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Distributed Optimal Heating Control of Building Zones

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***Abstract*—This paper deals with distributed control of building zones heating with individual room mathematical model and corresponding controller, and iterative coordination towards the joint energy efficiency goal of a single building floor consisted of 23 rooms. The control structure exploits model predictive control, which is based on the calculation of the system control variable on the prediction horizon while respecting the imposed constraints. Such control approach is applied individually to each room and further coordinated together through the concept of distributed optimal control and the asymmetric projection algorithm. The distributed room temperature control is implemented within a realistic scenario of a living lab case study. The presented results show that model predictive control enables a trade-off between comfort and required energy. The distributed model predictive control provides the same results as the centralized version while additionally enables fast digitalization of the room, simple thermal model, and high user privacy.**

***Index Terms*—building thermal model, RC model, model predictive control, distributed optimal control, energy efficiency**

I. Introduction

Looking from the global perspective, buildings are one of the largest energy consumers in the world, accounted for one-third of total energy use in 2019 [1]. Heating and cooling are among the most significant energy consumers in buildings with 35% of the consumption share [2]. Satisfying a large number of specific comfort requirements, located in separated, yet thermally interdependent zones, results in excess energy consumption. The aim of the control approach elaborated in this paper is to satisfy individual comfort requirements while minimizing the total heating energy consumption.

In recent times, the emphasis is put on model predictive control (MPC) of heating and cooling. The approach implies minimizing the energy consumption by using a forecast of external disturbances and user behavior. Based on the same algorithm premises, it is here applied on the heating and colling of the building zones, i.e. the building climate control utilizing the model of the system. It is possible to obtain such a model via resistive-capacitive (RC) approach in modeling. This method provides RC electrical circuit for the given thermal model, which can be written into state-space representation suitable for incorporation in the predictive control.

Centralized control in buildings implies a single model predictive control, that accepts temperature setpoints from every zone and then calculates suitable thermal energy by means of systems parameters while taking into account the predicted external disturbances, as it is done in [3]. Afterwards, controller sends requested temperature back to the zones. Fully digitalized and networked building zones are a pre-requisite of the approach, often available only in newer buildings and business sector.

Distributed optimal control implies that every zone has own controller with a corresponding MPC problem. Therefore, the zone controllers are considered as subunits of the same level and, with the coordinated operation of all the subsystems, it is possible to achieve significant increase of energy savings. The most important advantages of this type of control are lower computional complexity as well as shorter actuation latency under certain conditions [4]. Simulations published in [5] show that distributed MPC performed with 48% less computation time than centralized one. There are lot of different distributed control algorithms and all consist of a loop in which the monetary cost is calculated using the predicted energy consumption and that process is repeating until it has converged [6]. Distributed system is obtained by e.g. using methods like Benders' decomposition [7] or a dual decomposition [8] that split a single large-scale problem into several independent problems, which are coordinated by supervisor. In this paper, simple distribution is made by separating the joint RC model network and making mutual variables as constants. Central supervisor collects local requests and fix them using bundle method [9], 'Trim and respond method' [10] or asymmetric projection algorithm (APA), as we opted for in this paper. Distributed control utilizes the game theory premises, where an equilibrium point is reached such that all participants maximize their gain. In MPC terms, the individual controllers iteratively solve problems and jointly converge to the optimum [13]. Such optimum is coinciding with the Nash equilibrium [11], while respecting both local and global constraints. The APA is originally proposed in [15] and is successfully applied in distributed optimal batteries charging control [12]. Based on the same algorithm premises, we apply it here on the heating and colling of the building zones, i.e. the building climate control.

This paper is organised as follows. In Section II, RC modeling of thermal system is explained. Section III presents centralized model predictive control of heating and cooling of the building. In Section IV, theory and implementation of the distributed control is explained. In Section V, simulation results are shown and discussed. Section VI concludes the paper.

