The absence of reciprocity in active structures using direct velocity feedback

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The absence of reciprocity in active structures using direct velocity feedback

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Abstract

A variation of direct velocity feedback, often referred to as skyhook damping, is discussed in this paper. Skyhook damping cannot be regarded as collocated control method since only the action force component is collocated with the velocity sensor mounted onto the receiving part of the structure. The reaction control force component reacting off the source part of the structure does not have a collocated sensor. Depending on the characteristics of the passive structure under control, the feedback loop may be quite insensitive to the effects produced by the non-collocated reaction control force component, and maintain stability properties that are otherwise characteristic only for collocated control. Moreover, there exist additional effects related to the response of structures activated by the application of skyhook damping. It is shown in this paper that the structure subjected to such active control, although exhibiting stable response and linear input-output relationships, no longer complies with the reciprocity principle. The absence of reciprocity is interesting given the recent efforts in developing metamaterial cloaks, where one of the critical issues is how to design material structures or systems that demonstrate non-reciprocal behaviour.

Keywords: active structures; velocity feedback; non-reciprocal behaviour; metamaterial cloaks

1. Introduction

Direct velocity feedback can be used for active vibration control in mechanical structures. It has been shown that if collocated sensor-actuator pairs are used, the control method extracts energy from vibrating mechanical systems, and the feedback loop is in principle unconditionally stable [1]. The frequency response of practical sensor-actuator pairs can disrupt the stability of the feedback loop [2]. Nevertheless it has been demonstrated that in particular situations large feedback gains can be applied and significant active damping effects can be achieved [2–6].

One possible practical situation where the direct velocity feedback can be considered is the problem of vibration isolation. In such a case, the control scheme is as follows. The velocity sensor is placed at the receiving part of the structure. Its output is augmented by a negative feedback gain and fed back to a force actuator reacting between the receiving part of the structure and the source part of the structure [7]. This scheme with reactive force actuators driven with signals proportional to the absolute velocity of the receiving structure is often referred to as skyhook damping [5–7].

Skyhook damping is not a strictly collocated control method since only the action force component is collocated with the velocity sensor mounted at the receiving substructure. The reaction control force component reacting off the source substructure does not have a collocated sensor. A number of studies suggest stability problems related to the absence of the source body velocity sensor [8,9].

Nevertheless, there exist a class of vibration isolation problems that are suitable for the implementation of skyhook damping. Problems belonging to this class are characterised by the fact that the fundamental natural frequency of the receiving substructure, when uncoupled from the rest of the structure, is lower than the fundamental natural frequency of the source substructure. Such structures have been referred to as supercritical vibration isolation problems [8]. With supercritical vibration isolation problems, the feedback loop, with regard to its stability, is "tolerant" to the non-collocated reaction component of the control force [8].

However, as discussed in this paper, there exist additional effects related to the response of structures activated by the application of skyhook damping. The structure subjected to such active control, although exhibiting stable response and linear input-output relationships, no longer complies with the reciprocity principle. The absence of reciprocity may be interesting given the recent developments in the area of acoustic metamaterials, where one of the critical issues is how to design acoustic devices or materials which generate non-reciprocal behaviour, see for example [10] and the references therein.

2. Discussion

An active structure *S* equipped with a direct velocity feedback loop is shown in Figure 1. It is assumed that the structure is linear elastic. Velocity sensor is placed at point 2 of the structure and its output is fed back via a negative gain $-g$ to the control actuator reacting between points 2 and 1.

