp_i} (T_{w_i} - T_{p_i}). \quad (4)$$

Equations (2)-(4) capture the dynamics of: T_{w_i}, E_{lat_i}, and T_{p_i}, which are water temperature, latent phase-change energy stored in the PCM and the PCM temperature, respectively for the i-th layer. Water temperatures of the layer above ($T_{w_{i-1}}$) and the layer below ($T_{w_{i+1}}$) also have effect on the i-th layer. Variable $q_{in,i}$ represents the water flow that enters the storage from the grid and $T_{w_{in,i}}$

represents the temperature of the inlet water. Also, $q_{out,i}$ stands for the water flow that leaves the i-th layer. Unique inner flow exists between every two neighbouring layers (e.g. $q_{i-1,i}$) and all are determined by:

$$\begin{bmatrix} q_{1,2} \\ \vdots \\ q_{i-1,i} \\ q_{i,i+1} \\ \vdots \\ q_{L-1,L} \end{bmatrix} = \begin{bmatrix} 1 & -1 & \dots & -1 & -1 & -1 & \dots & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & -1 & -1 & \dots & -1 & -1 \\ 1 & 1 & \dots & 1 & 1 & -1 & \dots & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & -1 \end{bmatrix} \begin{bmatrix} q_{in,1} \\ q_{in,2} \\ \vdots \\ q_{in,i-1} \\ q_{in,i} \\ q_{in,i+1} \\ \vdots \\ q_{in,L-1} \\ q_{in,L} \end{bmatrix}$$

$$+ \begin{bmatrix} -1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & -1 & \dots & -1 & 1 & 1 & \dots & 1 & 1 \\ -1 & -1 & \dots & -1 & -1 & 1 & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & -1 & \dots & -1 & -1 & -1 & \dots & -1 & 1 \end{bmatrix} \begin{bmatrix} q_{out,1} \\ q_{out,2} \\ \vdots \\ q_{out,i-1} \\ q_{out,i} \\ q_{out,i+1} \\ \vdots \\ q_{out,L-1} \\ q_{out,L} \end{bmatrix}. \quad (5)$$

Auxiliary variable $\delta_{i-1,i}$ determines direction of the inner flow between the i-th layer and the (i-1)-th layer. Modelling of inner flow direction logic is explained in the subsection 3.3. Heat transfer between different layers is described by thermal conduction with the coefficient k_c. Thermal convection describes energy transfer between water and PCM of each layer which depends on h_{wp}. At every moment, tank loses energy by dissipating it through the storage envelope to the outside wall. Integer variable δ_i assures transitions from solid phase to liquid and vice versa. Considering conditions that are described in the next section, it decides which of the two PCM's variables is changeable: energy or temperature. Constants used in the model are shown in Table 1.

Table 1. Model parameters

Parameter	Unit	Description
m_{w_i}	kg	mass of water in layer i
m_{p_i}	kg	mass of PCM in layer i
k_c	W/(mK)	conduction coefficient of water
h_{wa}	W/(m^{2}K)	convection between water and air
h_{wp}	W/(m^{2}K)	convection between water and PCM
Δx	m	length between centers of layers
A_h	m^2	contact surface between layers
A_{wa}	m^2	contact surface between water and air
A_{wp}	m^2	contact surface between water and PCM

While PCM layers are modelled by three equations, non-PCM layers are expressed with only one. Equation that represents the temperature of the water of non-PCM layer is obtained by excluding parts related to the PCM from (2), such as $h_{wp}A_{wp_i}(T_{w_i} - T_{p_i})$ in this case.

3. MIXED LOGICAL DYNAMICAL MODELLING

Heating system nonlinearities are mostly caused by binary nature of PCM and inner flows, and also by system relays. These nonlinearities are modelled by using MLD modelling.

3.1 Phase-change material modelling

Regardless of the material that is used in the latent TES, here we propose a general model for PCM dynamics in the framework of MLD systems. This model distinguishes 2 states of the PCM:

- State 1 (phase change): The latent energy is changeable while the temperature is constant.
- State 2 (no phase change): The temperature is changeable while the latent energy is constant.

