

Market Power of Coordinated Hydro-wind Joint Bidding: Croatian Power System Case Study

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Abstract— The paper analyses the coordinated hydro and wind power generation considering joint bidding in the electricity market. The impact of mutual bidding strategies on market prices, traded volumes, and revenues have been quantified. The coordination assumes that hydro generation is scheduled mainly to compensate differences between actual and planned wind power outputs. This coordination has exceptional potential to achieve market power and utilization of this market power is explored here. The market equilibrium of asymmetric generation companies is analyzed using a game theory approach. The assumed market situation is imperfect competition and non-cooperative game. A numerical approximation of the asymmetric supply function equilibrium is used to model this game. An introduced novelty is the application of an asymmetric supply function equilibrium approximation for coordinated hydro-wind generation. The model was tested using real input data from the Croatian power system.

Index Terms— Asymmetric firms, bidding strategy, coordination, hydro-wind, hydropower, imperfect competition market power, non-cooperative game, optimal bidding strategies, supply function equilibrium, wind power.

ACRONYMS

GAMS	General Algebraic Modelling System
MU	Monetary unit
NLP	Non-linear programming
SFE	Supply function equilibrium
SRMC	Short-run marginal cost

SYMBOLS

$c_i(y_{ik})$	Total short-run cost of the electricity generation for a firm i at the output y_i .
$c'_i(y_{ik})$	Short-run marginal cost of the electricity generation for a firm i at the output y_i .
D	Deterministic and price-insensitive (perfectly inelastic) part of the total demand, $D \in \mathbb{R}_+$.

$D(\varepsilon_k)$

$D_r(\pi_k, \varepsilon_k)$

ε_k

$\{1, \dots, I\}$

$k_{Tu,i}$

$n_{Tu,i}$

π_k

$\tilde{\pi}_{ik}$

$[\underline{\pi}, \bar{\pi}]$

s_i

s_f

y_{ik}

$\{1, \dots, K\}$

β_{ik}

ξ_{ik}

κ_{ik}^{Tu}

v_{ik}^{Tu}

ρ

Total electricity demand consists of deterministic part D and a stochastic part ε_k , $D(\pi_k) = D + \varepsilon_k$, (MW·h).

Residual demand curve for k^{th} outcome, which is obtained by subtracting the fringe supply function, $s_f(\pi_k)$, from total electricity demand $D(\varepsilon_k)$.

Demand shock is the demand value of the k^{th} outcome of some random variable defined for stochastic demand on probability space $(\Omega_\varepsilon, \mathcal{F}_\varepsilon, P_\varepsilon)$, $\varepsilon_k \in \mathbb{R}_+$.

Index set of electricity generation firms participating in the electricity market, $i \in \{1, \dots, I\}$.

Maximum power output of firm i , $k_{Tu,i} \in \mathbb{R}_+$ (MW).

Minimum power output of firm i , $n_{Tu,i} \in \mathbb{R}_+$ (MW).

Electricity price for k^{th} residual demand outcome, $\pi_k \in [\underline{\pi}, \bar{\pi}] \subset \mathbb{R}$.

Supply curve break (intersection) point $\tilde{\pi}_{ik} \in (\pi_{ik}, \pi_{ik+1})$.

The electricity price range at the analyzed market, $\forall \pi \in [\underline{\pi}, \bar{\pi}] \subset \mathbb{R}$.

Firm i supply/bidding curve $s_i : [\underline{\pi}, \bar{\pi}] \rightarrow [n_{Tu,i}^l, k_{Tu,i}^l]$.

Fringe supply curve $s_f : [\underline{\pi}, \bar{\pi}] \rightarrow [0, ATC]$.

Power output of firm i for k^{th} demand shock outcome (MW·h).

The index set of possible demand shocks (i.e. number of possible outcomes), $k \in \{1, \dots, K\}$.

The slope of the linear segment of supply curve at the equilibrium price π_k , $(\beta_{ik} = s'_i(\pi_k))$, $\beta_{ik} < \infty$.

The ancillary variable used to avoid strict inequalities which enable that $\tilde{\pi}_{ik}$ falls strictly within (π_k, π_{k+1}) , $0,0001 \leq \xi_{ik} \leq 0,9999$.

The shadow price related to the limits of the maximum output of the company at the k^{th} residual demand outcome, $\kappa_{ik}^{Tu} \in \mathbb{R}^+$ (€/MW·h).

The shadow price related to the constraints of the company's minimum power output at the k^{th} residual demand outcome, $v_{ik}^{Tu} \in \mathbb{R}^+$ (€/MW·h).

Right-hand side value of inequality constraints used for relaxation of the problem.

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I. INTRODUCTION

WHEN a hydro power plant is scheduled to balance the differences between actual and planned wind power outputs, coordination of hydro-wind participation in the electricity market is made possible. This coordination may have the opportunity to achieve and utilize market power by choosing optimal bidding strategies. The paper examines this phenomenon and the impact of these decisions on other electricity market participants. It has been shown that some beneficial operating strategies are emerging from short run coordinated hydro and wind generation. These advantages are reflected in lower, short run marginal costs (SRMC) of hydrogeneration [1]; consequently, this implies that the joint SRMC curves of coordinated hydro and wind generation are more competitive [2]. The paper also explores possibilities for potential synergies that may arise from this joint bidding. Similar operational synergies of different units or in different markets were studied in [3, 4]. These joint bidding strategies mainly arise from low marginal costs of hydro and wind generation on the one hand and operational synergies on the other hand. All of this implies that coordination has the potential to exploit market power to increase market revenue. An example of exerting market power by using capacity restraint is analyzed in [5].

Typically, bidding strategies and curves are based on the skills and expertise of traders and operators of the power units. Literature dealing with the systematic analysis and creation of optimal bidding strategies is extensive and mainly deals with optimality conditions intended for symmetric solutions (firms are identical competitors and offer the same supply curve) or usually implies price-taker participants. There are also examples where both price-maker roles (in the day-ahead market) and price-taker roles (balancing market) were studied [6]. A mixed approach assuming both symmetric supply function equilibrium (SFE) and asymmetric equilibria can be found in [7]. Challenges remain for models with asymmetrical firms (different cost functions) [8, 9]. An overview of mathematical models of imperfect competition in electricity markets can be found in [10, 11].

A. Related works

In this paper, the focus is on the electricity market state of imperfect competition and non-cooperative game. For this purpose, game theory is utilized and SFE is used because it is considered suitable for modelling competition in electricity markets. This part of the paper gives a very brief overview of articles which discuss this topic i.e., imperfect competition and market power in the electricity market. Compared to Cournot's models of competition, SFE is more suitable for electricity markets as it enables competing with supply curves (pairs of volumes and prices) rather than just with volumes. More about Cournot models and SFE in the electricity markets can be found in [12-16] and [7, 17-19] respectively. A comparison of the Cournot and SFE model is given in [20], where a test also provided results that concluded which model best suits the German electricity market of oligopolistic interaction--see more in [21]. In [22], SFE is applied to the case of an identical convex cost function (symmetric firms) and concave demand

curves. This was the first comprehensive analysis of SFE that showed existence of equilibrium in the case of uncertain elastic demand. This enabled the firm to ex-ante create supply curves in an environment of uncertain demand, which is the case in reality. This case was not applicable to electricity markets as they mostly consist of asymmetric firms and, almost perfectly, short-term inelastic demand.

