

Technical work in WP2 and WP5

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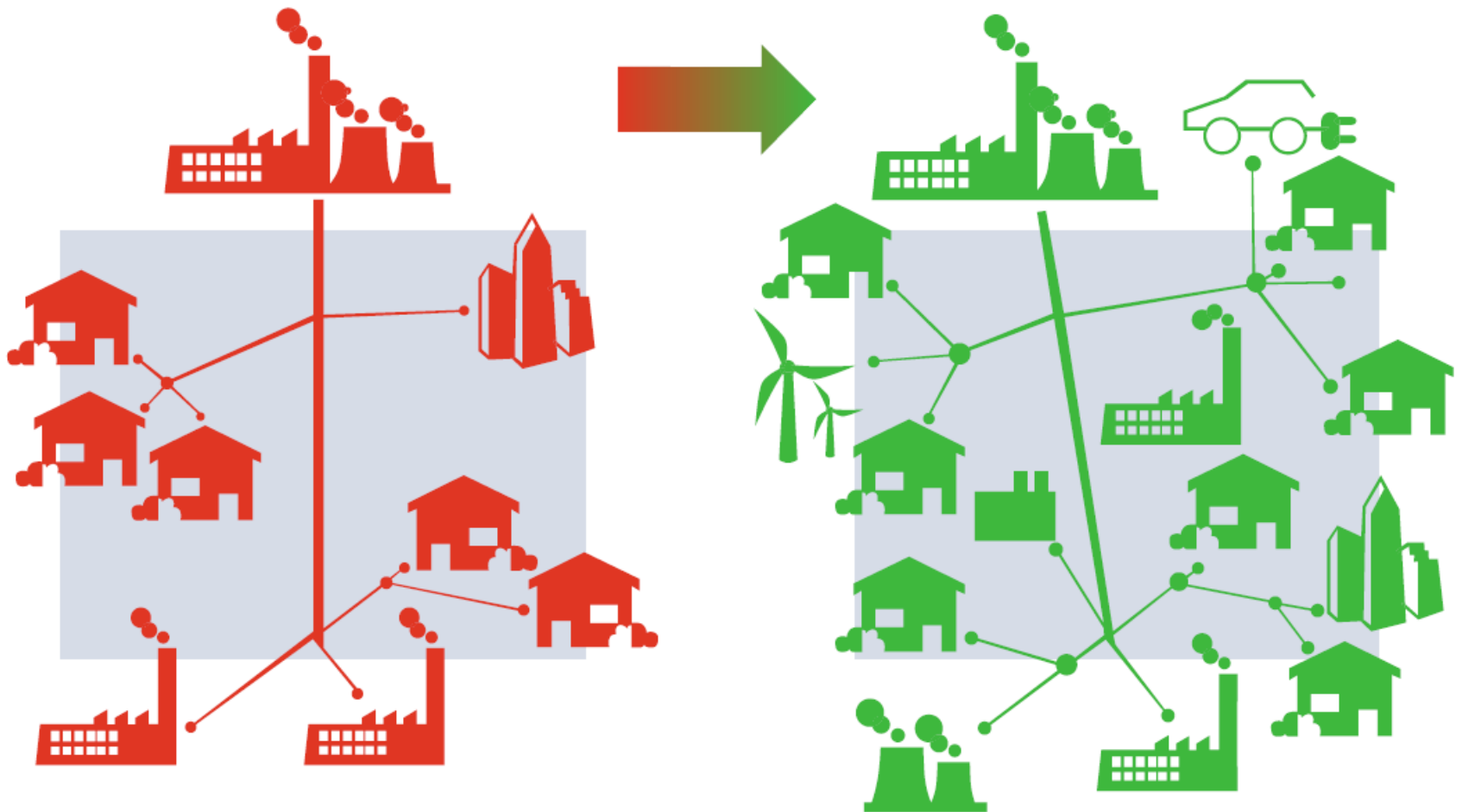
WP2 Work Plan (Task 2.4)