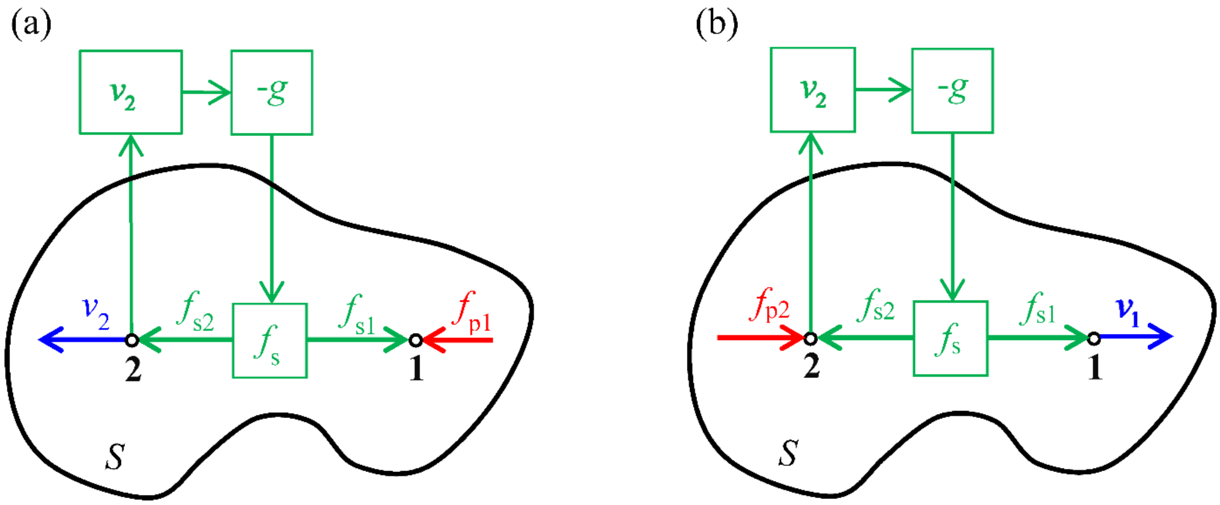

Figure 1: An active liner elastic structure (a) excited from point 1 and responding at point 2. (b) excited from point 2 and responding at point 1.

Provided that the feedback loop is stable, velocity response v_2 at point 2, due to the primary forcing f_{p1} at point 1, can be calculated as the sum of contributions from the primary force and the secondary (control) forces f_{s1} and f_{s2}:

$$v_2 = Y_{2,1} f_{p1} + Y_{2,2} f_{s2} + Y_{2,1} f_{s1} \,. \tag{1}$$

$Y_{2,1}$ is the transfer mobility of the passive system between points 2 and 1, and $Y_{2,2}$ is the driving point mobility of the passive structure at point 2. The secondary forces f_{s1} and f_{s2} generated by the control actuator are given by the control law:

$$f_{s2} = -g v_2 \,, \tag{2}$$

$$f_{s1} = g v_2 \,. \tag{3}$$

Substituting (2) and (3) into (1) and isolating for v_2, yields the transfer mobility function of the active structure S between the force f_{p1} and the velocity v_2:

$$Q_{2,1} = \frac{Y_{2,1}}{1 + g\left(Y_{2,2} - Y_{2,1}\right)} \,. \tag{4}$$

Considering now the situation shown in Figure 1b, where the structure *S* is excited by the primary force f_{p2} at point 2, and assuming again a stable controller, velocity v_2 can be calculated using (2) and (3) as:

$$v_2 = Y_{2,2} f_{p2} - Y_{2,2} g v_2 + Y_{2,1} g v_2 \,, \tag{5}$$

whereas velocity v_1 is given by:

$$v_1 = Y_{1,2} f_{p2} + Y_{1,1} g v_2 - Y_{1,2} g v_2 \,. \tag{6}$$

$Y_{1,2}$ is the transfer mobility of the passive structure *S* between points 1 and 2, and $Y_{1,1}$ is the driving point mobility of the passive structure *S* at point 1.

Eliminating v_2 from equations (5) and (6) yields the transfer mobility function of the active structure between the force f_{p2} and the velocity v_1:

$$Q_{1,2} = \frac{g\left(Y_{1,1}Y_{2,2} - Y_{2,1}^2\right) + Y_{2,1}}{1 + g\left(Y_{2,2} - Y_{2,1}\right)} \,. \tag{7}$$

As can be seen by comparing equations (4) and (7), $Q_{1,2} \neq Q_{2,1}$ unless g=0, thus the reciprocity principle does not hold if the system is made active.

The above formulation is valid if the active system is stable. The stability depends on the properties of the passive system, namely on the properties of the four mobility functions of the passive system $Y_{i,j}$. Therefore, an example system is considered next so that the mobilities of the passive and active systems can be calculated and the stability discussed. The example system is as shown in Figure 2.