The first case of solving the issue of short-term inelastic demand in SFE was found in [23]. The same author considered two aspects in [24]. The first was the potential impossibility of continuous equilibrium while an infinite set of discontinuous equilibria exists, while the other is convergence to a linear equilibrium through learning in a linear supply system. Consequently, in [25] it was shown that there is a unique solution of SFE in the case of inelastic demand and symmetric firms. The first step towards the asymmetric case was made in [26] where the simplest asymmetric firm problem was addressed and where the SRMC functions are identical but with different capacity constraints. Another important step towards asymmetric SFE was done in [27] where the constant SRMC functions framework was discussed. Another example of an SFE based model for utilizing market power by generation withholding can be found in [28]. Although SFE was significantly used in the analysis of the electricity markets [29], only the research from [17] made a real breakthrough in SFE in the analysis of the electricity markets because it solved two main problems. The first issue was finding an equilibrium solution in SFE problems, numerically or analytically, even when it is known to exist. The second issue was the problem of many possible equilibrium solutions, which resulted in the successive problem of choosing a certain equilibrium solution. The paper [17] proved that firms with capacity constraints in most cases have a unique equilibrium solution and, if there is a family of equilibrium solutions, then they form an ordered family. Due to the elimination of these problems, SFE has become a suitable tool for modelling competition in the electricity markets. In [30], the objective function regarding constraints on generation capacity and price cap (based on a monotonically increasing unique supply curve) was studied. The goal was to identify the Nash supply function equilibrium that maximizes firms' profit. The competition in the electricity market in England and Wales was analyzed by applying the Nash equilibrium in [31]. The same author in [32] analyzed the issue of increased competition in the electricity market in England and Wales with SFE (with linear supply functions of asymmetric firms). In [33] he also analyzed another example of SFE with linear supply functions in the contract electricity market in England and Wales. A similar approach of SFE was also applied in [34] and [35] in the case of electricity markets and mixed duopoly. Market power has traditionally been evaluated by Herfindahl - Hirschman Index (HHI) and Lerner Index (LI) [36]. HHI estimates market power with market concentration measures [37-40], while LI measures the deviation of price from a firm's marginal cost [41]. In [42] it is argued that the traditional HHI approach correlates poorly with market prices and proposes a new market power index - Residual supply index (RSI). RSI was also used in previous works [43] and [44].

B. The novelty of the paper

The novelty in this paper is application and adaptation of the asymmetric SFE approximation, developed in [17], for coordinated hydro-wind electricity market bidding. This coordination was already introduced in [2] but, more generally, it can also be suitable for thermo-wind coordination, with slight modifications. Based on this, an original model was developed and used for analyzing effects from the hydro-wind joint bidding strategy and its market power on the electricity market prices, traded volumes, its revenues, and the revenues of competing firms.

The model was tested on a real case using the input data of the Croatian power system. The developed procedure and findings of this paper could be generally useful whenever the impacts of bidding strategies on market prices, volumes, and revenues need to be quantified.

Also, this paper can provide support to those who need to analyze and quantify the potential of market power utilization for various types of firms with arbitrarily defined SRMC functions and capacity constraints.

In Section III, the numerical approximation of asymmetric SFE is briefly described and is applied to model competition in the Croatian electricity market. Section IV presents a case study with real input data from the Croatian power system to model competition. In this Section, the joint bidding strategies, market power analysis, and an analysis of the impacts on electricity market prices, traded volumes, and revenues are presented and discussed. Section V provides conclusions of the paper.

II. METHOD

The concept assumes that producers simultaneously submit supply functions to a uniform-price auction in a one-shot game. This is a common electricity market structure in Europe called day-ahead electricity market. Under this concept, it is assumed that hydro and wind power plants are owned by the same firm which jointly bids on the market.

In the market state of perfect competition, an optimal supply curve that firm i sends to the short-term electricity market (such as day-ahead market) is a SRMC curve of electricity generation which lies above the average short-term variable cost curve. The supply curve obtained in such a way should not be manipulated since it already guarantees maximal revenue. On the other hand, in the state of imperfect competition there is a possibility for a firm with significant market share to exercise market power which is achieved by manipulating (shifting) the SRMC curve horizontally and/or vertically. This curve shifting results in an optimal supply curve which means more revenue for a firm. In this research, the focus is on *strongly optimal supply curves*. These curves are obtained via numerical approximation of an SFE defined as a nonlinear optimization problem.

The idea is to use strongly optimal supply curves derived from an SFE to determine the possibility of joint bidding strategies and their impact on the other electricity market participants. Here, the electricity market state of imperfect competition and the non-cooperative game is assumed, which is modelled using a numerical approximation of the asymmetric SFE developed in [17].

Let's first examine the basic idea behind the SFE form in equations (1)–(9) before examining its numerical approximation in equations (10)–(31). This is a short overview of SFE basics and for a full explanation of SFE and its approximations refer to [17,18] and [30]. Symbols are defined separately for general form and numerical approximation due to slight differences in indexation in some cases.

The supply function of firm i , s_i , is strongly optimal if, in response to the competing firms' supply curves, $s_j, j \neq i$, and for any realization of demand, D , provides such output for firm i that ensures the highest possible revenue while considering generation and price constraints. Equilibrium is called *strong equilibrium* if all of the supply curves, $s_i \forall i$, are strongly optimal. It is assumed that firms behave rationally, in other words, they try to guess the expected behavior of competing firms and find the best response. Consequently, over time they usually end up identifying the strategies of other firms, therefore, reaching an equilibrium state.

Only strong equilibrium is analyzed here. The supply curves analyzed here are only valid for the ranges $[\varepsilon_{\min}, \varepsilon_{\max}]$ and $[\underline{\pi}, \bar{\pi}]$ of demand shock, ε , and prices, π , respectively. The ε_{\min} and ε_{\max} represent minimal and maximal possible value of demand shock ε , while $\underline{\pi}$ and $\bar{\pi}$ represent minimal and maximal possible value of electricity price π in monetary units per MW·h. In strong equilibrium, the supply curves of a firm i are pointwise and always lie above its SRMC function, meaning, no supply is offered at prices below marginal cost. For more about this issue, see Lemma 1 and related proof in [17].

Let's assume an oligopolistic electricity market with a maximum of I competing firms where each firm i has a maximum and minimum output capacity $k_{Tu,i} \in \mathbb{R}_+$ and $n_{Tu,i} \in \mathbb{R}_+$ where $i \in \{1, \dots, I\}$. The $c_i(y_i)$ is a short-run total cost of electricity generation $y_i \in \mathbb{R}_+$, of firm i . It is assumed that c_i is a convex, differentiable function that has a bounded first derivative, $c_i'(y_i) < \infty$, and is expressed in monetary units per MWh. Another assumption is that the SRMC function $c_i'(y_i) \geq 0$.