- Analysis of applicability and estimation of computational complexity of different uncertainty management strategies for SoS. (M15 - M20)
- Development of scenario-based robust dynamic management for SoS. (M18 - M24)
- Development of a stochastic methodology for the coordination of systems using SoS model subject to Gaussian distribution of uncertainties and disturbances. (M21 - M27)
- Development of a stochastic approach to the coordination of systems using SoS model subject to uncertainties and disturbances with arbitrary distributions, using both explicit and scenario-based approach. (M24 - M30)
- Development of methods for representation of the resulting SoS performance in stochastic sense, for merging with its neighboring systems. (M30 - M33)

WP5 Work Plan (Task 5.2)

- Mathematical modeling of the case studies. (M07-M18)
- Implementation of the models. (M09-M18)
- Derivation of simplified models and implementation in MATLAB. (M12-M22)
- Documentation. (M22-M24)

Motivation



Simulation framework – a quick reminder

- Requirements:
 - MATLAB (tested on version 8.3.0.532, R2014a)
 - Windows (tested on Windows 8.1), should work on Mac OS X, Linux (e.g. Ubuntu), and older Windows (7, XP)
 - SimPowerSystems - standard Simulink toolbox for simulation of electrical power systems
 - For the OPF routines additional MATLAB packages are needed:
 - YALMIP
 - A dedicated solver (e.g. SeDuMi, SDPT3, Mosek, CPLEX)
- Advantages:
 - Out-of-box integration with MATLAB
 - Dynamic simulation of power system (phasor, continuous, discrete)
 - Many dynamic models already implemented in the toolbox

Simulation framework – key features

- Automatic generation of simulation models
 - The development of (large-scale) power network models is very tedious, time-consuming and prone to human error
 - A network is already described in a structured file format
 - Write MATLAB script/function that can interpret a network description and automate the creation of the simulation model
- Automatic initialization
 - Initialization of the dynamic simulation model of the network
 - Use OPF solution as an initial steady state operating point
- Full advantage of SimPowerSystems model libraries

Constraints and target (1)

- Objective: minimize total active power losses over a prediction horizon N

$$\sum_{t=0}^N \sum_{i \in \mathcal{V}} P_{i,t}^I$$

- Power balance constraints

$$P_{i,t}^I = P_{i,t}^G - P_{i,t}^D = \sum_{j \in N(i)} \delta_{ij,t} P_{ij,t}, \forall i \in \mathcal{V}$$

$$Q_{i,t}^I = Q_{i,t}^G - Q_{i,t}^D = \sum_{j \in N(i)} \delta_{ij,t} Q_{ij,t}, \forall i \in \mathcal{V}$$

$$P_{ij,t} = g_{ij} V_{i,t}^2 - V_{i,t} V_{j,t} (g_{ij} \cos(\theta_{ij,t}) + b_{ij} \sin(\theta_{ij,t}))$$

$$Q_{ij,t} = -b_{ij} V_{i,t}^2 + V_{i,t} V_{j,t} (b_{ij} \cos(\theta_{ij,t}) - g_{ij} \sin(\theta_{ij,t}))$$

Constraints and target (2)

- Voltage constraints

$$\underline{V} \leq V_{i,t} \leq \bar{V}, \forall i \in \mathcal{V}$$

- Current constraint

$$I_{ij,t}^2 = \delta_{ij,t} (g_{ij}^2 + b_{ij}^2) (V_{i,t}^2 + V_{j,t}^2 - 2V_{i,t}V_{j,t} \cos(\theta_{ij})) \leq \bar{I}_{ij}^2$$

- Battery storage constraints

$$x_{i,t+1}^{BAT} = x_{i,t}^{BAT} - \eta_i P_{i,t}^{BAT},$$

$$\eta_i = \begin{cases} \eta_i^c, & P_{i,t}^{BAT} < 0 \text{ (charging)} \\ 1/\eta_i^d, & \text{otherwise (discharging)} \end{cases}$$

Constraints and target (3)

