MULTI-CRITERIA DECISION-MAKING MODEL DESIGN FOR EFFICIENT SELECTION OF INVESTMENT PROJECTS

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1 Abstract

Investments are a basic prerequisite of economic growth and development, as well as increase in employment and population's living standard. Assessment of investment projects' efficiency is the focus of each enterprize's microeconomic analysis. Selection of investment projects is observed as a multi-criteria decision-making issue. The paper uses several different criteria classified into 2 basic categories - the static criteria (financial indicators from basic accounting statements) and the dynamic criteria (net present value, internal rate of return, etc.). The decision-maker demonstrates the importance of each criterion by using weights or by prioritizing the criteria. The greatest difficulty encountered by the arbiter is the assignment of numerical values to weights. This is why the model prioritizes the criteria, which causes the least problems for the arbiter. For this purpose, a mathematical model is formed for each investment project where the weights are variables with lower and upper boundaries set on them. The optimal solution for the mathematical model is the weight values for the relevant investment project. This way, investment projects are assigned different weight values. The weight values so obtained are then used to determine equal weights for all investment projects. The objective function combining all criteria is formulated as a weighted sum of the nominal criteria values. The assumption in the formulation of the mathematical model is that all criteria are positively oriented and normalized.

Keywords:*Investment Project, Static And Dynamic Criteria, The Arbiter, Multi-Criteria Decision Making Mathematical Model. Weight Variables, Optimal Solution*

2 Introduction

Managers use different methods to evaluate the efficiency of different investment projects in business and investment banks. This way they decide on authorizing credit placement to their comities. It is known that quantitative support is rarely used among economists and managers because of their lack of certain mathematical knowledge. One of the goals of this paper is to emphasize the simplicity of this procedure. Because of this the procedure has a very practical application and it will be available to managers and economists. Up to this point the manager was coerced into learning to use different specific multi-criteria decision making software (EXPERT CHOICE, DECISION LAB, ELECTRE III,...). Another problem arose while choosing weights of criteria, as you needed to apply various numerical values.

In a multi-criteria decision making the arbiter has to decide the weight value or relative preference degree as it is usually done at other multi-criteria decision making methods (AHP, PROMETHEE, ELECTRE,...).

Instead, the new approach requires business financing knowledge. The knowledge should manifest through the arbiters capability to rank the importance of the criteria in the model.

The process of selecting an investment has a couple of phases: In the first phase we formulate the criteria (nine were used in this paper). Then we rank the criteria in a descending order. The arbiter can alter the criteria and even add new criteria. In this phase the arbiter formulates his strategy by choosing his criteria and ranking them. The criteria ranking depends on the market and the arbiters policy. The next phase is the construction of a quantitative procedure to evaluate the investment project. The procedure is based on multi-criteria decision making in which the weight criteria is evaluated.

In this paper we propose a mathematical model in which the variables are the weight of criteria.

The formulated model is an extension of multi-criteria decision making by Ng (1.) by incorporating lower and upper bounds. The result of the mathematical model is the optimal weight values for every investment that applies its optimal score. We should also notice that the weights are not equal for every investment. In this step we should rank the investments and eliminate the ones with the lowest score.

Then we use the criteria values and the weights of the remaining investments for generating random criteria values and random weights of fictional proposals.

For every fictional investment we solve its associated problem and weight values. The weight values we use to calculate equal weights for every investment project and determine the score and the ranking of every investment.

This paper is organized with four more sections. Section 2 is about determination of the criteria, while in the third we formulate the model and its solutions. The fourth section we analyze the results, randomly generate proposals and determine weight values and investment ranking. The final section is the conclusion.