Regarding the electricity demand function, it is in the form $D(\pi, \varepsilon) = D(\pi) + \varepsilon$. The price-sensitive part $D(\pi)$ is a strictly decreasing, smooth and concave function. The part, ε , is a demand shock which is the stochastic element with positive density function $f(x) > 0$ for $x \in (\varepsilon_{\min}, \varepsilon_{\max})$. The supply curves $s_i : [\underline{\pi}, \bar{\pi}] \rightarrow [n_{Tu,i}, k_{Tu,i}]$ (see Fig. 1) are defined over the possible market prices $[\underline{\pi}, \bar{\pi}]$ which are mapped to the operation range of firm i .

Simply put, in SFE, after all firms "have chosen" their optimal supply curves s_i , the market is cleared for demand $D(\pi^*) + \varepsilon^*$, where "*" denotes the optimal/equilibrium solution. Therefore, the market equilibrium constraint $D(\pi^*, \varepsilon^*) = D(\pi^*) + \varepsilon^* = \sum_{i \in I} s_i(\pi^*)$ in equation (6) clears the market at the price π^* and volume $D(\pi^*, \varepsilon^*)$ and gives the unit dispatch by using the right-hand side expression $\sum_{i \in I} s_i(\pi^*)$. Therefore, each firm supplies the amount $s_i(\pi^*)$ and earns a profit of $\pi^* \cdot y_i(\pi^*) - c_i(y_i(\pi^*))$ as described in equation (2). The hardest obstacle in SFE is finding the optimal supply curves s_i and in order to obtain them the following steps

are necessary: 1) set up the profit maximization problem of each firm i as shown in equations (1) - (6). Supply curves obtained in this way are not strongly optimal since they do not account for the optimal behavior of other firms; 2) in order to account for the competing behavior of other firms, the problem in equations (1) - (6) is defined for each firm; 3) in order to find the strongly optimal supply curves, the profit function of each firm (equation (2)) is derived by a price variable as shown in expression (8). In this way, the family of optimality conditions is obtained which consists of the number of I optimality conditions, one for each firm i as shown in equation (9); and 4) in the final step, the family of optimality conditions (equation (9) for all firms) needs to be solved simultaneously. In this way, optimality conditions, also called best response functions, are changed to *strongly optimal supply functions* which are needed to determine equilibrium prices, volumes, and optimal dispatch. In short, these four steps explain the SFE basics.

It should be noted that it is suitable to index market prices with index set $i \in \{1, \dots, I\}$, π_i . The reasoning behind this is the fact that all firms are *price-makers* and can influence equilibrium market prices; since they all manipulate the same market price, equation (7) is assumed.

Given

$$(k_{Tu,i}, n_{Tu,i}; \bar{\pi}, \underline{\pi}) \in \mathbb{R}_+^2 \times \mathbb{R}^2 \quad (1)$$

Maximize

$$\Pi_i(\pi, \varepsilon) = \pi \cdot y_i - c_i(y_i) \quad (2)$$

Over

$$\pi \in \mathbb{R} \quad (3)$$

Subject to

$$\bar{\pi} \leq \pi \leq \underline{\pi} \quad (4)$$

$$n_{Tu}^i \leq y_i \leq k_{Tu}^i \quad (5)$$

Where

$$y_i := D(\pi) + \varepsilon - \sum_{j \neq i} s_j(\pi) \quad (6)$$

$$\pi_1 = \dots = \pi_i = \dots = \pi_I = \pi \quad (7)$$

To put it simply, the family of optimality conditions defined with equation (9) for $\forall i \in \{1, \dots, I\}$ are solved simultaneously to obtain equilibrium solution in which these optimality conditions become strongly optimal supply curves.

$$\frac{\partial \Pi_i(\pi, \varepsilon)}{\partial \pi} = 0 \quad (8)$$

$$s_i(\pi) = [\pi - c'_i(s_i(\pi))] \cdot \left[\sum_{j \neq i} s'_j(\pi) - D'(\pi) \right] \quad (9)$$

It is important to note that with the expressions (1) - (7) the problem is formulated for one demand shock ε . Also, this formulation needs to be approximated in order to be ready for computer implementation and this is done according to [17]. Therefore, in the rest of the Section, the numerical approximation of SFE, extended to K demand shocks, is defined.

The procedure used to quantify the impact of joint bidding strategies and market power of coordinated hydro-wind

generation on electricity market prices, volumes and competing firms is the optimization problem defined with equations (13) – (31). Final optimal values of decision variables from optimization are consequently used to construct supply curves with equations (10) – (12). In the final step, supply curves, revenues, and market prices were plotted to visualize the effect of joint hydro-wind bidding for different installed capacities and short-run marginal costs of competing firms.

In order to explain the whole procedure, the optimization problem set by equations (13) – (31)—which represents a numerical approximation of SFE [17] used for the construction of the curve—is first explained.

Let us first examine firm i and find one point on its supply curve s_i as shown in Fig. 1. To do so, it is necessary to calculate the optimal power output y_{ik} and the optimal supply price π_{ik} for the k^{th} scenario of aggregate demand $D_r(\pi_k, \varepsilon_k)$ that can be realized in the electricity market and for the identified supply curves of other competitors, $s_j \forall j \neq i$. The y_{ik} is generation output at the k^{th} outcome of the residual aggregated demand $D_r(\pi_k, \varepsilon_k)$. In this way, pair (π_{ik}, y_{ik}) is obtained which is one point on the supply curve s_i . Since a few more points on the supply curve are needed, let us simulate the K number of electricity demand scenarios and repeat the calculation K times to obtain the number of K pairs of (π_{ik}, y_{ik}) that form the points on the supply curve of firm i . Besides these pairs, the breaking points $\tilde{\pi}_{ik}$ for scenario k and the slopes of the linear segments connecting the two breaking points (see Fig. 1) are also calculated. In this way, a family of K quadruplets $(\pi_{ik}, \tilde{\pi}_{ik}, y_{ik}, \beta_{ik})$ is obtained, which is used in equations (10) – (12) to construct a final supply curve called the strongly optimal supply curve s_i of firm i . Since there are I competing firms with different total short-run costs $c_i(y_{ik})$ and operating ranges, the calculation is simultaneously computed for all of the competing firms. In this way, a family of I strongly optimal supply curves, $s_i, \forall i \in \{1, \dots, I\}$ is obtained which forms a strong equilibrium. To increase their expected revenue in this non-cooperative game, firms intend to manipulate market prices and volumes by changing their supply price π_{ik} which, due to strong equilibrium, converges to the single price π_k called equilibrium price. Therefore, the indexation i can be omitted from the offer price symbol of each firm i.e., $\pi_{ik} = \pi_{jk} = \dots = \pi_{Ik} = \pi_k$. Let us assume that the equilibrium price in this non-cooperative game with imperfect competition is equivalent to the market-clearing price in the real electricity market.