- Battery storage constraints (cont'd)

$$\underline{X}_i \leq x_{i,t}^{BAT} \leq \overline{X}_i$$

$$\underline{P}_i^{BAT} \leq P_{i,t}^{BAT} \leq \overline{P}_i^{BAT}$$

$$\underline{Q}_i^{BAT} \leq Q_{i,t}^{BAT} \leq \overline{Q}_i^{BAT}$$

- Generator constraints

$$x_{i,t+1}^G = Ax_{i,t}^G + Bu_{i,t}^G$$

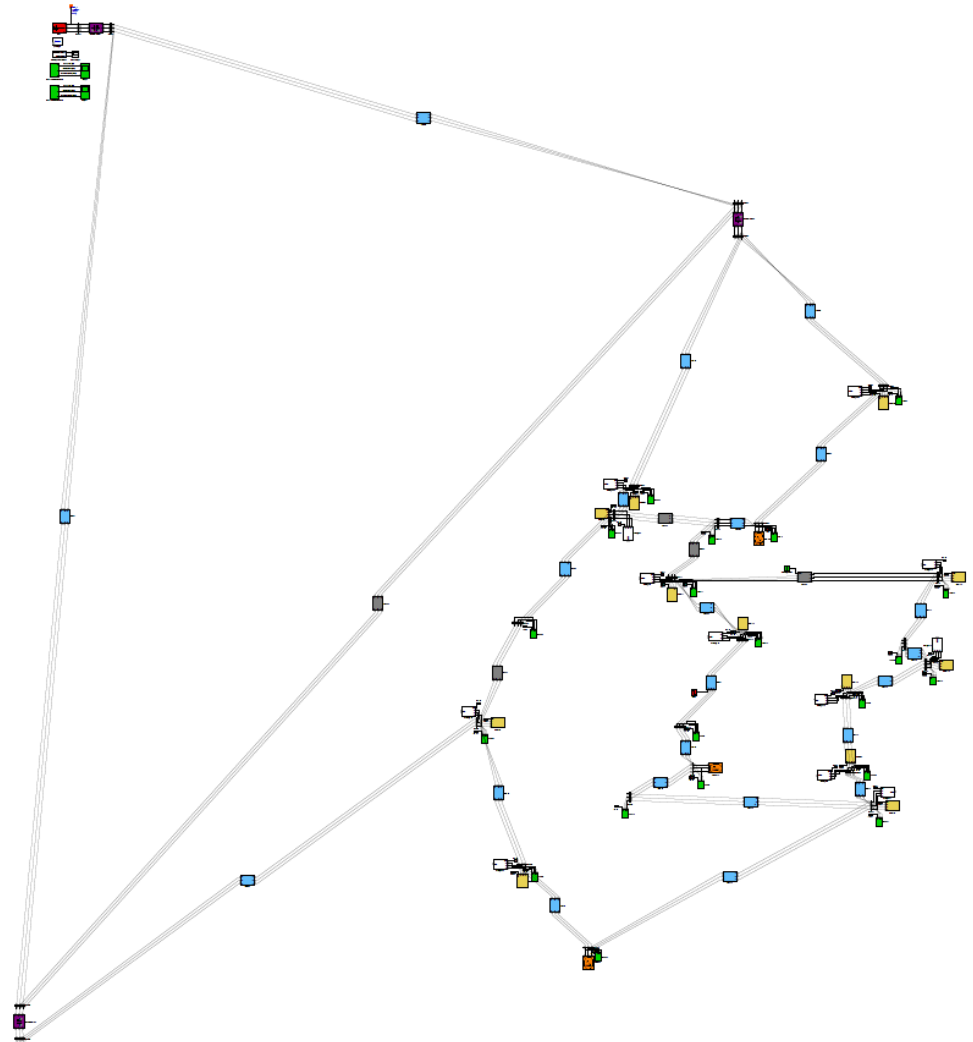
$$y_{i,t}^G = Cx_{i,t}^G + Du_{i,t}^G$$

$$\underline{P}_i^G \leq P_{i,t}^G \leq \overline{P}_i^G$$