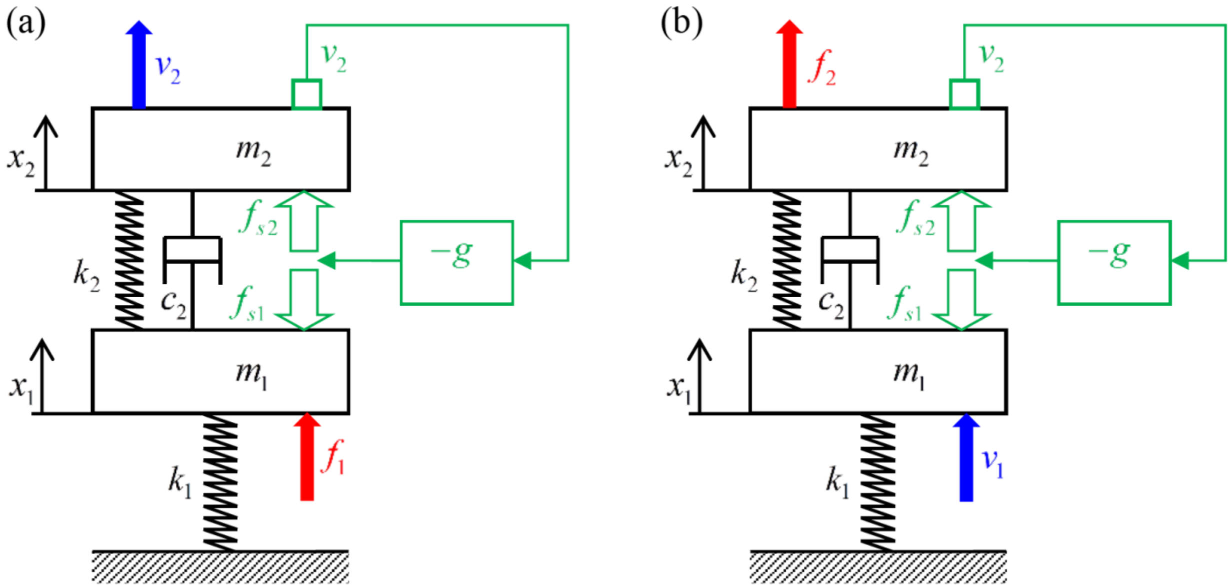

Figure 2: (a) An example active system excited from the base mass. (b) same as (a) but excited from the top mass.

The equations of motion for the passive system ($g=0$) shown in Figure 2 are:

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{f} \tag{8}$$

where **M**, **C**, and **K** are the mass matrix, the damping matrix and the stiffness matrix, respectively:

$$\mathbf{M}=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \tag{9}$$

$$\mathbf{K}=\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \tag{10}$$

$$\mathbf{C}=\begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, \tag{11}$$

and $\mathbf{x}=\begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^{\mathrm{T}}$ is the displacement vector and $\mathbf{f}=\begin{bmatrix} f_1(t) & f_2(t) \end{bmatrix}^{\mathrm{T}}$ is the force vector.

By assuming simple harmonic motion and the steady-state response, Eq. (8) becomes:

$$\mathbf{S}(\mathrm{i}\omega)\mathbf{x}(\mathrm{i}\omega)=\mathbf{f}(\mathrm{i}\omega), \tag{12}$$

where, after substituting $s=\mathrm{i}\omega$:

$$\mathbf{S}(s)=\begin{bmatrix} s^2m_1+sc_2+k_1+k_2 & -sc_2-k_2 \\ -sc_2-k_2 & s^2m_2+sc_2+k_2 \end{bmatrix}. \tag{13}$$

The receptance matrix of the passive system is thus given by:

$$\mathbf{X}(s)=\mathbf{S}^{-1}(s)\,. \tag{14}$$

The mobility matrix of the passive system is obtained by differentiating Eq. (14):

$$\mathbf{Y}=s\mathbf{X}\,. \tag{15}$$

By inverting the matrix **S,** the four elements of the mobility matrix are obtained:

$$Y_{1,1}=\frac{s\left(s^2m_2+sc_2+k_2\right)}{s^4m_1m_2+c_2\left(m_1+m_2\right)s^3+\left(m_2\left(k_1+k_2\right)+m_1k_2\right)s^2+k_1sc_2+k_1k_2}, \tag{16}$$