II. Building Thermal Model

Thermal model is based on the laws of thermodynamics, which describe heat transfer in the rooms. Such model can be used to determine temperature of indoor air in relation with external weather conditions, namely the temperature and the irradiance. The process of heat transfer is described by a partial differential equations, that contain spatial distribution of temperature. By replacing spatially distributed parameters with a single-point approximation for room temperatures, an acceptable approximation of the physical process is achieved, leading to a simple model of ordinary differential equations.

In the RC approach, parameter of the heat flow is analogous to the electric current and temperature is analogous to the voltage. For modeling purposes, equations are set for the calculation of the corresponding elements in the circle. Thermal resistance is calculated by the following equation:

$$R = \frac{\Delta T}{H} = \frac{1}{hA}, \tag{1}$$

where ΔT is temperature difference, H is thermal flow, h is thermal conductivity and A is the contact surface. The thermal capacity is described with:

$$C = \rho C_p V, \tag{2}$$

where ρ is density, C_p is heat capacity and V is volume of the zone (room, hall, office).

In this paper, a thermal model of two rooms is presented. In the procedure of RC modeling, it is necessarily to collect parameters that are related to the characteristics of building materials. Also, various room dimensions, such as walls, doors and windows thickness, surface and construction materials are required. Table I provides construction materials and corresponding layers used in the model.

All of the above parameters, by means of equations (1) and (2), are represented with equivalent electrical resistance and capacity. In modeling, it is assumed that heat transfer is one-dimensional and linear. Therefore, exterior wall is modeled by two capacitors, each covering one side of the wall and representing the corresponding wall envelope. On the other hand, interior wall is modeled with a single capacitor. Doors and windows include only resistors, so they are modeled without dynamics. Respecting those premises, an RC model is designed. In Fig.1, an RC model of two rooms is shown as a simple example.

The model from Fig. 1 is represented by 22 differential equations, related to 22 temperature points in the rooms. The temperatures T_{r1} and T_{r2} represent the air temperatures, which are directly incorporated in the MPC objective function.

TABLE I
Wall layers specifications

Internal brick walls	Layer thickness [cm] Plaster	Full brick	Plaster
10 cm thick wall	2	6	2
12 cm thick wall	0	6	0
16 cm thick wall	2	6	2

Internal reinforced brick walls	Layer thickness [cm] Plaster	Reinforced concrete	Plaster
28 cm thick wall	2	24	2
38 cm thick wall	0	34	2
41 cm thick wall	0	39	2

External reinforced concrete wall	Layer thickness [cm] Reinforced concrete	Plaster
41 cm thick wall	39	2

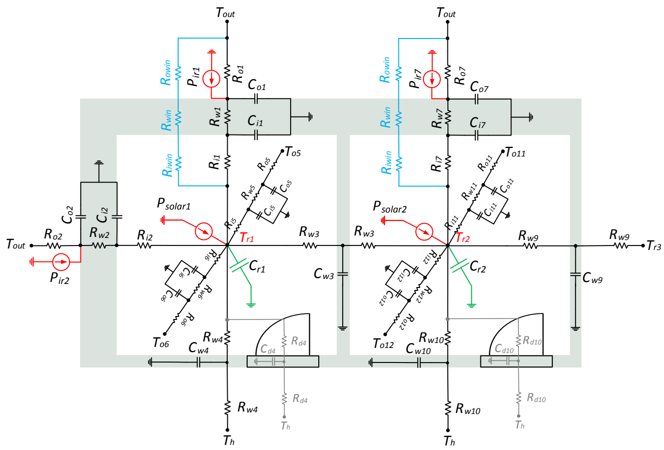

Fig. 1. RC model of two rooms

The mathematical model expressed by differential equations is reformulated into state-space representation suitable for model predictive control, as follows:

$$\dot{x} = Ax + Bu, \tag{3}$$
$$y = Cx. \tag{4}$$

Vector x contains all zone temperatures including wall temperature, window temperature, door temperature, etc. Vector y represents room air temperature. Vector u contains control variables, i.e. fan-coil unit heat energies.