The rest of the Section explains the procedure in detail. In an optimization problem defined by equations (13) – (31), let each firm i have a maximum and a minimum output $k_{Tu,i} \in \mathbb{R}_+$ and $n_{Tu,i} \in \mathbb{R}_+$ defined by (23) where $i \in \{1, \dots, I\}$ and I is the number of competing firms in the oligopolistic electricity market. The minimum and maximum possible market prices are $\underline{\pi}$ and $\bar{\pi}$ and are defined by (22). The π_k is the price of electricity in the time interval $[0, T]$ in monetary units per MW·h (MU/MW·h) for certain residual aggregate demand realization $D_r(\pi_k, \varepsilon_k)$ on the electricity market. This residual aggregate demand is obtained by subtracting the elastic fringe supply curve, s_f from $D(\varepsilon_k)$ in (31) which is perfectly inelastic. The final demand $D_r(\pi_k, \varepsilon_k)$ in (32) is called ‘residual’ as it is

residual when competitive fringe supply is subtracted from it. This also enables it to become elastic, which is a necessary condition for the strong equilibrium defined in Lemma 1 in [17]. This lemma stated that in order to achieve a strong equilibrium, the demand curve must be strictly decreasing, smooth, and concave (the assumption used in the proof of Lemma 1).

The inelastic part $D(\varepsilon_k)$ in (31) consists of the deterministic part D and the stochastic part ε_k called the demand shock for outcome k . For the deterministic part D , a single forecast or expected value of demand for the analyzed period can be used. This part is perfectly inelastic. The second part is also perfectly inelastic, but it is stochastic. It is called the demand shock and is noted with ε_k where k indicates the outcome number. Generally, the demand shock ε_k is the demand value of the k^{th} outcome of a random variable defined for a stochastic demand in the probability space $(\Omega_\varepsilon, \mathcal{F}_\varepsilon, P_\varepsilon)$, with a cumulative distribution function, $F_\varepsilon(\varepsilon)$, on which demand shocks can be sampled using $\varepsilon_k = F_\varepsilon^{-1}\left(\frac{k-1}{K-1}\right)$ with positive density function, $f(x) > 0$, $x \in (\varepsilon_{\min}, \varepsilon_{\max})$ where ε_{\min} and ε_{\max} are the smallest and the largest possible demand shocks. With respect to Theorem 2 in [17] and since strong equilibrium is required here, the results are independent of the distribution function and the probability of occurrence of each ε_k . This greatly simplifies the procedure, and the probabilities are omitted. Discretization over the demand range $[\varepsilon_{\min}, \varepsilon_{\max}]$ is performed with expression (28) and is indexed with $\{1, \dots, K\}$ where the first demand shock is $\varepsilon_1 = \varepsilon_{\min}$ and the last $\varepsilon_K = \varepsilon_{\max}$. The set $\{1, \dots, K\}$ is the index set of possible demand shocks, $k \in \{1, \dots, K\}$. Also, the inequality: $\varepsilon_1 \leq \varepsilon_2 \leq \dots \leq \varepsilon_K$ should be maintained.

In continuation, the elastic part of the residual aggregated demand in (31) is discussed, which is called the competitive fringe supply function s_f . A *competitive fringe* is a large price-taking firm which aggregates all the small, non-cooperative, price-taking firms competing against each other and which independently have an insignificant market share and, therefore, must constantly adjust their market position to the position of dominant firms. Although each small, price-taking fringe firm has a small market share, all fringe firms together have a significant market share thus why we aggregated them in one large price-taking firm called competitive fringe represented by *competitive fringe supply function* s_f .

Here, it is assumed that the import of electricity to Croatia from Hungary and Slovenia (Fig. 5) is a competitive fringe, with the supply function s_f . In the short-term, electricity demand is almost completely inelastic, which is the case in this study; hence, fringe supply function, s_f will introduce elasticity into the demand by equation (31) and thus a necessary condition for strong equilibrium defined in Lemma 1 in [17] will be retained. This residual demand in (31) is then balanced by the dominant firms in the model by equation (17).

The $c_i(y_{ik})$ is the total short-run cost of electricity generation for firm i at the power output y_{ik} . The assumption is that the first derivative of the cost function, $c_i'(y_{ik})$, defined by (29) is always positive, $c_i'(y_i) \geq 0$. This is ensured by its convexity in the operating range $[n_{Tu,i}, k_{Tu,i}]$. These costs are

ordered according to $c_1'(n_{Tu,1}) < c_2'(n_{Tu,2}) < \dots < c_N'(n_{Tu,I})$ (see [17] for more about SFE conditions).

Supply curves s_i (Fig. 1) need to be positive, monotonous, and piecewise smooth, as is often the case in real markets.

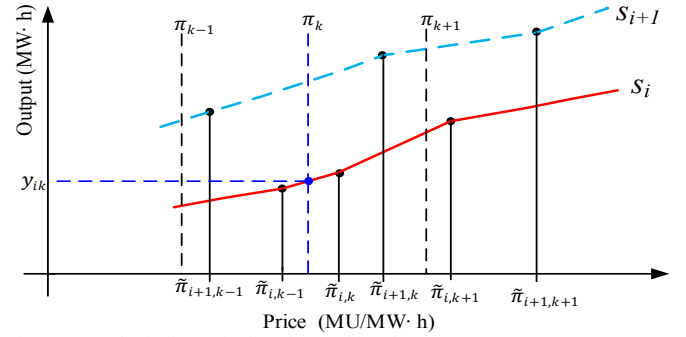


Fig. 1 Hypothetical supply function and its elements

The slope β_{ik} is calculated by $\beta_{ik} = s_i'(\pi_k)$. The pair (π_{ik}, y_{ik}) and the breakpoints, $\tilde{\pi}_{ik}$ of each linear segment are calculated with (16) – (27) for all outcomes of residual demand i.e., $D_r(\pi_k, \varepsilon_k) \forall k$. Therefore, k^{th} linear segment passes through point (π_{ik}, y_{ik}) with slope β_{ik} . Neighboring linear segments k and $k+1$ will always strictly intersect somewhere between price points (π_{ik}, π_{ik+1}) i.e. $\pi_{ik} < \tilde{\pi}_{ik} < \pi_{ik+1}, \forall i, k$ which is ensured by equations (18) and (21). The requirement for strictness assures that the linear segments are smooth functions and that the non-differentiable break (intersect) points, $\tilde{\pi}_{ik}$, do not belong to the linear segments. The requirements in equations (10) and (11) ensure that the supply function (12) is non-decreasing. The supply function in (12) consists of generation and price pairs (y_{ik}, π_k) defined for all demand shocks $\forall k \in \{1, \dots, K\}$.

$$\pi_k < \pi_{k+1}, \forall k \quad (10)$$

$$y_{ik} < y_{ik+1}, \forall k \quad (11)$$

$$s_i(\pi) = \begin{cases} y_{i1} + \beta_{i1}(\pi - \pi_1), & \underline{\pi} \leq \pi \leq \tilde{\pi}_{i1} \\ y_{ik} + \beta_{ik}(\pi - \pi_k), & \tilde{\pi}_{i,k-1} \leq \pi \leq \tilde{\pi}_{ik}, k = 2, \dots, K-1 \\ y_{iK} + \beta_{iK}(\pi - \pi_K), & \tilde{\pi}_{i,K-1} \leq \pi \leq \bar{\pi} \end{cases} \quad (12)$$

The final numerical approximation of SFE for K demand shocks and I competing firms used for hydro and wind coordinated generation follows in the next Section.