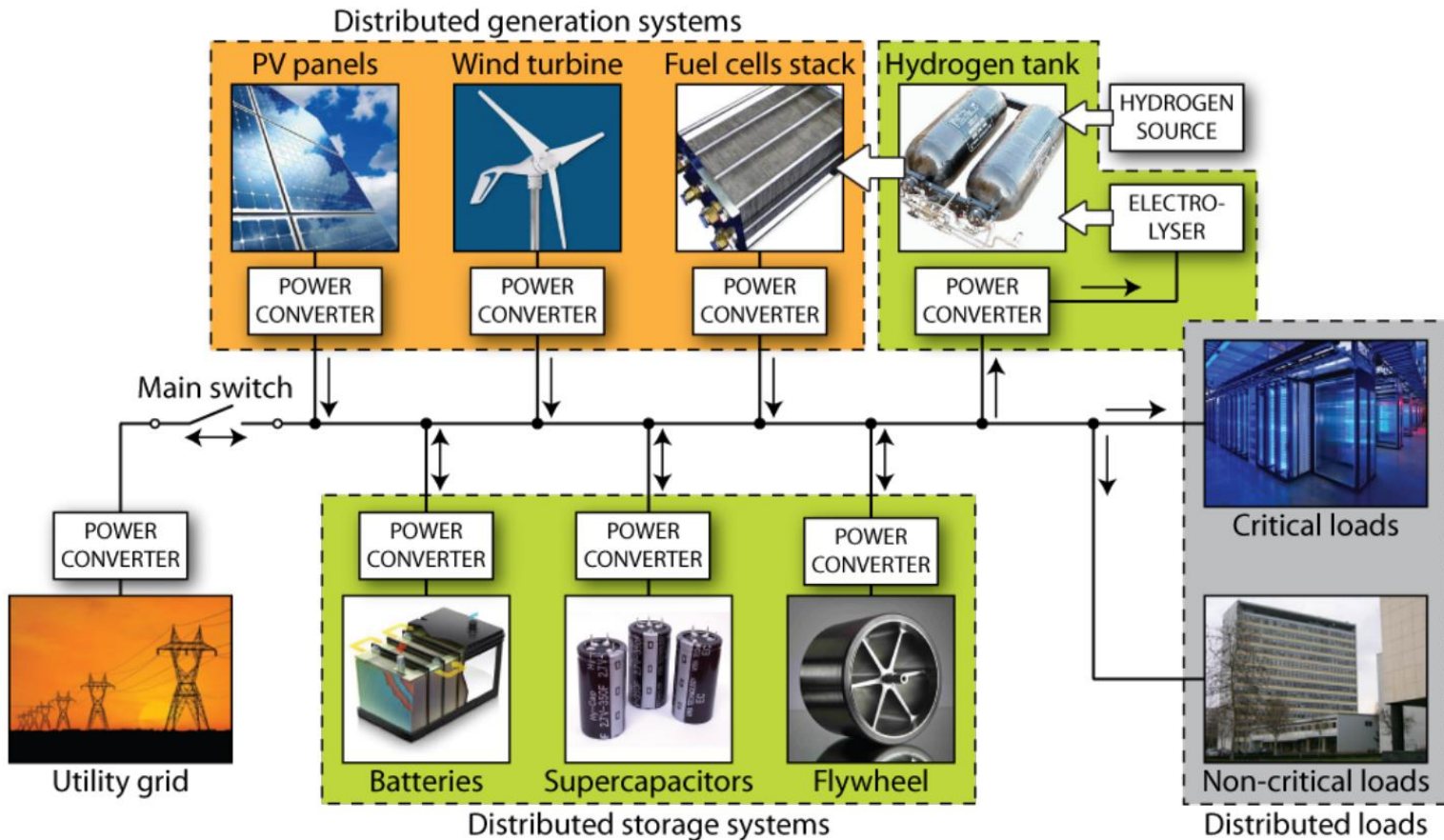
$$\underline{Q}_i^G \leq Q_{i,t}^G \leq \overline{Q}_i^G$$

Simulation scenarios

- Line fault
- Generator outage
- Major change of the overall system demand



Microgrid



Objective function and constraints

- Economic criterion

$$J(u, x_0, c, d) = -c_N \bar{x}_N + \sum_{k=0}^{N-1} c_k P_k^G \Delta T$$
$$c_N = \frac{p_{pct}}{100} \max_k c_k, \quad t \leq k \leq t + N - 1$$