$$Y_{1,2}=\frac{s\left(sc_2+k_2\right)}{s^4m_1m_2+c_2\left(m_1+m_2\right)s^3+\left(k_1m_2+k_2\left(m_1+m_2\right)\right)s^2+k_1sc_2+k_1k_2}, \tag{17}$$

$$Y_{2,1}=Y_{1,2}=\frac{s\left(sc_2+k_2\right)}{s^4m_1m_2+c_2\left(m_1+m_2\right)s^3+\left(k_1m_2+k_2\left(m_1+m_2\right)\right)s^2+k_1sc_2+k_1k_2}, \tag{18}$$

$$Y_{2,2}=\frac{s\left(s^2m_1+sc_2+k_1+k_2\right)}{s^4m_1m_2+c_2\left(m_1+m_2\right)s^3+\left(k_1m_2+k_2\left(m_1+m_2\right)\right)s^2+k_1sc_2+k_1k_2}\,. \tag{19}$$

By substituting from Eqs. (16)-(19) into (4) and (7) the two transfer mobilities of the active system are obtained as:

$$Q_{2,1}=\frac{s\left(sc_2+k_2\right)}{s^4m_1m_2+\left(\left(m_1+m_2\right)c_2+gm_1\right)s^3+\left(k_1m_2+k_2\left(m_1+m_2\right)\right)s^2+k_1\left(c_2+g\right)s+k_1k_2}, \tag{20}$$

$$Q_{1,2}=\frac{s\left(\left(c_2+g\right)s+k_2\right)}{s^4m_1m_2+\left(\left(m_1+m_2\right)c_2+gm_1\right)s^3+\left(k_1m_2+k_2\left(m_1+m_2\right)\right)s^2+k_1\left(c_2+g\right)s+k_1k_2}, \tag{21}$$

so $Q_{1,2}\neq Q_{2,1}$ if $g\neq 0$, and the reciprocity principle does not hold.

The last step in the analysis is to show that the system is stable. According to the Routh-Hurwitz stability criterion, a necessary condition is that all coefficients $A_n,\, n=0...4$ multiplying increasing powers of s in the characteristic equation of the system have the same sign. Further to this, all principal diagonal minors of the Routh-Hurwitz array must be positive. The characteristic equation is the denominator of the right hand side of any of Eqs. (20),(21), and the coefficients A_n are $A_0=k_1k_2, A_1=k_1\left(c_2+g\right),..., A_4=m_1m_2$. Thus it can be seen from (21) that the necessary condition is certainly satisfied for any $g>0$. The sufficient condition is given by:

$$\Delta_1 > 0 \quad \Rightarrow \quad (c_2 + g)k_1 > 0, \tag{22}$$

$$\Delta_2 > 0 \quad \Rightarrow \quad (m_1 + m_2)c_2 + gm_1 > 0, \tag{23}$$

$$\Delta_3 = \begin{vmatrix} A_3 & A_4 \\ A_2 & A_1 \end{vmatrix} > 0 \quad \Rightarrow \quad (m_1 + m_2)\left((m_1 + m_2)c_2 + gm_1\right)k_2 + c_2 m_2^2 k_1 > 0, \tag{24}$$

$$\Delta_4 = \begin{vmatrix} A_3 & A_4 & 0 \\ A_1 & A_2 & A_3 \\ 0 & A_0 & A_1 \end{vmatrix} > 0 \quad \Rightarrow \quad k_1\left(k_2^2 (c_2 + g)k_1^2 + c_2 m_2^2 (c_2 + g)k_1 + k_2\left((c_2 + g)m_1 + c_2 m_2\right)m_2 g\right) > 0, \tag{25}$$

where Δ_{1-4} are the first four principal minors of the Routh-Hurwitz array and the vertical brackets in equations (24),(25) denote the determinant. The fifth principal minor is proportional to the fourth via an always positive number. Thus, it can be seen from Eqs. (22)-(25) that the active system is stable for any positive feedback gain g (this means negative velocity feedback). It should be mentioned that this is due to the fact that ideal velocity sensor and reactive actuator have been assumed. With real transducer dynamics the unconditional stability is not possible. Nevertheless, relatively large feedback gains have been implemented resulting in significant active control effects [5,6].

The amplitudes of the mobilities $Q_{1,2}$, $Q_{2,1}$, and $Y_{2,1}$ are depicted in Figure 3 for an example system with m_1=1 kg, m_2=1 kg, k_1=1 N/m, k_2=1 N/m, c_2=0.1 Ns/m, and g=1 Ns/m.