3 Choosing the criteria

When choosing an investment project, the commercial or investment banks, use a number of criteria. The procedure is described on a particular problem of one specific commercial bank whose data we have at our disposal. The bank needs to make a choice between five different investment projects. In these paper we choose nine criteria of which five are dynamic(Net Present Value - NPV, Internal Rate of Return - IRR, Payback Period - PBP, Accounting Rate of Return - ARR, Cumulative Cash Flows - CCF) and four static(Return on Investment - ROI, Net Profit Margin - NPM, Interest Coverage Ratio - ICR, Current Ratio - CR). The reasons for choosing those particular criteria are closely associated with the definition of the three intermitted elements of financial management: profitability, risk and liquidity. The balanced relationships between these three elements are crucial for achieving the fundamental goal of every commercial bank: successful business transactions. All three elements are mutually dependent and are in a conflicting relationship. That means that alteration in one of these elements brings alteration to the other two. For instance: the increase of profitability brings forward the increase of risk and decrease of liquidity. To each and every criterion in the mathematical model every single one of the three elements is defined. This way we achieve a table in the form of a data matrix, with the final order of all three elements given by their relative importance in the view of priorities of commercial banks:

1.Profitability; 2.Risk; 3.Liquidity

The final order of the criteria will not only depend on the presence of these three elements, but also whether the chosen criterion is dynamic or static and whether the dynamic criterion takes into account the use of discounted cash flows or not. The arbiter must decide the importance of every single criterion and we recommend the following ranking:

1. Net Present Value - NPV
2. Internal Rate of Return - IRR
3. Payback Period - PBP
4. Accounting Rate of Return - ARR
5. Cumulative Cash Flows - CCF
6. Return on Investment - ROI
7. Net Profit Margin - NPM
8. Interest Coverage Ratio - ICR
9. Current Ratio - CR

In order to better understand the model it is necessary to explain the reasons for choosing the specific dynamic or static criteria. It is known that the five dynamic criteria are fully adequate for the assessment of the investment projects efficiency, but the four chosen static criteria are more apt for the analysis of business transactions up to this point(based on available accounting reports), rather than future investment projects. For this model it is taken into consideration a specific combination of both static and dynamic criteria. The results of dynamic criteria are based on budgeted values of cash flows, while the results of static criteria are realistic values taken from accounting reports of the last reporting period. Because of this only two static indicators were chosen, profitability and liquidity, which possess crucial characteristics fundamental for the models forming.

4 Mathematical model

It is necessary to select one of the five investment projects. After the criteria are chosen and their measures obtained, the assessment is based on the weighted sum of normalized and positively related measures.

The measure of investment i under criteria j is denoted as:

$$x_{ij}(i = 1, ...5; j = 1, ...9)$$

All of the criteria except the PBP (Payback Period) are the benefit criteria, which mean that achieving their maximum values is in the interest of the investor. We say that those criteria are positively related. It is necessary for the table to consist of solely benefit (positively related) or cost (negatively related). Because of this the cost criterion (PBP) needs to be treated as a benefit criterion in the following way:
We enter the transformed value x_{i3}, it's reciprocal values $\frac{1}{x_{i3}}$ for every investment i.
All nine positively related criteria, that are sequenced by importance and will be calculated in to the mathematical model. In the first step they are written in the following Table 1, where X_j is the criterion and Y_i is an investment:

	Y_1	Y_2	Y_3	Y_4	Y_5	The Criteria
X_1	85,80	75,68	75,14	70,82	53,63	1.NPV
X_2	33,58%	37,93%	18,07%	18,24%	12,76%	2.IRR
X_3	0,1816	0,1690	0,1135	0,1105	0,0841	3.$\frac{1}{PBP}$
X_4	6,47%	6,96%	6,49%	8,89%	38,24%	4.ARR
X_5	180,60	157,98	255,20	244,26	322,59	5.CFF
X_6	5,40 %	6,23%	6,40%	8,13%	9,57%	6.ROI
X_7	29,78%	35,69%	25,77%	28,88%	31,03%	7.NPN
X_8	2,56	5,18	2,80	2,46	3,93	8.ICR
X_9	1,64	1,95	2,14	1,75	1,63	9.CR