- Constraints

$$x_{min} \leq x_k \leq x_{max}$$

$$u_{min} \leq u_k \leq u_{max}$$

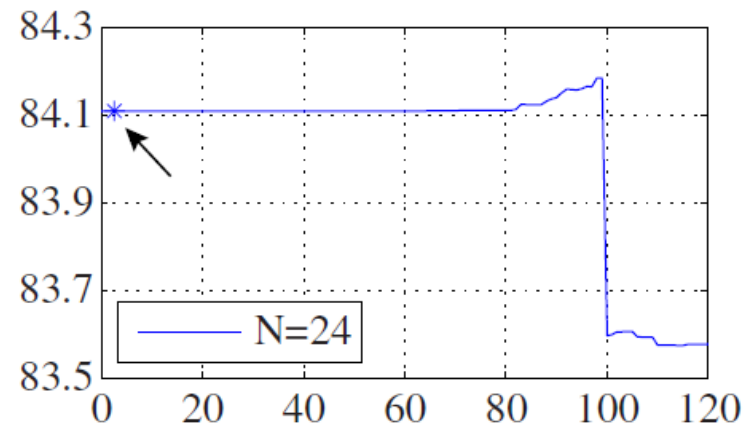
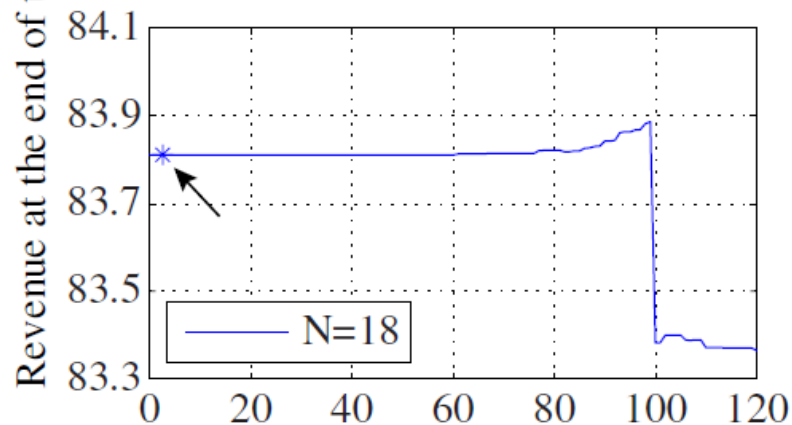
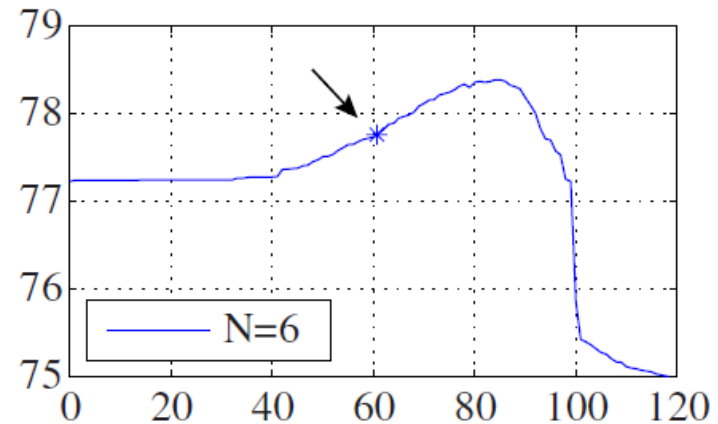
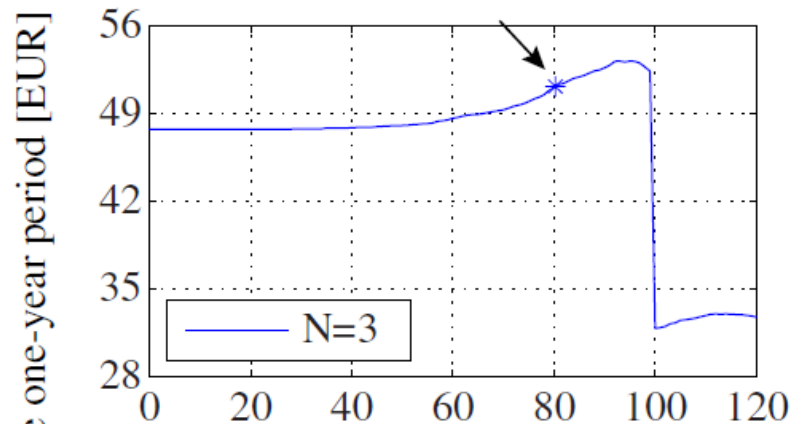
$$P_{min}^G \leq P_k^G \leq P_{max}^G$$