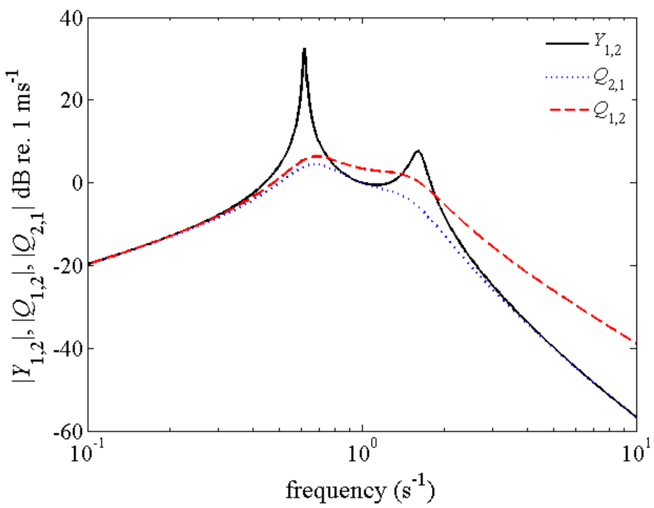

Figure 3 The amplitudes of the transfer mobilities of the passive system (solid), and of the active system (dashed, dotted)

It can be seen that the amplitude of mobility $Q_{1,2}$ is larger than the amplitude of mobility $Q_{2,1}$ and that the difference increases with frequency. Thus, although the controller is stable, and the active system is linear, the reciprocity principle does not hold. This is because external excitations exerted either at point 1 or point 2 of the activated structure become asymmetrically redistributed internally by the control forces f_{s1} and f_{s2}. If the distributed parameter structure *S*, shown in Figure 1, is immersed in an acoustic medium, then the corresponding sound transmission loss of the structure will differ depending on the direction of sound propagation through it in a broad frequency band. This is interesting given the recent research efforts in the area of acoustic metamaterials, where one of the critical issues is achieving non-reciprocal behaviour. For example, Fleury et al. demonstrated theoretically and experimentally an active metamaterial cell that is entirely transparent to tonal sound propagating through it from left to right and highly reflective to sound propagating in the opposite direction [10]. In order to achieve this, a pair of loudspeakers was placed in a 1D acoustic waveguide (rectangular pipe) at a subwavelength distance. The left (absorbing) loudspeaker was shunted with a passive electrical circuit, whereas the right (lasing) loudspeaker was shunted with a carefully tuned non-Foster electrical circuit (negative impedance circuit). Therefore the system has been made active without the use of sensors or feedback loops, and the authors clearly demonstrated the feasibility of an acoustic sensor invisible to 250 Hz sound.

Referring now again to Figure 3 and considering the roll-off of the amplitude of mobility $Q_{1,2}$ from the active vibration isolation point of view, the velocity sensor could have been placed on the mass m_1 while keeping everything else the same. However, such a feedback loop would exhibit conditional stability since the system would behave as the subcritical one (for more details see [8]). Consequently the control performance would be rather limited [8].

If the total force in the isolator (the elastic force due to stiffness k_2, plus the passive damping force due to damping c_2 plus the active force f_s) can be measured with a sensor, this sensor signal can be time-integrated and fed back to the reactive actuator. Such an approach to vibration isolation has been referred to as integral force feedback (IFF) [9]. This would again enable an unconditionally stable feedback loop (assuming ideal sensor and actuator) and more convincing vibration isolation performance. It should be mentioned that in situations where the structure is characterised by distributed parameters (i.e. the structure *S* shown in Figure 1), measuring the total force between points 1 and 2 may not be practical.

Conclusions

A variation of direct velocity feedback, skyhook damping, is considered for active vibration isolation. It is shown that the feedback loop generates effects that disrupt the reciprocity although a stable, linear controller is used. This is related to the fact that the external excitations exerted at the activated structure become redistributed internally by the control forces proportional to the absolute velocity of the receiving point. Therefore, the sound transmission loss of structures implementing a stable skyhook damping loop would become different depending on the direction of sound propagation through it. This is an interesting behaviour in the spirit of recent developments in the area of active acoustic metamaterial cloaks.

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