# Statistical Analysis of Missile's Trajectory Deviation Due to Production Errors

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The paper describes the model for assessing the effect that manufacturing errors have on the missile's trajectory and the position of its impact point. Manufacturing errors are considered to be non-deterministic variables. One typical error is analyzed: the erroneously manufactured missile's warhead. To test the resulting trajectory deviation, a realistic 3D CAD model of a missile is built and linked to the improved 6DOF model. A simulation using the Monte-Carlo method shows the resulting dispersion of missile's impact points.

## 1 Introduction

Because of the contradiction between the criteria of accuracy and cost, it is necessary to find a satisfying optimum. The traditional approach determines production tolerances empirically, which does not lead to satisfactory results. We suggest direct linking of the missile's 3D CAD model and the 6DOF flight model, to identify the most critical steps of the production. Earlier works deal with the same problem, but without linking the final dispersion to the manufacturing process [1]. The 6DOF model of an asymmetric projectile has been previously developed [2], but it is ony applicable to classical artillery projectiles. Our model also introduces the thrust force errors.

## 2 The 3D CAD Model

A developed 3D CAD model of the projectile is parametrically given, which allows the introduction of manufacturing errors after which the model gives modified projectile's geometric and inertia characteristics. The production process is this way simulated together with its immanent imperfection. As a result comes the real, imperfect projectile. Inertia characteristics relevant for the later analysis are mass $m$, inertia tensor $\mathbf{I}$, and center of gravity position $\boldsymbol{\rho} = [x\ y\ z]^T$, all calculated inside the CAD program for a particular set of manufacturing errors. In this paper, model's capabilities are demonstrated over the analysis of the erroneously manufactured warhead. If the warhead is not made perfectly, there will be a misalignment of the inner surface's axis of symmetry $x_E$, relative to the outer surface's axis of symmetry $x_H$.

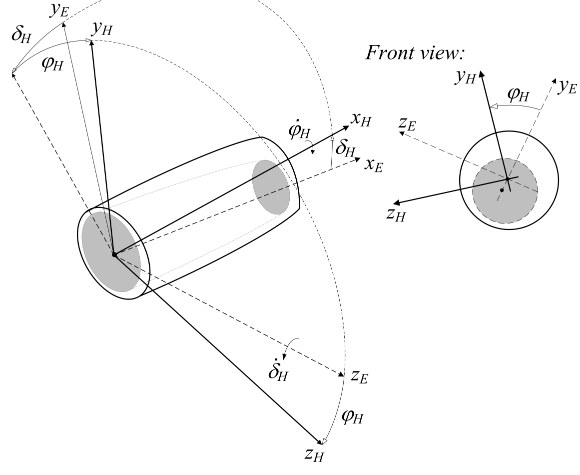

**Fig. 1:** The misalignment between the warhead's inner and outer surface.

The angle between two axes is $\delta_H$, situated in the disturbance plane $x_E - y_E$ (Figure 1). The angle $\varphi_H$ defines the rotation of this plane relative to the reference plane $x_H - y_H$. Both $\delta_H$ and $\varphi_H$ are nondeterministic variables. Because of the manufacturing error, aforementioned inertia characteristics are modified, leading to the deviation of the missile impact point.

## 3 The 6DOF Model of Flight

The 6DOF model of flight is here adjusted for projectiles with geometrical or inertia asymmetry. The 6DOF model generally follows from four differential equations for position $\mathbf{r}$, for ground velocity $\mathbf{V}_K$, for angular momentum $\mathbf{H}$ and for attitude $\mathbf{s}$:

$$
\begin{aligned}
&\dot{\mathbf{r}} = \mathbf{V}_K^G \qquad && m\left(\dot{\mathbf{V}}_K^G + \widetilde{\boldsymbol{\Omega}}_G^G \cdot \mathbf{V}_K^G\right) = \mathbf{F}_A^G + \mathbf{F}_T^G + \mathbf{L}_{GO} m \mathbf{g}^O - m\mathbf{a}_c^G \\
&\dot{\mathbf{s}} = \mathbf{R}^{-1} \cdot \left(\boldsymbol{\Omega}_G^G - \boldsymbol{\Omega}_O^G\right) && \dot{\mathbf{H}}^G + \widetilde{\boldsymbol{\Omega}}_G^G \cdot \mathbf{H}^G = \mathbf{M}_A^G + \mathbf{M}_T^G\ ,
\end{aligned}
\tag{1}
$$