- LP formulation

$$\min_u \mathbf{f}^T \mathbf{u} + const$$

$$s. t. \mathbf{E}_x \mathbf{x}_0 + \mathbf{E}_u \mathbf{u} + \mathbf{E}_d \mathbf{d} \leq \mathbf{g}$$

Penalization of residual storages state



Share of the maximum price value over the prediction horizon p_{ct} [%]

Power distribution system

- Sources of uncertainty:
 - Power demand prediction
 - Uncontrollable generation prediction (e.g. wind, sun)
- Robust Optimal Power Flow (ROPF)
 - Compute optimal power references such that constraints are robustly satisfied for all possible realizations of uncertainty
- Objective
 - E.g. nominal objective (total system losses)
- Constraints
 - Must be satisfied for every realization of uncertainty

Exponential growth of num. of constraints

- Let $P_i^D = P_{i0}^D + \Delta P_i^D, \Delta P_i^D \in [\underline{\Delta P_i^D}, \overline{\Delta P_i^D}]$
- Then $\mathbf{P}^D = [P_1^D \quad P_2^D \quad \dots \quad P_n^D]^T \in \mathbb{W}_P^D$
 - $\mathbb{W}_P^D = \text{co}\{\mathbf{P}_1^D, \mathbf{P}_2^D, \dots, \mathbf{P}_m^D\}$, where \mathbf{P}_i^D is one extreme realization of demand uncertainty
- The same goes for reactive power demand \mathbf{Q}^D
- Constraints must be satisfied for every combination of extreme realizations \mathbf{P}_i^D and \mathbf{Q}_i^D !
- The number of constraints grows exponentially with number of nodes in the network!

Tube MPC (1)

- System dynamics:

$$x^+ = Ax + Bu + w, \quad (x, u) \in \mathbb{X} \times \mathbb{U}, \quad w \in \mathbb{W}$$

- Idea: Separate control into two parts

1. A portion that steers the nominal system to the origin

$$z^+ = Az + Bv$$

2. A portion that compensates for deviations from the nominal system $e^+ = (A + BK)e + w$

- The linear feedback controller K is fixed offline (such that $A + BK$ is stable)
- Keep “real” trajectory close to the nominal

$$u_i = K(x_i - z_i) + v_i$$

Tube MPC (2)

- Bound maximum error (how far is the “real” trajectory from the nominal)
 - Compute minimum robust positive invariant set \mathcal{E}
 - $x_i \in z_i \oplus \mathcal{E}$
- Compute tightened constraints on nominal system
 - $z_i \in \mathbb{X} \ominus \mathcal{E}$
 - $v_i \in \mathbb{U} \ominus K\mathcal{E}$
- Formulate as convex optimization problem
 - Optimize the nominal system with tightened constraints
 - The cost is with respect to tube centers z_i
 - Choose terminal set and terminal objective function to ensure robust feasibility and stability

Scenario-based approach (1)

- Taking into account every possible combination of extreme uncertainty realization leads to an untractable optimization problem
- Scenario approach: take into account only N uncertainty realization scenarios (instead of all possible combinations of extreme uncertainty realizations)
- Constraint violation is tolerated, but with some probability level ε
 - E.g instead $x \in \mathbb{X}$ we have $\mathbb{P}\{x \in \mathbb{X}\} \geq 1 - \varepsilon$
- Additionally, we can discard k out of N scenarios in order to improve the objective value

Scenario-based approach (2)

- Theoretical guarantees that the solution obtained by inspecting N scenarios only is a feasible solution for the chance-constrained optimization problem with high probability $1 - \beta$, provided that N and k fulfill the following condition:

$$\binom{k+d-1}{k} \sum_{i=0}^{k+d-1} \binom{N}{i} \varepsilon^i (1 - \varepsilon)^{N-i} \leq \beta$$

where d is the number of optimization variables.

- Choose N within the computational limit of the used solver, ε according to the acceptable level of risk, β small enough to be negligible (e.g. $\beta = 10^{-10}$), and compute the largest k number of scenarios that can be discarded

Acknowledgments

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