The same approach is applied on the considered case study of ninth floor of the building of Faculty of Electrical Engineering and Computing in Zagreb, resulting in model that contains 23 rooms, includes 391 state variables, 23 inputs and 23 outputs. Prediction of disturbances are obtained by measuring outside temperature and solar irradiance in the year of 2014.

III. Model predictive control

The MPC requires a discrete model given in (5)-(7):

$$x_{k+1} = Ax_k + B_u u_k + B_d d_k, \tag{5}$$

$$y = Cx_k, \tag{6}$$

$$\begin{aligned} x_{k+1} =& Ax_k + B_u u_k + B_T T_{o,k} \\ &+ B_{T+1} T_{o,k+1} + B_d d_k, \end{aligned} \tag{7}$$

where $k \in Z$ is sampling time. It is obtained from (3) and (4) by applying a ZOH discretization. The B_u describes relates state variables with inputs and B_d with disturbances. The $T_{o,k}$ is vector of outside air temperature and d_k is vector of disturbances that act on the system such as solar irradiance, internal gains, etc. For improving the model accuracy, influence of outdoor temperature (T_o) variation within the sampling time interval is included by linear interpolation of temperature between the current and the next sampling time instant (k and $k+1$) as a first-order hold extrapolation [16].

A prediction horizon of 24h is chosen together with a sampling time of 1h as proven adequate to capture the relevant dynamics for such high-level temperature control. With limitation on the system input, individual room controllers determine the total amount of heat energy that can be requested by the system, for simultaneous heating of all the rooms. It is very important, because systems are often designed not to withstand the peak operation or may yet be downscaled to save the costs and still deliver the required heat, controlled by MPC [16]. Each of the room actuators are also constrained in possible heat delivery as (8):

$$u_{min} \leq u_i \leq u_{max}, \tag{8}$$

where i is the index representing rooms. Parameter u_{min} is maximum cooling energy that can be delivered into zone within an hour (cooling has negative sign). Similar to that, u_{max} represents maximum heating energy.

Cost function J of the problem is a linear quadratic function in form of a reference w tracking problem:

$$J_k = J_{k-1} + (y_k - w_k)^T q_k (y_k - w_k) + u_k^T r_k u_k. \tag{9}$$

Function takes two criteria into consideration. First one is setpoint tracking, so minimization between currently temperature and setpoint is made. Second criterion is minimization of the energy. In this way, a trade-off is accomplished between users comfort and energy savings. By increasing parameter q, an emphasis is put to the comfort, and controller energy efficiency is less significant. On the other hand, by increasing parameter r, controller saves more energy but at the cost of user comfort.

Last part of the MPC implementation is solving the problem. We used a solver IBM ILOG CPLEX [14]. After that, optimal vector of inputs is calculated on the time horizon.

IV. Distributed optimal control

Distributed optimal control is based on dividing one problem to many smaller ones. Each problem is then solved separately and independently. Solutions are passed to the central supervisor that makes additional decisions on global level and steers the distributed solutions towards the joint optimum. Ultimately, it is expected to achieve the same results as the centralized MPC. The main advantage is that solving few smaller problems is sometimes computationally faster, than solving a large one. In this particular application, distributed approach enables fast zone digitalization as a practical benefit, with a usage of simple RC models. Also, another advantage is to avoid the sharing of private data as the central supervisor knows only what is the total amount of the requested thermal energy, without information about the individual zone models or room temperature setpoints.

A. Asymmetric projection algorithm

Asymmetric projection algorithm (APA) is a method of decentralized control originally proposed in [15]. The APA is based on iterative convergence of control actions and constraints, along their individual gradients, until the equilibrium point is reached, i.e. the global optimum. Such an equilibrium point is aligned with game theory premises of minimizing the risk of a negative joint decision outcome for all the players, leading to a common best interest strategy, which is called the Nash equilibrium. In this case, particular zone MPCs of rooms are considered as the players. Global, joint constraints of the system from (8), are formulated as:

$$G\boldsymbol{u} \leqslant W. \tag{10}$$

where $\boldsymbol{u}$ is a vector of requested thermal energies extended on the time horizon. Constraint matrices W and G assure that total thermal energy does not exceed the defined limits. Local constraints correspond to the (8), keeping thermal energy of each zone within the allowed levels. Constraints for each zone are set independently, hence the name of asymmetric projection.