A. Model

A brief overview of the SFE approximation from [17], used for hydro and wind coordinated power generation, is presented in (13) – (31) and is modified using the fringe competition supply function, s_f , and residual demand, D_r .

By minimizing the objective function (14), a quadruple $(\pi_k, \tilde{\pi}_{ik}, y_{ik}, \beta_{ik})$ in (15) is obtained, which characterizes the supply function (slope and position) in (12):

Given $\rho \rightarrow 0^+$ and

$$(I, K; (c_i)_{i \in \{1, \dots, I\}}; D; \varepsilon_{\max}, \varepsilon_{\min}; k_{Tu,i}, n_{Tu,i}; \bar{\pi}, \underline{\pi}) \quad (13)$$

minimize:

$$\sum_{\forall i} \sum_{\forall k} (\xi_{ik} - 0.5)^2 + \sum_{\forall i} \sum_{\forall k} \beta_{ik} \quad (14)$$

over:

$$(\pi_k, \tilde{\pi}_{ik}, y_{ik}, \beta_{ik}; \varepsilon_{ik}, \kappa_{ik}^{Tu}, \nu_{ik}^{Tu}, \xi_{ik}) \forall i, \forall k \quad (15)$$

The objective function and asymmetric equilibrium. The minimization of the objective function (14), while satisfying the family of equilibrium optimality conditions defined in (16) and other constraints, results in an asymmetric equilibrium. Consequently, the equilibrium solution that holds for all of them was found. Equations (16) – (27) are defined for all i and k i.e. $\forall i \in \{1, \dots, I\}$ and $\forall k \in \{1, \dots, K\}$

$$y_{ik} - (\pi_k - c'_i(y_{ik})) \cdot \left(\sum_{j \neq i} \beta_{jk} - D_r'(\pi_k, \varepsilon_k) \right) + \kappa_{ik}^{Tu} - v_{ik}^{Tu} = 0 \quad (16)$$

$$\sum_{i=1}^I y_{ik} = D_r(\pi_k, \varepsilon_k) \quad (17)$$

$$y_{ik+1} - y_{ik} - \beta_{ik+1} \cdot \pi_{k+1} + \tilde{\pi}_{ik} \cdot (\beta_{ik+1} - \beta_{ik}) + \beta_{ik} \cdot \pi_k = 0, k \neq K \quad (18)$$

$$\kappa_{ik}^{Tu} \cdot (k_{Tu,i}^t - y_{ik}) \leq \rho \quad (19)$$

$$v_{ik}^{Tu} \cdot (y_{ik} - n_{Tu,i}^t) \leq \rho \quad (20)$$

$$\xi_{ik} \cdot (\pi_{k+1} - \pi_k) = \pi_{k+1} - \tilde{\pi}_{ik} + \rho, k \neq K \quad (21)$$

$$\underline{\pi} \leq \pi_k \leq \bar{\pi} \quad (22)$$

$$n_{Tu,i} \leq y_{ik} \leq k_{Tu,i} \quad (23)$$

$$\beta_{ik} \geq 0 \quad (24)$$

$$\kappa_{ik}^{Tu} \geq 0 \quad (25)$$

$$v_{ik}^{Tu} \geq 0 \quad (26)$$

$$0,0001 \leq \xi_{ik} \leq 0,9999 \quad (27)$$

Handling complementarity constraints. Replacing the equations: $\kappa_{ik}^{Tu} \cdot (k_{Tu,i}^t - y_{ik}) = 0$ and $v_{ik}^{Tu} \cdot (y_{ik} - n_{Tu,i}^t) = 0$ with expressions (19) and (20) helps the nonlinear programming (NLP) solver address complementarity constraints and reduce computation time. Therefore, the predefined positive initial value of ρ is scaled down at each iteration, $\rho \rightarrow 0^+$, and, at each iteration, equations (13) – (31) are solved until the problem is unfeasible. The ρ in (21) allows the reduction of computing time [17]. Equations (25) and (26) ensure that the dual variables are positive as needed according to the definition of Lagrangian dual of convex problems. The variables κ_{ik}^{Tu} and v_{ik}^{Tu} are shadow prices (dual variables) associated with the constraints for the maximum and minimum power output of firm i and k^{th} demand outcome.

Handling strict inequalities. In order for $\tilde{\pi}_{ik}$ to fall strictly within (π_k, π_{k+1}) and to avoid strict inequalities that violate the standard NLP form, the relation: $\pi_k < \tilde{\pi}_{ik} < \pi_{k+1}$ is replaced by expression (21), which unfortunately needs another strict inequality: $0 < \xi_{ik} < 1$, which is again replaced with expression (27).

$$\varepsilon_k = \varepsilon_{min} + \frac{(k-1) \cdot (\varepsilon_{max} - \varepsilon_{min})}{K-1} \quad (28)$$

$$c'_i(y_{ik}) = \frac{\partial c_i(y_{ik})}{\partial y_{ik}} \quad (29)$$

$$D(\varepsilon_k) := D + \varepsilon_k \quad (30)$$

$$D_r(\pi_k, \varepsilon_k) := D(\varepsilon_k) - s_f(\pi_k) \quad (31)$$

III. CASE STUDY

The expressions (13) – (31) form a nonlinear programming problem solved using the COIN-OR IPOPT solver under the General Algebraic Modelling System (GAMS) Rev 239. COIN-OR IPOPT solver for nonlinear programming (NLP) optimization was chosen because it has provided the best overall performance in terms of results quality and computational time regarding other NLP solvers available in GAMS. The case study was conducted for 5th December 2013. That week of December 2013 was by expert judgement considered the worst-case scenario from the standpoint of wind forecasting error and natural water inflow in hydropower systems of Croatia. The European regional block consisting of Austria, Slovenia, Hungary, and Croatia has been Modelled, as shown in Fig. 2.

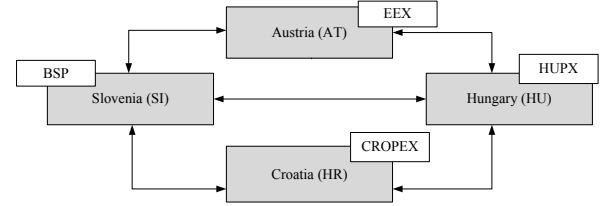


Fig. 2 Depiction of the regional block consisting of Austria (AT), Slovenia (SI), Hungary (HU) and Croatia (HR) and possible power flows between countries.

The electricity market in Croatia is based on the model of a bilateral market, upgraded with the model of balance groups. Further, in 2016, the operation of the Croatian Power Exchange (CROPEX) started. It began with day-ahead trading and, in 2017, intraday trading was introduced, which is an important factor for cost-efficient balancing and integration of renewable energy sources. For 2013 which was analyzed, the total available capacities of all power plants amounted to 4,132 MW. Of this, 1,671 MW were thermal power plants, 2,187 MW hydro power plants, 254 MW wind power plants, and 20 MW solar power plants. There was also 348 MW in the nuclear power plant Krško (50% of the total available capacity) used for the Croatian power system. Typically, the system load ranges from 1,100 to 3,000 MWh/h [45] and annual net consumption in 2013 was 16.0 TWh [46]. Also, Croatia has a substantial net cross-border transmission capacity towards Bosnia and Herzegovina, Serbia, Slovenia and Hungary, which in total ranges from about 3,000 MW to 4,000 MW [47], i.e., usually during peak load.