Integer variable δ_p decides which of the states is currently active. If the value of δ_p is 1 then state 1 is active, while state 2 is active when the value of δ_p is 0. Constant T_{chg} denotes the temperature at which the phase change occurs, while E_{max} stands for required latent energy for the full phase transition (complete solid to liquid). Model variable E_{lat} is an obvious indicator of the PCM phase. Non-positive value of E_{lat} means that the PCM is solid, while it is liquid when E_{lat} exceeds E_{max}. After all, state 1 is active if: T_p is higher than T_{chg} while PCM is fully solid, T_p is lower than T_{chg} while PCM is fully liquid or if the value of E_{lat} is somewhere in the range between zero and E_{max}. These can be written with two conditions, where δ_1 and δ_2 are auxiliary binary variables:

$$T_p \geqslant T_{chg} \quad \text{AND} \quad E_{lat} < E_{max} \Rightarrow \delta_1 = 1, \qquad (6)$$

$$T_p \leqslant T_{chg} \quad \text{AND} \quad E_{lat} > 0 \Rightarrow \delta_2 = 1. \qquad (7)$$

To write these conditions in a form of a MLD model, firstly we define auxiliary binary variables δ_T, δ_{E1}, and δ_{E2}. Parameter δ_T is 1 when temperate of PCM is lower than T_{chg}, δ_{E1} is 1 when PCM's energy is non-positive number and finally δ_{E2} is 1 when E_{lat} is not greater than E_{max}. By the rules of the MLD that are proposed by Bemporad et al. (1999), following inequalities are given:

$$T_p - T_{chg} \leqslant M_T(1 - \delta_T), \qquad (8)$$
$$T_p - T_{chg} \geqslant \epsilon + (m_T - \epsilon)\delta_T, \qquad (9)$$
$$E_{lat} \leqslant M_{E1}(1 - \delta_{E1}), \qquad (10)$$
$$E_{lat} \geqslant \epsilon + (m_{E1} - \epsilon)\delta_{E1}, \qquad (11)$$
$$E_{lat} - E_{max} \leqslant M_{E2}(1 - \delta_{E2}), \qquad (12)$$
$$E_{lat} - E_{max} \geqslant \epsilon + (m_{E2} - \epsilon)\delta_{E2}, \qquad (13)$$

where constants M_T, M_{E1}, and M_{E2} stand for the maximum value of the left sides of inequalities, while m_T, m_{E1}, and m_{E2} imply their minimum. Constant ϵ is a small tolerance, typically the machine precision. Variables δ_1 and δ_2 are unambiguously determined by following inequalities:

$$-(1 - \delta_T) + \delta_1 \leqslant 0, \qquad (14)$$
$$-\delta_{E2} + \delta_1 \leqslant 0, \qquad (15)$$
$$1 - \delta_T + \delta_{E2} - \delta_1 \leqslant 1, \qquad (16)$$
$$-\delta_T + \delta_2 \leqslant 0, \qquad (17)$$
$$-(1 - \delta_{E1}) + \delta_2 \leqslant 0, \qquad (18)$$
$$\delta_T + (1 - \delta_{E1}) - \delta_2 \leqslant 1. \qquad (19)$$

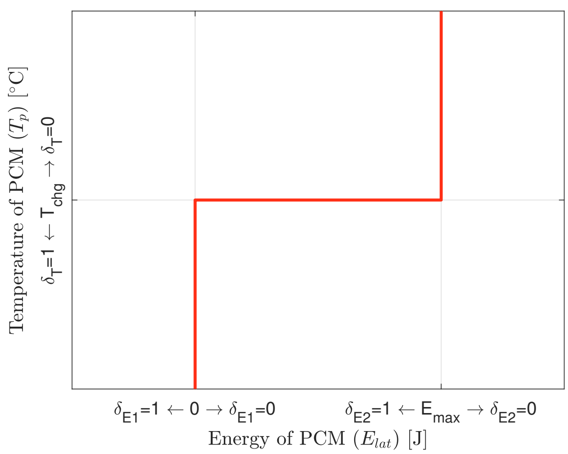

Fig. 2. PCM temperature and energy relation

Finally, in the case when either (6) or (7) is satisfied, state 1 is active, which is described by equation (20).

$$\delta_p = \delta_1 + \delta_2. \tag{20}$$

Relation between auxiliary variables that are used in this model can be seen in Fig. 2.

3.2 Partial submodels decision

Thermal systems often include some ON/OFF units e.g. valves, pumps, fan-coils which create nonlinearity. Moreover, PCM also has a binary nature of behavior as it is described in the previous subsection. In this paper, we use a decomposition approach to divide one large model into many smaller ones where each of them is linear. Number of partial submodels (S) depends on the number of combinations that integer variables create. Latent storages have integer variable that represent PCM state (δ_{p_i}) for each layer, but besides that they also include integer variables which decide the direction of inner flow, variables that describe openness of valves, etc. Number of submodels is always greater or equal to the number of decision variables (D). For each submodel, ZOH discretization is made, whereupon model is written in a state-space form. System variables in every time step k are represented by a vector z_k which is described by the following equation:

$$z_k = z_{k,1} + z_{k,2} + ... + z_{k,S}, \quad k \in [0, N]. \tag{21}$$

Only one submodel ($z_{k,i}$) is active at a time and it represents real system values while others include zero elements. Decision of choosing right submodel can be problematic. This problem can be solved using MLD which is proposed here and is based on Bemporad et al. (1999). Model that is, for example, active when all decision variables (δ_1, δ_2,...,δ_D) are equal to 1 is expressed by the following inequalities:

$$z_{k,i} \leqslant \delta_1 M, \tag{22}$$
$$z_{k,i} \geqslant \delta_1 m, \tag{23}$$
$$z_{k,i} \leqslant \delta_2 M, \tag{24}$$
$$z_{k,i} \geqslant \delta_2 m, \tag{25}$$
$$\vdots$$
$$z_{k,i} \leqslant \delta_D M, \tag{26}$$
$$z_{k,i} \geqslant \delta_D m, \tag{27}$$
$$z_{k,i} \leqslant A_{k-1,i} z_{k-1} + B_{k-1,i} u_{k-1} ... \\ ... - m[(1-\delta_1) + (1-\delta_2) + ... + (1-\delta_D)], \tag{28}$$
$$z_{k,i} \geqslant A_{k-1,i} z_{k-1} + B_{k-1,i} u_{k-1} ... \\ ... - M[(1-\delta_1) + (1-\delta_2) + ... + (1-\delta_D)], \tag{29}$$

where vector M contains constant values that are higher than a maximum of $z_{k,i}$, while values of m need to be lower than a minimum of $z_{k,i}$ divided by D. Furthermore, M is not allowed to contain negative numbers as well as m is not allowed to contain positive. Reason for this is not related to the physical model, but to the modelling logic. In theory, infinitely large values can be used in these vectors. However, that is not practical because this approach extends allowed hyperspace, which ultimately leads to the slower computations. Vector z_{k-1} used here is actually written using (21). $A_{k-1,i}$ and $B_{k-1,i}$ are suitable state-space matrices of the i-th submodel in the $(k-1)$-th time step. Finally, vector u represents system inputs (thermal power, supplying water temperature, etc.).

As it is already explained, M is not allowed to be a negative number as well as m is not allowed to be positive. This constraints can easily lead to a model limitation, because lower bounds of the system variables are usually required to be positive numbers (e.g. minimum temperature of supplying water in a heating system is certainly higher than 0°C). This problem can be solved by adding constraint given in (30), where z_{min} is a constant vector of system minimums (physical limitations, equipment limitations, etc.). The same principle is applied on model's upper bound.

$$z_{max} \geqslant z_{k,1} + z_{k,2} + ... + z_{k,S} \geqslant z_{min}, \quad k \in [0, N]. \tag{30}$$

3.3 Inner flows modelling

Inner flows of the storage are modelled for every pair of neighbouring layers. E.g. variable $\delta_{i-1,i}$ decides does water flow from $(i-1)$-th layer to the i-th layer ($\delta_{i-1,i} = 1$) or in the opposite direction ($\delta_{i-1,i} = 0$) by following inequalities:

$$q_{i-1,i} \leqslant -\epsilon + (\epsilon + M_{i-1,i})\delta_{i-1,i}, \tag{31}$$
$$q_{i-1,i} \geqslant M_{i-1,i}(\delta_{i-1,i} - 1), \tag{32}$$

where $q_{i-1,i}$ is actual inner flow and it is calculated by (5) and $M_{i-1,i}$ is maximal value of the corresponding inner flow.

4. CASE STUDY

The MLD modelling proposed in this paper is used to create a suitable mathematical model for a system shown in Fig. 3 that is actually based on a real pilot library building in Lendava, Slovenia. As it can be seen in Fig. 3, the system consists two energy sources. One of them is conventional (oil boiler), while the other one is renewable

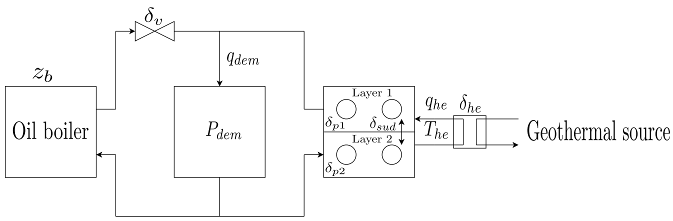

Fig. 3. Heating system in pilot building

(geothermal energy source). Considering that geothermal energy is more affordable than the oil boiler energy and also CO_2 free, the emphasis is put on using it as often as possible. However, the temperature of the geothermal side depends on many natural factors and sometimes it is too low, so the geothermal source cannot be used directly. Because of that, a better utilization of geothermal source is achieved by using MPC, which requires a suitable model, like the one proposed in this paper.