The APA consists of these 3 steps:

Step 1: Central operator average update:

$$\sigma_j \leftarrow \frac{1}{N} \sum_{n=1}^{N} u_j^i,$$

Step 2: Local individual strategy update:

$$u_{(j+1)}^i \leftarrow \Pi_{\chi^i}[u_j^i - \tau(\mathcal{F}^i(u_j^i, \sigma_j) + G_{(:,i)}^T \lambda_j)],$$

Step 3: Central operator multiplier update:

$$\lambda_{(j+1)} \leftarrow \Pi_{R^m_{\geqslant 0}}[\lambda_j - \tau(W - 2Gu_{(j+1)} + Gu_j)].$$

First, algorithm initialization is performed where starting value of number of iterations j is set up to be zero. Parameter τ represents a size of the step taken in the direction of the gradient. Larger value of τ means that step rate is also higher, but it can potentially lead to the algorithm divergence. Setting up τ as a very small number secures the convergence, but gives needlessly long calculations. Therefore, it is recommended to use τ that is somewhere in between these two extremes. Multiplier vector λ is a non-negative parameter that secures adherence to the global constraints. If λ is equal to the zero, global constraints are satisfied and if λ is larger than the zero, the constraints are not triggered. In the beginning of the algorithm execution λ is filled with small positive numbers.

In the first step, central supervisor accepts all the requested energy demands from every zone and calculate the average thermal energy demand σ. As a central operator of the algorithm. Then in Step 2, the central supervisor sends central operator to each zone, so zones can adjust theirs initial demand using central operator within gradient of cost function. Also, factor that includes λ is added so the global constraints will be respected. Newly obtained demand, i.e. the input variable, is potentially placed outside of the allowed hyperspace due to arbitrary selection of the initial conditions. That means that local constraints are no more satisfied. Therefore, projection is made to find a new demand within the allowed hyperspace as the closest point to the original one, defined by the Euclidean norm. Projection is given by (11):

$$\Pi_{\chi} y = \operatorname{argmin}_{x\in\chi} \|y - x\|_2. \tag{11}$$

In the third step of APA, central supervisor accepts new energy demands from all the zones and obtains the central operator λ to assure the respecting of the global constraints. It is possible that λ is outside the feasible hyperspace and therefore the projection (11) is performed here again to avoid the negative values of λ.

With the iterative execution of the algorithm steps, the control variable is moving toward the Nash equilibrium and the difference in energy demands between two iterations becomes smaller. The algorithm is therefore executed until the convergence is satisfied, defined as the e.g. end condition:

$$\|u_{(k+1)} - u_k\| \leqslant \varepsilon_{toll}, \tag{12}$$

where ε_{toll} is a small arbitrary number defining the allowed suboptimallity of the solution, i.e. with $\epsilon_{toll} \to 0$, the solution is optimal and coinciding with the result of the centralized MPC. The choice of ϵ_{toll} is also a trade-off between the suboptimality and the execution time.

B. APA implementation

Decentralized system implies that each zone has their own model that is unique and independent. State variables of every model are temperatures of a room, which that model describes. Temperatures of other nearby zones are represented as system disturbances alongside with outside temperature, solar irradiance, etc. Energy demand of every zone i is calculated by their own local MPC.