This paper assumes an oligopoly in the electricity market, where the following firms are analyzed: hydro and wind coordination of the wind farm Vrataraša and the hydropower system Vinodol in Croatia; the dominant firm representing a former monopoly; competitive fringe which represents imports from Hungary and Slovenia; and Croatian electricity demand. To simplify the model, the oligopoly is reduced to a duopoly where hydro-wind coordination and the dominant firm compete and whose bidding strategies are optimized. This is possible since imports are aggregated and presented as a competitive fringe. The installed capacity in the hydro and wind coordination consists of the Vinodol hydropower plant and the Vrataraša wind power plant. The installed capacity is iteratively scaled up from 132 MW (90 MW of hydro capacity and 42 MW

of wind capacity) to 4124 MW (this is done with a constant hydro-wind ratio 68/32 in order to retain initial conditions of the current situation and conduct the analysis of scaling up of hydro-wind production in Croatia, which is also close to the hydro-wind ratio for 2030 defined by the Croatian Energy Strategy for the 2nd Scenario of moderate energy transition [48], which is 67/33).

The SRMC curve of the hydro-wind coordination used in the model is shown in Fig. 3. For more information on the calculation and construction of SRMCs for coordinated hydro-wind generation, see [1, 2] where a method for constructing joint SRMC curves for hydro-wind coordination is presented. The stepwise SRMC curve is approximated by a 2nd-degree polynomial approximation that is differentiable and convex to satisfy formal conditions for a strong equilibrium.

The exponential function shown in blue in Fig. 4 is used to approximate a merit-order owned by the dominant firm. It is also a differentiable and convex function that satisfies the formal conditions for strong equilibrium. Although these approximations involve errors in the solution, these errors are tractable and enable convexity, meaning that the global optimum can be easily achieved. It is preferable that 2nd-degree polynomial approximation has no negative value in the range of 250 MW to 1250 MW, but this does not affect the final solution to a great extent. On the other hand, it reduces the computation time and ensures convexity of the problem (due to 2nd polynomial) compared to the exponential function and higher-order polynomials.

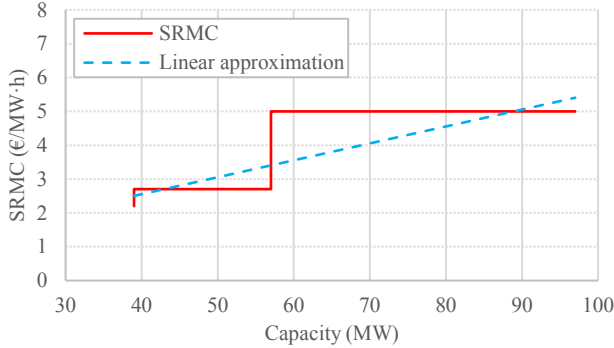


Fig. 3 SRMC function of the hydro-wind coordination (red stepwise function) and its linear approximation (dashed blue).

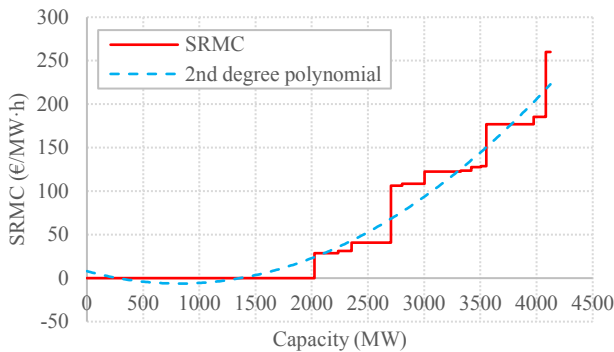


Fig. 4 The dominant firm production portfolio is shown in a merit order way (red stepwise function of SRMC) and its 2nd-degree polynomial approximation (dashed blue).

The supply functions in Fig. 5 describe the dependence of electricity imports to Croatia on nodal electricity prices in Croatia. These functions represent the competitive fringe supply functions s_f of the imports in the model. The aggregate demand function is perfectly inelastic and consists of a deterministic and a stochastic part. The deterministic part, D is equal to 2307 MW for all demand shocks, $\forall k$, which is an average electricity demand on December 5th, 2013. The stochastic part, demand shocks, ε_k , are bounded by $\varepsilon_{min} = -2307$ MW and $\varepsilon_{max} = 300$ MW. The minimum, $\underline{\pi}$, and maximum, $\bar{\pi}$, possible equilibrium prices are set at -500 €/MW·h and 3000 €/MW·h according to the CROPEX.

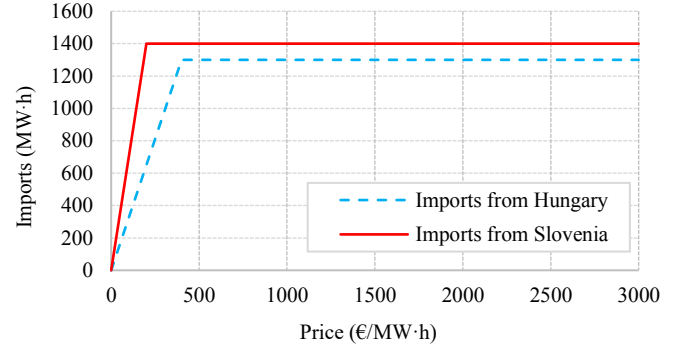


Fig. 5 Function of the electricity imports to Croatia from Hungary (dashed blue) and from Slovenia (red) as a function of nodal prices in Croatia, obtained by two-stage least squares (2SLS) regression according to [20].

A. Results

The results of the case study are shown in Fig. 6-9. In Fig. 6a), the family of strongly optimal supply curves for hydro-wind coordination is shown. Fig. 6b) shows the strongly optimal supply curves of the dominant firm. Each supply curve corresponds to one different level of installed capacity in hydro-wind coordination (from 132 MW to 4124 MW). In this way, it is possible to assess how a different size of hydro-wind coordination affects market prices and volumes.

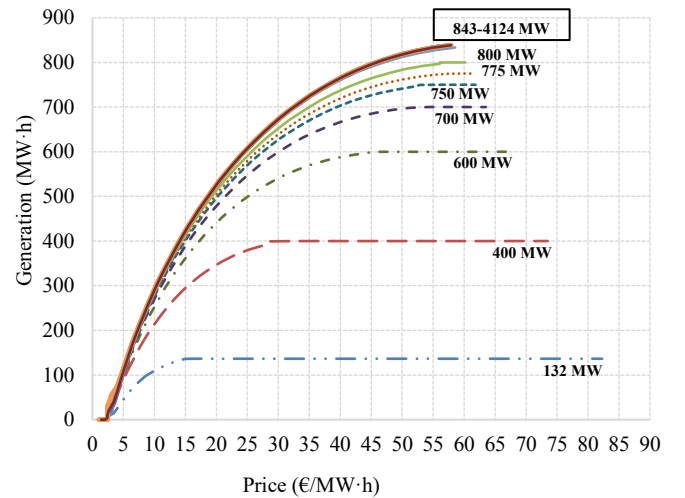


Fig. 6a) Strongly optimal supply curves of the hydro-wind coordination for different hydro-wind capacities.