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where $\mathbf{F}_A$ and $\mathbf{F}_T$ are aerodynamic and thrust force respectively, and $\mathbf{M}_A$ and $\mathbf{M}_T$ are their moments. Solving equations (1), especially derivation of the angular momentum equation, becomes complex for bodies of changing mass and produced with an asymmetry (hence dynamically imbalanced). If 6DOF would be developed in the frame coordinate system, the aerodynamic must be constantly recalculated. Therefore, the improved G6DOF is developed in the geometrical coordinate system $G$, which is linked to the outer surface just as aerodynamics is. For G6DOF model equations (1) are written in $G$ coordinate system, using Earth fixed spherical coordinate system for the position $\mathbf{r} = [\phi_E \; \lambda \; h]^T$ and aeronautical three-angles notation for the attitude $\mathbf{s} = [\phi_G \; \theta_G \; \psi_G]^T$. Vector $\mathbf{a}_c^G$ presents Coriolis acceleration in $G$ and for gravity acceleration $\mathbf{g}$ a transformation is introduced from north-east-down coordinate system $O$ to the $G$ with the transformation matrix $\mathbf{L}_{GO}$. In the state vector $\mathbf{X} = [\phi_E \; \lambda \; h \quad u_K \; v_K \; w_K \quad H_x \; H_y \; H_z \quad \phi_G \; \theta_G \; \psi_G]^T$ the angular momentum is introduced instead of commonly used angular velocity, which is determined from $\mathbf{\Omega}_G^G = \text{inv}\left(\mathbf{I}^G\right) \cdot \mathbf{H}^G$.

## 4 Results

The analysis of the warhead manufacturing error effect is performed by the Monte-Carlo simulation. For the case study, an unguided missile of the well-known characteristics [3] is analyzed. Following parameters have been chosen:

- the angle $\delta_H$ is dispersed according to the Rayleigh probability density function (PDF); 95% of $\delta_H$ angles are inside the interval $0° < \delta_H < 0.5°$, which corresponds to the medium production quality;
- the angle $\varphi_H$ is uniformly dispersed around the $x_H$;
- the target is at the distance of 13700 m (2/3 of the maximum range);

For the Monte-Carlo simulation the 3D CAD model is linked to the G6DOF flight model, as described earlier. The obtained results are compared with the Firing Tables. Figure 2 shows the results of the Monte-Carlo simulation, i.e. impact points of 300 missiles, each produced with some random error described by $\delta_H$ and $\varphi_H$.

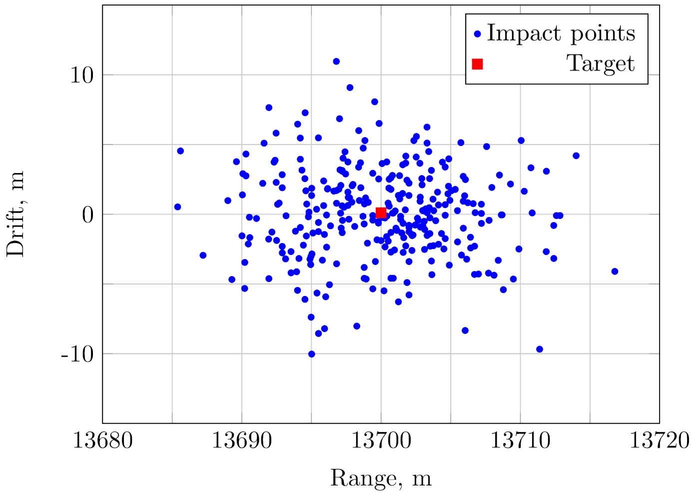

**Fig. 2:** Impact points for 300 missiles with simulated warhead manufacturing error.

The dispersion of the impact points shows a trend of concentrating around the target's position, decreasing in frequency as the distance from the target increases. This corresponds well with ballistic theory, as well as the data from the Firing Tables. The statistical analysis gives the estimated midpoint insignificantly shifted from the target position (< 0.1 m). The standard deviation was estimated at $s_X = 5.5$ m for range and $s_Z = 3.2$ m for drift, resulting in a small area of impact point dispersion.

## 5 Conclusion

The Firing Tables give the dispersion area 830 × 630 m with 99,5% confidence, which leads to the conclusion that dispersion is mostly caused by other disturbances. Based on this, the conclusion is made that warhead manufacturing is not a critical step in production. Our broader preliminary study shows indeed that some other manufacturing errors cause a significantly larger dispersion. Therefore, for warhead manufacturing, lower manufacturing quality and wider tolerances can be applied. The effect of other manufacturing errors should be investigated by the same methodology. Presented analysis helps to single out sensitive stages of production, where particularly strict manufacturing tolerances must be imposed.

## References

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