Cost function from (9) is put in the form (13):

$$J^i = \boldsymbol{u^i}^T \mathcal{H}^i \boldsymbol{u^i} + f^i \boldsymbol{u^i} + g^i. \tag{13}$$

The gradient of (13) is obtained as:

$$\mathcal{F}^i = \nabla J^i = \frac{\partial J^i}{\partial \boldsymbol{u^i}} = (\mathcal{H}^i N\sigma + f^i)^T, \tag{14}$$

where N is number of zones. Matrices $\mathcal{H}^i$ and f^i are given in (15) and (16), respectively:

$$\mathcal{H}^i = \boldsymbol{B_u^i}^T \boldsymbol{C^i}^T Q \boldsymbol{C^i} \boldsymbol{B_u^i} + R, \tag{15}$$

$$\begin{aligned} f^i =& 2x_0^T \boldsymbol{A^i}^T \boldsymbol{C^i}^T Q \boldsymbol{C^i} \boldsymbol{B_u^i} \\ &+ 2\boldsymbol{d^i}^T \boldsymbol{B_d^i}^T \boldsymbol{C^i}^T Q \boldsymbol{C^i} \boldsymbol{B_u^i} \\ &- 2\boldsymbol{r^i}^T Q \boldsymbol{C^i} \boldsymbol{B_u^i}. \end{aligned} \tag{16}$$

where Q and R are weight parameters contained in the model predictive control of the room temperature.

V. Simulation results

The described algorithm is applied to the case study of ninth floor of the building of Faculty of Electrical Engineering and Computing in Zagreb and corresponding 23 rooms (offices). The rooms are equipped with two-pipe fan coil units that are capable of seasonal heating or cooling, depending on the medium temperature that is conditioned in the central system. Prediction horizon of 24 hours is selected as a relevant to capture long-term dynamics, and as usually available weather forecast period. The MPC cost function parameters q and r are chosen to be 1000 and 10^{-5}, respectively with such a large difference to co-measure the temperature setpoint tracking and heat energy transfer. Three of the rooms have two fan coil units, while each of the remaining rooms has one. All of the fan coil units are identical and have a maximum thermal power of 3.5 kW. Thermal power is also defined as a positive number, so cooling is not possible. The parameters for the APA algorithm τ and ϵ_{toll} are set to 0.1 and 10, respectively, and corresponding to watts over the discrete time step, i.e. having the significance of energy.

Figure 2 shows the outside temperature and solar irradiance from two main directions, north and south, corresponding to the building orientation. The presented day is January 2, 2014. The simulations are performed as closed-loop with corresponding MPCs, model matrices from (5)-(7), and weather data as disturbances.

Figure 5 shows temperatures and energy consumptions for all 23 rooms with distributed control on January 2nd. During working hours, 6:00-18:00, setpoint is set to 22 $^\circ C$ and to 15 $^\circ C$ for the rest of the day to simply switch-off the system. Additionally, apart from the working hours, reference tracking is neglected and parameter q is set to zero since there is no need to maintain the temperature setpoint. The successful reference tracking of the 23 zones is evident in Fig. 5, together with the corresponding heating powers. Figure 3 shows comparison of individual, local MPCs in each room where the local optimum is achieved, without specified joint constraint or the effort towards the reduction of overall control action, i.e. global energy efficient operation, and the distributed MPCs that are jointly coordinated through the iterations such that the global conditions are satisfied. The figure shows the comparison for selected e.g. rooms 5 and 21, corresponding to south and north position, respectively. Local MPC zone controllers are denoted as 'Lz3' and 'Lz21', and distributed