Fig. 7a) shows strongly optimal supply curves for hydro-wind coordination for situations when the SRMC of hydro-wind coordination varies from 0 €/MW·h to 60 €/MW·h while

the hydro-wind capacity is fixed at 132 MW. In these situations, the supply curves of the dominant firm are also shown in Fig. 7b). This was done to evaluate how different production costs of hydro-wind coordination affect market prices and volumes. Fig. 8 shows a comparison of market prices for cases when hydro and wind jointly bid with 843 MW of capacity in addition to the case of separate market bidding (non-coordinated generation) with 268 MW of wind and 575 MW in hydro. A comparison is made for different realizations of demand. This is done to quantify the effects of joint bidding on market prices. It can be observed that optimal bidding of coordination tends to smooth the price curve for different demands. In Fig. 9, a comparison of revenues is shown for the same cases when wind and hydro jointly bid and in the case of separate market bidding. The positive aspects of joint optimal bidding of coordinated hydro-wind generation have been quantified i.e., the synergy in the form of increased revenues.

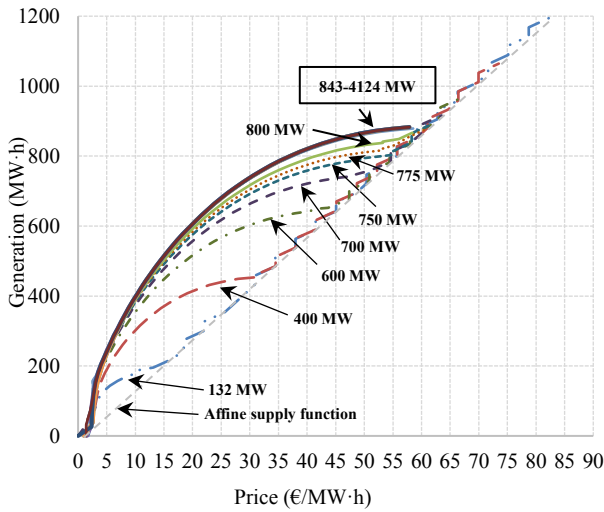


Fig. 6b) Strongly optimal supply curves of the dominant firm for different hydro-wind capacities.

B. Discussion

In Fig. 6 and Fig. 7, supply functions of the hydro-wind coordination and the dominant firm show some distinctive features, such as:

1. *Increasing hydro-wind installed capacity: convergence of equilibrium solutions.* The supply functions of the hydro-wind coordination strongly converge during increase of the installed power in hydro-wind coordination from 132 MW to 843 MW. After the 843 MW of capacity in coordination, the supply curves are practically the same up to 4124 MW. This means that the optimal market strategy for the capacity range between 843 MW and 4124 MW is the same. Therefore, there is no need for installing hydro-wind capacity higher than 843 MW, as it will not achieve any growth of market power i.e. profits (this is seen in Fig. 6a), where the supply curves from 843 MW to 4124 MW are equal.

2. *Increasing hydro-wind installed capacity: the market power of the hydro-wind coordination.* In Fig. 6b), the supply functions of the dominant firm are shown for cases where the capacity in hydro-wind coordination increases from 132 MW to 4124 MW. The figure illustrates that the supply functions of the dominant firm deviate from *affine supply function* which represents supply function of the dominant firm when it has a

monopoly, (Fig. 6b)) whenever hydro-wind coordination is introduced into the electricity market. This deviation also increases in magnitude as the capacity in hydro-wind coordination increases.

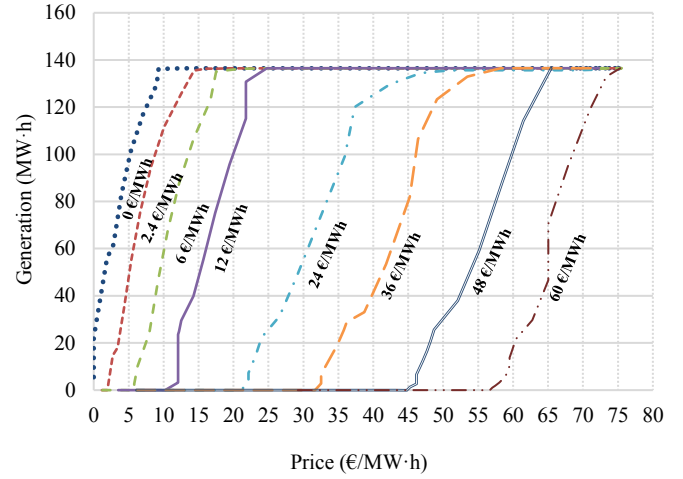


Fig. 7a) Strongly optimal supply curves of the hydro-wind coordination for its different SRMC. This is the initial case when hydro-wind coordination capacity equals 132 MW.

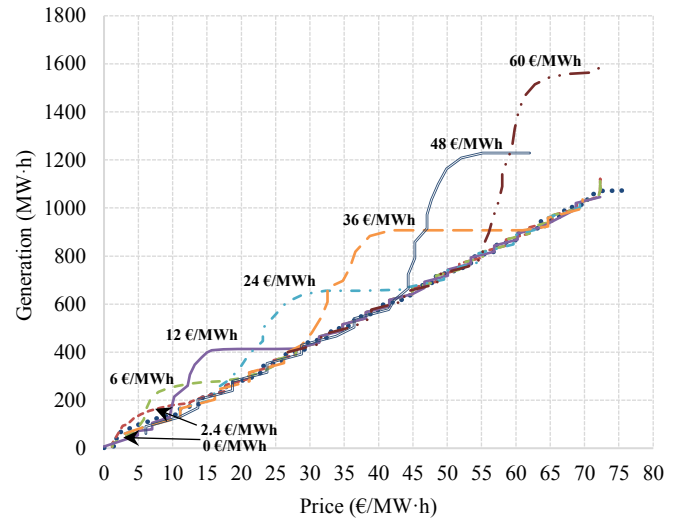


Fig. 7b) Strongly optimal supply curves of the dominant firm for different values of hydro-wind SRMC. This is the initial case when hydro-wind coordination capacity equals 132 MW.

Namely, in the range of 38 MW to 60 MW, the market power of coordination is negligible. In this range, coordination acts as a competitive fringe. The market power of hydro-wind coordination becomes visible for capacities from 66 MW to 132 MW. If the installed power in coordination increases more (from 132 MW onwards), then the dominant firm deviates extensively from its monopolistic values. Therefore, at 132 MW, the coordination has limited market power since it is at low market prices (up to 15 €/MWh) which is a rare situation in the analyzed power market (Fig 6b). As its installed power increases, it becomes clear (in Fig 6b) that market power of hydro-wind coordination becomes more significant and can be utilized at a higher price (up to 60 €/MWh) and a higher demand. After achieving a value of 843 MW in coordination, the supply

functions of the dominant firm are the same (deviations from monopolistic values are the same). This means that there is no need for more than 843 MW in the capacity of hydro-wind coordination as revenues of coordination would not increase (coordination cannot influence prices above 60 €/MWh and they are the same as in affine supply function). At 843 MW of installed power, coordination achieves a market share of 49% at residual demand. The generally accepted empirical values of market power are 50% or more [49], which means that the coordination of 843 MW achieves market power and is the dominant market player.