The second step is the transformation of values for the positively related criteria. Here we use percentage transformation. This transformation is done because it leads to proportional changes.In Table 2 we denote all transformed measures with r_{ij}(i=1...5,j=1...9) as shown:

	Y_1	Y_2	Y_3	Y_4	Y_5	Total	The Criteria
X_1	0,2376	0,2096	0,2081	0,1961	0,1486	1,000	1.NPV
X_2	0,2785	0,3145	0,1499	0,1512	0,1058	1,000	2.IRR
X_3	0,2757	0,2566	0,1722	0,1677	0,1277	1,000	3.$\frac{1}{PBP}$
X_4	0,0965	0,1038	0,0968	0,1326	0,5704	1,000	4.ARR
X_5	0,1556	0,1361	0,2199	0,2105	0,2779	1,000	5.CFF
X_6	0,1511	0,1744	0,1791	0,2276	0,2677	1,000	6.ROI
X_7	0,1970	0,2361	0,1705	0,1911	0,2053	1,000	7.NPN
X_8	0,1509	0,3059	0,1657	0,1455	0,2320	1,000	8.ICR
X_9	0,1802	0,2142	0,2351	0,1914	0,1791	1,000	9.CR

The following are the parameters of the mathematical model:

The formulation of the mathematical model is with the assumption off positively related and normalized criteria. The criteria are arranged by importance-from most important to least important, the first criterion is more important than the second, the second, from the third and so on.

After that we formulate the following notations for the mathematical model:

1. The Indexes

   i-investment$(i = 1, ..., m)$
   j-criterion $(i = 1, ..., n)$

2. The Parameters

   r_{ij}-the value of criterion j of investment i,$(i = 1, .., m; j = 1, ..., n)$

3. The Variables

   w_{ij}-the weight of the investment i under the criterion j $(i = 1, ..., m; j = 1, ..., n)$

Now we formulate a mathematical model for every investment $i(i = 1, ..., m)$ as follows:

$$\max \sum_{j=1}^{n} r_{ij} w_{ij}$$

$$w_{ij} - w_{i(j+1)} \geq 0, j = 1, ..., n-1 \quad (1)$$

$$l_j \leq w_{ij} \leq u_j, j = 1, ..., n \quad (2)$$

$$\sum_{j=1}^{n} w_{ij} = 1 \quad (3)$$

$$w_{ij} \geq 0, j = 1, ..., n \quad (4)$$

For every investment i objective function represents the value of the weighted sum of normalized values of criterion j. The first group of constraints (1) represents the criteria ranking and the ranking of weights by importance. The second group of constraints (2) represents lower and upper bounds on the weighted values of criterion j in investment i. The third constraint (3) is normalization. The fourth group of constraints (4) is that the weights are non-negative.

We are given a linear programming problem in which the optimal solution is the weight value for which investment i has the largest score. In this paper we have n=9,m=5. We also use $l_j = \frac{9-j}{45}$ and $u_j = \frac{11-j}{45}$ for every j. We now solve five linear problems for each of the five investments(i=1,...,5). Then we rank the investments based on the scores of the nine criteria(j=1,...,9).
Objective functions maximum values S_i and the optimal values of the variables w_{ij} for every five investments are in the following Table 3:

	w_{i1}	w_{i2}	w_{i3}	w_{i4}	w_{i5}	w_{i6}	w_{i7}	w_{i8}	w_{i9}	Score	Rank
Y_1	0,2220	0,2000	0,1780	0,1110	0,0890	0,0780	0,0780	0,0220	0,0220	0,2165	3
Y_2	0,2000	0,2000	0,1760	0,1110	0,0890	0,0670	0,0670	0,0670	0,0210	0,2266	2
Y_3	0,2220	0,1560	0,1560	0,1110	0,1110	0,1110	0,0450	0,440	0,0440	0,1768	5
Y_4	0,2220	0,1560	0,1330	0,1225	0,1225	0,1110	0,0890	0,0220	0,0220	0,1811	4
Y_5	0,1780	0,1560	0,1560	0,1560	0,1330	0,1110	0,0500	0,0500	0,0100	0,2422	1

Notice that the investment with the largest score is investment Y_5, whose maximal objective function S_5 is 0,2422.