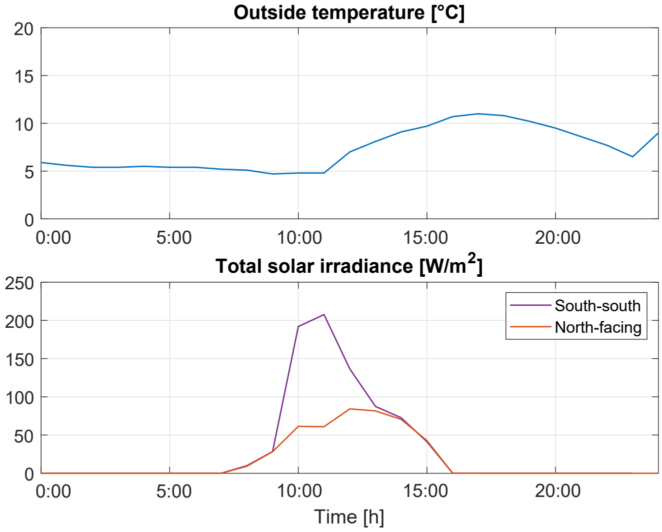

Fig. 2. System disturbances

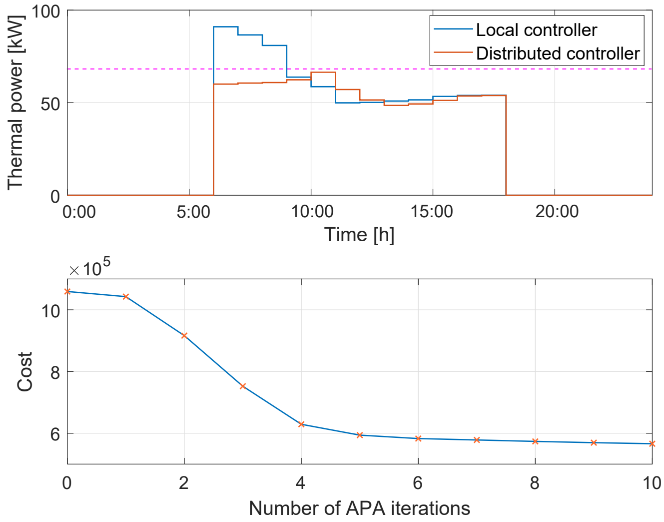

Fig. 4. Distributed control improvements

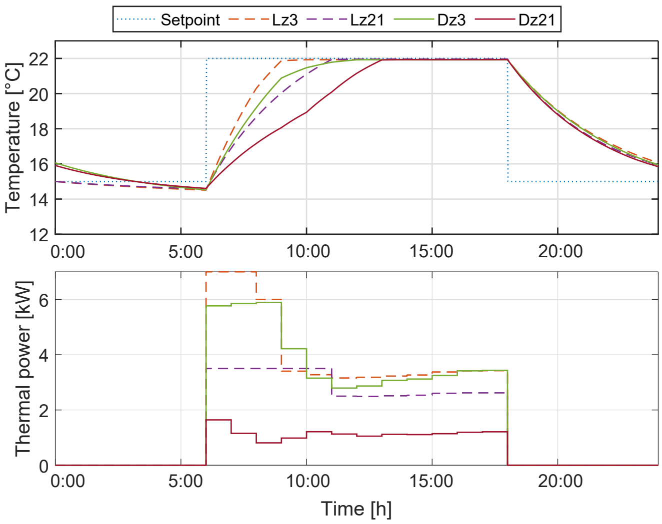

Fig. 3. Comparison of local control and distributed control

MPCs as 'Dz3' and 'Dz21'. Although local control realizes better reference tracking, distributed control improves overall floor energy savings, which ultimately leads to the joint energy efficiency goal. The subject of reference tracking emphasis over energy saving trade-off may be further tuned by selecting MPC cost weighting coefficients q and r.

The coordinated MPC control and APA algorithm respect the joint, global constraint as described in previous sections, which can be imposed by heat station limitations and possibility to very accurately determine the heating need. In our simulations, the joint constraint is set to 70 kW, while the maximum possible requested power, without the joint constrain, is 91 kW. Respecting the joint constraint may be observed in Fig. 4, where the constraint is denoted with the dashed line. The central supervisor of the distributed control adjusts zone thermal powers through iterations, so that total power is never higher than 70 kW. The figure also shows how the distributed MPC decreases the overall system cost function through the iterations, while the sum of local MPC costs are equivalent to the starting cost of the APA, i.e. the cost when the number of iterations is zero.