It is possible that these sources deliver energy simultaneously, while there is always an option to use each of them separately. A heat-exchanger connects geothermal grid with the four identical latent storages, as it is described in section 4.1., from which hot water flows to the building system at a demanded flow (q_{dem}). In the case when valve is open, one half of the demanded flow is delivered from the latent TES while another is covered by the boiler. To simplify the model, it is mathematically correct to represent four storages as one large storage that is stratified by height in two layers. System is expressed by 7 differential equations following the rules that are described in section 2.2, while one additional equation stands for the oil boiler temperature. Besides that, two more equations that describe the energy cost are added in sections 4.1 and 4.2. Finally, the obtained mathematical model contains five integer variables (δ_{he}, δ_v, δ_{sud}, δ_{p1} and δ_{p2}). δ_{p1} and δ_{p2} represent the state of the PCM in two layers of the tank, as it is shown in section 3.1., while the other are described in the following text. At the end, by applying the method proposed in section 3.2, 26 linear submodels are obtained by the combinations of these five variables. Although combining five integer variables leads to 32 submodels, some of combinations are not physically achievable. Therefore, these submodels are excluded from consideration.

4.1 Geothermal source model

Hot water is supplied from the geothermal source to the heat exchanger at a temperature that needs to be higher than the temperature of water in the storage, so that the geothermal source can be useful. Water from the bottom layer of the tank enters the exchanger and returns to the top layer, after it is warmed up, at the constant flow of q_{he}. The heat exchanger provides power P_{he}, so the temperature of geothermal water (T_{he}) is limited with:

$$T_{he} \leqslant T_{w_2} + \frac{P_{he}}{c_w q_{he}}, \tag{33}$$

where T_{w_2} stands for water temperature of the bottom layer and c_w is the specific heat of water. Therefore, differential equation (34) that calculates the amount of geothermal energy that is spent (E_{ge}), is added to the model, where δ_{he} describes ON/OFF state of the heat exchanger and η_{he} is the heat exchanger efficiency.

$$\frac{dE_{ge}}{dt} = \delta_{he} q_{he} c_w \frac{1}{\eta_{he}} (T_{he} - T_{w_2}). \tag{34}$$

4.2 Oil boiler model

Oil boiler activation/deactivation is determined by integer variable δ_b. It is modelled as a disjunctive power source, which arises from the fact that if it is turned on, its power cannot be lower than P_{min}. Furthermore, new variable of boiler power Z_b is created by expressions:

$$Z_b \leqslant P_{max} \delta_b, \tag{35}$$

$$Z_b \geqslant P_{min} \delta_b, \tag{36}$$

$$Z_b \leqslant P_b - P_{min}(1 - \delta_b), \tag{37}$$

$$Z_b \geqslant P_b - P_{max}(1 - \delta_b), \tag{38}$$

where $Z_b \in \{0 \cup [P_{min}, P_{max}]\}$ and $P_b \in [P_{min}, P_{max}]$. Then the energy consumed by the oil boiler (E_b) is simply:

$$\frac{dE_b}{dt} = Z_b. \tag{39}$$

4.3 Valve model

Valve openness is determined by δ_v and it depends on the status of the oil boiler and on the temperature difference between boiler's outlet and inlet water temperatures ($T_{b_{out}}$ and $T_{b_{in}}$). If the boiler is turned on than the valve is open. If the boiler is turned off and the differences between temperatures is higher than Δ then the valve retains its previous state, otherwise in the case when the difference is lower, the valve is closed. It can be modelled with the following expressions:

$$f = T_{b_{out}} - T_{b_{in}} - \Delta, \tag{40}$$

$$\delta_{v,k} \geqslant \delta_{b,k}, \tag{41}$$

$$(M_v + \epsilon)\delta_{v,k} \leqslant (M_v + \epsilon)(1 + \delta_{b,k}) + f - \epsilon, \tag{42}$$

$$M_v \delta_{v,k} \leqslant M_v(1 + \delta_{b,k} + \delta_{v,k-1}) - f, \tag{43}$$

$$M_v \delta_{v,k} \geqslant -M_v(1 + \delta_{b,k} - \delta_{v,k-1}) - f, \tag{44}$$

where M_v is the maximal predictable temperature difference between outlet and inlet water and $k \in [0, N-1]$.