3. *Increasing hydro-wind marginal costs: a horizontal shift in hydro-wind supply functions.* Fig. 7a) shows the supply curves of the hydro-wind coordination for different SRMC (from 0 €/MWh to 60 €/MWh) of hydro-wind coordination. It is clear from Fig. 7a) that the supply curves are above the SRMC. Consequently, the supply curves of the hydro-wind coordination shift to the right as its SRMC rises. The shifting phenomenon is the result of Lemma 1 from [17] i.e., no supply is offered at prices lower than the marginal cost. The result of this phenomenon is a decrease in hydro-wind generation as its SRMC increases.

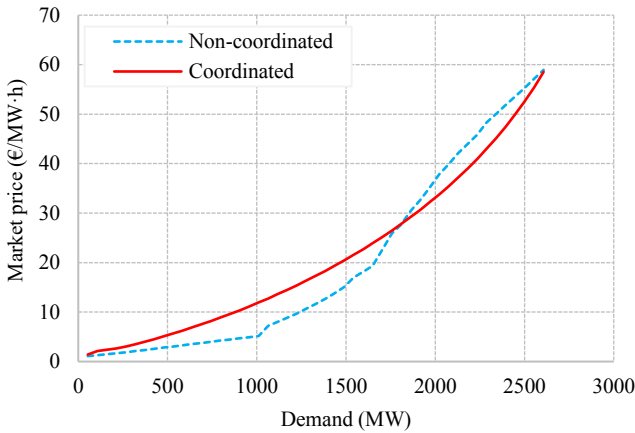


Fig. 8 Market price as a function of electricity demand for the case when: (red) hydro-wind generation jointly bid (coordinated generation), and (blue dash) non-coordinated hydro and wind generation.

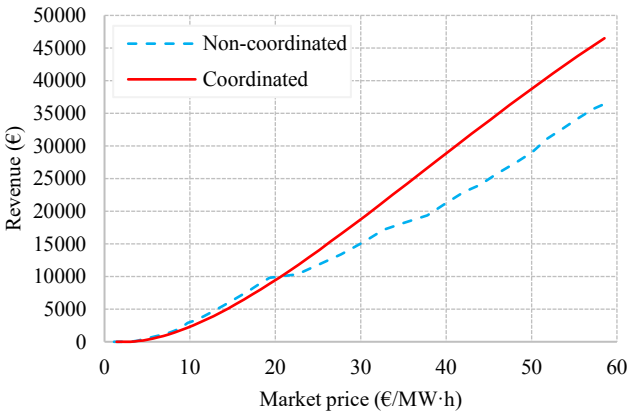


Fig. 9 Revenue curves as a function of electricity demand for the case when: (red) hydro-wind generation jointly bid (coordinated generation), and (blue dash) non-coordinated hydro and wind generation.

4. *Increasing hydro-wind SRMC: market power of the hydro-wind coordination.* Fig. 7b) shows the supply curves of the dominant firm for the different SRMC of the hydro-wind coordination. It can be observed that hydro-wind coordination reduces its generation to increase market price, π_k , as its SRMC increases (Fig. 7a). This is clearly visible in (Fig. 7a) and Fig. 7b) for one situation of market price of $\pi_k = 40$ €/MWh. At the same time, the dominant firm annuls that behavior by increasing its generation (dominant firm increased generation from 660 MW·h to 906 MW·h and prevented a rise of market prices which stayed constant at $\pi_k = 40$ €/MWh) as the hydro-wind reduced generation due to SRMC rise from 24 €/MWh to 36 €/MWh. This behavior of the dominant firm is attributed to the fact that hydro-wind withholding generation enables the dominant firm to produce more at the same equilibrium price.

5. *Increasing revenues and smoothing market prices.* The impact of bidding strategies on revenues and market prices for the case of coordinated and non-coordinated hydro-wind generation was quantified and shown in Fig. 8 (impact on market prices) and Fig. 9 (impact on revenues). As expected, Fig. 8 shows that market prices rise as demand grows in both cases, but with different magnitudes. It can be observed that optimal bidding of coordination tends to smooth the price curve for different demands (Fig. 8), lowering prices at higher demands and increasing prices at lower demands thus annulling the negative effects of low off-peak prices due to wind overproduction (when negative pricing is possible). This is the result of joint optimal bidding strategies. This is a positive effect of joint bidding of coordinated hydro and wind generation as it annuls the “missing money” problem in markets with high wind penetration (since in a coordinated case excess production is stored in a hydro reservoir, therefore avoiding wind curtailment).

Coordinated hydro-wind bidding achieves market power at lower demand levels and increases market prices at higher rates (Fig. 8). Due to joint bidding of coordinated hydro-wind generation, it can achieve higher production levels and higher revenues compared to the non-coordinated case (Fig. 9). In other words, coordinating hydro and wind generation is more effective in terms of revenue. At a market price of around 58.9 €/MWh, the joint hydro-wind bidding shows a profit higher by 26.8% compared to non-coordinated bidding (€ 46,504 compared to € 36,664 respectively) and this significant increase in profits is possible in a wide range of market prices (Fig. 9). A comparison of the impact of coordinated and non-coordinated hydro and wind generation on market prices (Fig. 8) and overall profit (Fig. 9) provides some justification for considering the hydro-wind coordination concept in the Croatian case.

IV. CONCLUSIONS

The paper analyses the market behavior of joint bidding of coordinated production of hydro and wind on the electricity market. Coordination assumes that hydro generation balances wind forecasting errors. The paper contributes to the analysis of market power and the optimal bidding of hydro-wind coordination using a numerical approximation of the supply

function equilibrium that models the oligopoly in Croatia and imports from Hungary and Slovenia.

This work can be useful in cases where the impact of bidding strategies on market prices, volumes, and revenues needs to be quantified. This paper can also help those who need to analyze the possibility of the hydro-wind joint bidding to achieve market power and the impact of the joint bidding on market prices and volumes.

A comparison of the impact of coordinated and non-coordinated hydro and wind generation on market prices and overall revenue justifies the consideration of the hydro-wind concept in the Croatian case. Specifically, coordinated hydro-wind generation achieves higher profits and annuls the effects of low off-peak prices caused by wind overproduction in a non-coordinated case. Therefore, missing money problems due to large wind penetration are avoided.

Future work includes the application of SFE and game theory for the coordination of the solar photovoltaic systems and batteries [50] in local electricity markets [51]. Furthermore, the potential of wind turbines and coordinated operation with other sources will be studied in order to balance the power system and mitigate active power disturbances [52, 53]. Finally, it must be noted that market coordination between two separately owned power plants is prohibited. Therefore, this paper assumes that hydro and wind power plants are owned by the same entity.

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