Finally, Fig. 5 shows the overall heating system results for January 2, 2014, for all 23 rooms of the considered building floor. The difference between north and south rooms may also be observed in the heating powers where the MPC for south-facing rooms tends to lower heating power during high-irradiance period (shown in Fig. 2).

VI. Conclusion

The model predictive control in room temperature control is proven as a promising possibility in increasing energy efficiency of the buildings. The main advantage of MPC is the fact that it allows shifting of trade-off between comfort and energy savings, to the more favourable scale, resulting in potentially significant improvements of the energy efficiency. Distributed model predictive control brings potential algorithm improvements such as reduced complexity of the problem and possibly faster execution. However, it also introduces significant practical potential in the particular terms of fast and cheap zone digitalization, and data privacy protection. Implementation of such a control was made for the given thermal model using the asymmetric projection algorithm with individual room controllers optimizing the local operation and iteratively bidding to the global goal of energy efficiency.

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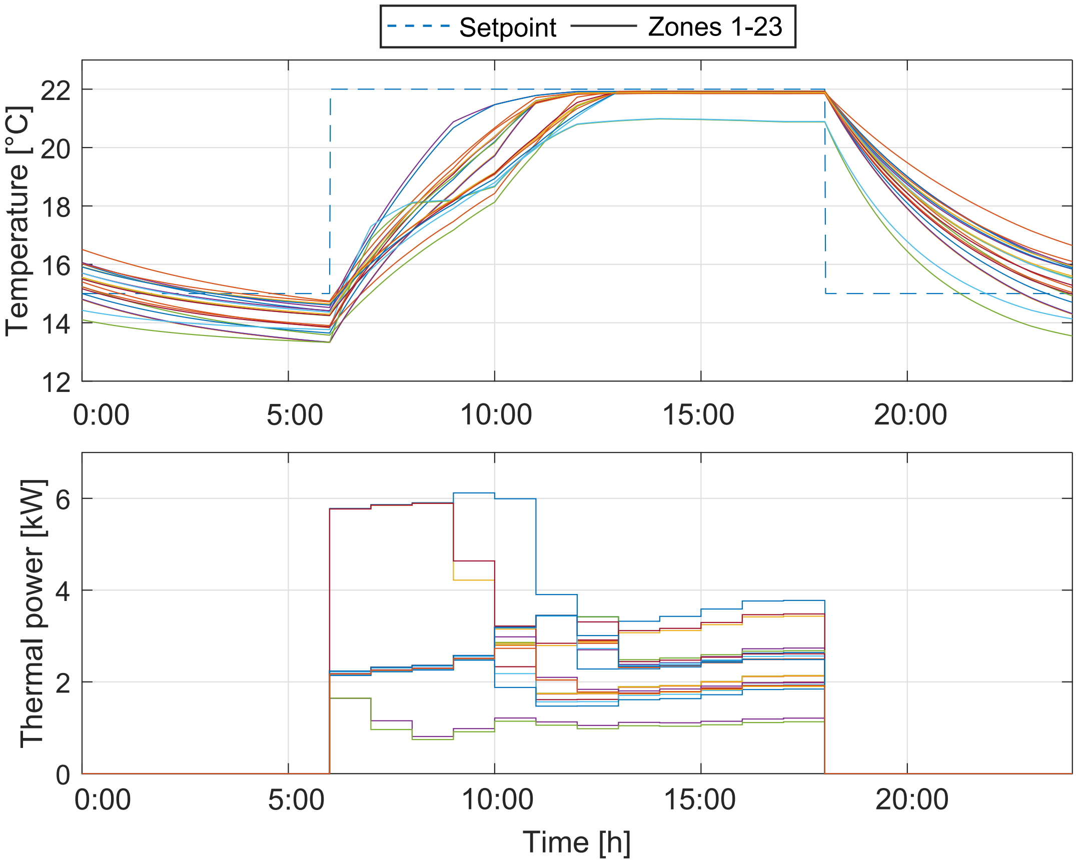

Fig. 5. Zone temperatures and heating power - distributed control

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