4.4 Storage inner flow model

Storage inner flow describes direction of water flow inside the storage and it is determined by variable δ_{sud}. This variable is easily obtained by using δ_{he} and δ_v within inequalities:

$$-q_{he}\delta_{he} - \frac{q_{dem}}{2}\delta_v + M_{sud}\delta_{sud} \leqslant M_{sud} - q_{dem}, \tag{45}$$

$$q_{he}\delta_{he} + \frac{q_{dem}}{2}\delta_v + (m_{sud} - \epsilon)\delta_{sud} \leqslant -\epsilon + q_{dem}, \tag{46}$$

where M_{sud} and m_{sud} are the maximal and the minimal possible inner flows, respectively.

4.5 Simulation results

Simulation for one day is performed, so comparison between the MLD model and the continuous non-linear model can be made. All input parameters and predictions (T_{he}, Z_b, P_{dem}, etc.) are chosen arbitrary to show the phase change of PCM (solid $\rightarrow$ liquid $\rightarrow$ solid). All figures

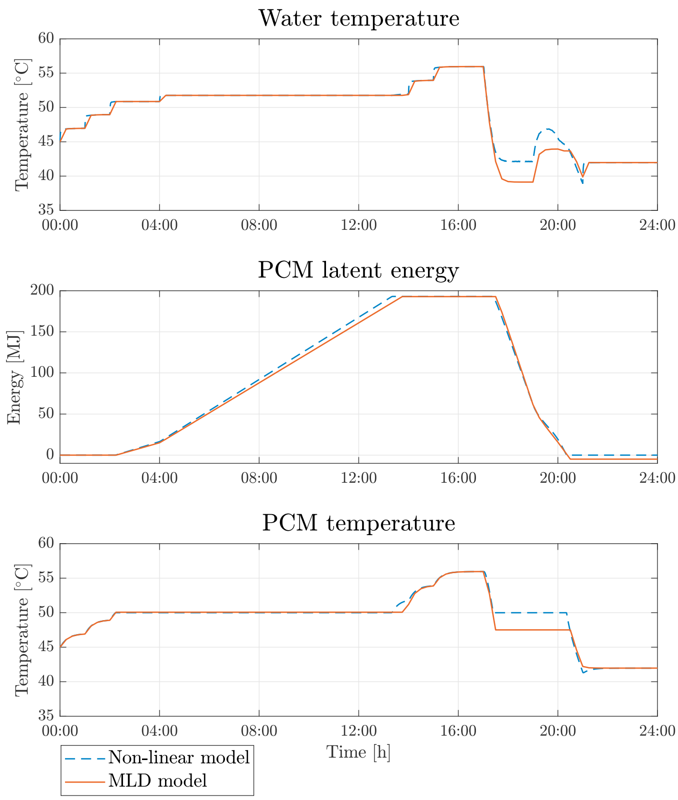

Fig. 4. Model comparison

present first layer of the thermal storage. MLD model is obtained by using the approach presented in this paper. Discretization time of 15 minutes is chosen, which is long enough to assure fast calculations, but still short to track the system dynamics. During one sampling time, all inputs are constant. It can be seen in Fig. 4 that MLD reponse accurately tracks the response obtained by the non-linear model with minimal delay. Figure 4 shows changing of the latent energy of PCM through the phase change. Increasing or decreasing of latent energy obtained by the non-linear model stops immediately when the latent energy reaches 0 or E_{max}. Latent energy of MLD model exceeds those limits because submodel decision wait for a next time step. However, these deviations are very small and can be neglected. Moreover, surplus or deficit of the energy will be compensated by the next phase change, so no free energy is granted. For the same reason, phase-changing temperature is not the same for both models. It can be seen in Fig. 4 that this temperature is exactly 50°C in the case of the non-linear model and around 50°C in the case of the MLD. After all, the model deviation is calculated by using root-mean-square error (RMSE). Due to the scale difference between temperature and energy, RMSE is normalized by using the difference between maximal and minimal value, so NRMSE is achieved. Both model deviations regarding water temperature, PCM temperature, and PCM energy from Fig. 4 are shown in Table 2.

Table 2. Model deviations

Variable	RMSE	NRMSE
T_w	0.94 °C	0.06
T_{pcm}	0.90 °C	0.06
E_{pcm}	3.88 MJ	0.02

5. CONCLUSION

The MLD modelling of heating system that contains latent heat storage is explained in this paper. The approach is used to obtain the model of a real heating system on a case study of library in City of Lendava, Slovenia. The proposed model is mathematically simple because it stratifies a tank in a minimal number of layers. Also, it allows existence of PCM in each layer. Results show that this model describes system dynamics very accurately, moreover normalized deviation is not higher than 0.07. The main advantage of this model is its suitability for usage in optimal control.

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