5 Weight determination

As the weight values depend on the investment, we use them to calculate a new score for every investment and then the new order. First we have to decide which investments, based on the ranking, need further analysis. Of the chosen criterion only three investments stand out based on their score values: Y_1, Y_2 and Y_5. Y_3 and Y_4 we leave out of further analysis. Next, by generating random numbers, we create fictional investments that have weight values w_{ij}. Because we choose the three best investments, we can enlarge the representative sample by formulating nine more investments($FY_1 - FY_9$) with random generated weight values.

The upper and lower bounds of the interval will depend on the three realistic investments. Of all the obtained criteria values we calculate the average values that then give the score and investment rankings. The values of all twelve investments are shown in Table 4:

	w_{i1}	w_{i2}	w_{i3}	w_{i4}	w_{i5}	w_{i6}	w_{i7}	w_{i8}	w_{i9}
Y_1	0,2220	0,2000	0,1780	0,1110	0,0890	0,0780	0,780	0,0220	0,0220
Y_2	0,2000	0,2000	0,1780	0,1110	0,0890	0,0670	0,0670	0,0670	0,0210
Y_5	0,1780	0,1560	0,1560	0,1560	0,1330	0,1110	0,0500	0,0500	0,0100
FY_1	0,2070	0,1710	0,1700	0,1610	0,0905	0,0820	0,0510	0,0390	0,0390
FY_2	0,1990	0,1890	0,1750	0,1440	0,1240	0,0610	0,0490	0,0330	0,0260
FY_3	0,2350	0,1750	0,1740	0,1180	0,0890	0,0823	0,0510	0,0510	0,0247
FY_4	0,1990	0,1950	0,1920	0,0970	0,0960	0,0950	0,0850	0,0350	0,0060
FY_5	0,2210	0,2091	0,1611	0,1268	0,0870	0,0790	0,0630	0,0440	0,0091
FY_6	0,1590	0,1480	0,1406	0,1340	0,1250	0,0998	0,0850	0,0770	0,0316
FY_7	0,2070	0,1680	0,1610	0,1090	0,0920	0,0870	0,0850	0,0680	0,0230
FY_8	0,1790	0,1770	0,1580	0,1520	0,1250	0,0950	0,0570	0,0350	0,0220
FY_9	0,2290	0,1880	0,1600	0,1540	0,1150	0,0660	0,0490	0,0310	0,0080
Total	2,4350	2,1761	2,0036	1,5738	1,2545	1,0032	0,7700	0,5520	0,2319
Average	0,2029	0,1813	0,1670	0,1312	0,1045	0,0836	0,0642	0,0460	0,0193

Investment rank:	Investment Y_5	Score:	S_5=0,224
	Investment Y_2		S_2=0,218
	Investment Y_1		S_1=0,209

Now we introduce nine fictional investments that with random generated criteria values are based on the criteria values of the chosen three realistic investments. The parameters of the nine mathematical models and the generated criteria values are given in Table 5:

	r_{i1}	r_{i2}	r_{i3}	r_{i4}	r_{i5}	r_{i6}	r_{i7}	r_{i8}	r_{i9}
Y_1	0,2376	0,2785	0,2757	0,0965	0,1556	0,1511	0,1970	0,1509	0,1802
Y_2	0,2096	0,3145	0,2566	0,1038	0,1361	0,1744	0,2361	0,3059	0,2142
Y_5	0,1486	0,1058	0,1277	0,5704	0,2779	0,2677	0,2053	0,2320	0,1791
FY_1	0,2249	0,1753	0,2636	0,1375	0,2315	0,2976	0,1397	0,1161	0,2342
FY_2	0,1760	0,1790	0,2850	0,5546	0,2934	0,2093	0,2275	0,1183	0,2053
FY_3	0,2118	0,2222	0,2868	0,3119	0,2339	0,2708	0,1050	0,2357	0,1906
FY_4	0,2158	0,2706	0,1403	0,5344	0,1713	0,2855	0,2898	0,1996	0,2668
FY_5	0,1194	0,2513	0,2614	0,1560	0,1273	0,2088	0,1432	0,2587	0,1344
FY_6	0,1415	0,2384	0,1960	0,1743	0,2367	0,1072	0,1357	0,1993	0,1676
FY_7	0,1008	0,2689	0,2744	0,2113	0,1733	0,1534	0,1314	0,1530	0,1160
FY_8	0,1248	0,2753	0,1295	0,1273	0,2903	0,1612	0,1623	0,1199	0,1330
FY_9	0,1399	0,1388	0,1699	0,1160	0,1647	0,2341	0,1939	0,2602	0,2400

For every fictional investment we solve its associated problem and weight values. Of all the obtained weight values we calculate the average values that then give the score and investment rankings. The value off all twelve investments ate shown in Table 6:

	w_{i1}	w_{i2}	w_{i3}	w_{i4}	w_{i5}	w_{i6}	w_{i7}	w_{i8}	w_{i9}
Y_1	0,2220	0,2000	0,1780	0,1110	0,0890	0,0780	0,780	0,0220	0,0220
Y_2	0,2000	0,2000	0,1780	0,1110	0,0890	0,0670	0,0670	0,0670	0,0210
Y_5	0,1780	0,1560	0,1560	0,1560	0,1330	0,1110	0,0500	0,0500	0,0100
FY_1	0,2220	0,1780	0,1780	0,1115	0,1115	0,1110	0,0440	0,0220	0,0220
FY_2	0,1780	0,1780	0,1780	0,1560	0,1330	0,0725	0,0725	0,0220	0,0100
FY_3	0,1780	0,1780	0,1780	0,1560	0,1110	0,1110	0,0440	0,0340	0,0100
FY_4	0,1885	0,1885	0,1560	0,1560	0,0890	0,0890	0,0890	0,0220	0,0220
FY_5	0,1945	0,1945	0,1780	0,1110	0,0890	0,0890	0,0670	0,0670	0,0100
FY_6	0,1955	0,1955	0,1780	0,1330	0,1330	0,0670	0,0440	0,0440	0,0100
FY_7	0,2000	0,2000	0,1780	0,1560	0,1230	0,0670	0,0440	0,0220	0,0100
FY_8	0,2000	0,2000	0,1330	0,1330	0,1330	0,0845	0,0845	0,0220	0,0100
FY_9	0,1780	0,1560	0,1330	0,1110	0,1110	0,1110	0,0890	0,0670	0,4400
Total	2,3345	2,2245	2,0020	1,6015	1,3445	1,0580	0,7730	0,4610	0,2010
Average	0,1945	0,1854	0,1668	0,1335	0,1120	0,0882	0,0644	0,0384	0,0168

Investment rank:	Investment Y_5	Score:	S_5=0,226
	Investment Y_2		S_2=0,217
	Investment Y_1		S_1=0,209

From the results we can observe that by values of objective function the best investment is Y_5, followed by Y_2, and finally Y_1. The resulting ranking suggests to the arbiter which investment should be chosen.

6 Conclusion

In this paper we formulated a multi-criteria decision making model for the optimal choice of an investment project. This procedure should have a significant amount of influence in choosing what investment projects should be financed. The advantages of this procedure are the lack of requiring specific multi-criteria decision making knowledge and its simplicity. It's required from the arbiter to define the importance of criteria by rank rather than weight values.

From the analysis of the results we can deduce that the investment Y_5 is the best investment in all three cases. It is important to note that the choice of the best investment depends on specific circumstances that are defined in the model- type and overall criteria. If at the beginning we take different assumptions and parameters in the model(different criteria ranking, introducing new criteria, different bounds on weight values, different calculation of average weight values...) ranking of the chosen investments could differ.

6.1 Literature

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