Coordinated Microgrid Control via Parametric Optimization

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***Abstract*—Microgrid is a cluster of distributed generation sources, storage units and loads, which, when controlled in an optimal coordinated fashion, can improve both the quality of the local power supply as well as the performance of the overall power distribution system. In this paper we present a coordinated control methodology for microgrids that utilizes parameterized costs of local operation declared by microgrid's constituent subsystems. The overall control problem is decomposed into smaller decoupled local problems related to individual subsystems and one global coordination problem that describes the coupling of the subsystems. Each local problem is cast as a multi-parametric quadratic optimization problem. When the dimension of the coordination parameter is one, a very efficient algorithm can be used to solve the coordination problem. Performance of the proposed control strategy is illustrated on a simulation case study.**

***Index Terms*—Coordinated control, microgrid, model predictive control, parametric optimization.**

I. Introduction

Microgrids are localized electricity distribution systems comprising loads and distributed energy resources, such as distributed generators and storage units, that can be operated in a coordinated way. Microgrids versatility of operation (they can be connected to the main grid or operate in an islanded mode) is the main driver for their increasing presence in modern distribution systems. As such they are becoming an active part of the power system contributing to its decentralization, thus increasing the reliability and efficiency of the energy supply. In this paper we consider a general microgrid concept comprising many different (renewable) energy sources and storage units. Clearly, a microgrid becomes increasingly complex system (with a large number of control variables, states, and constraints) as the number of local subsystems increases.

In order to coordinate the microgrid energy resources cost efficiently, the optimization of the microgrid operations is extremely important. The overall microgrid optimal control problem consists in determining how to optimally schedule generators, storage units, and controllable loads, to cover the microgrid demand or follow some externally defined power reference while minimizing the running costs of all constituent subsystems and satisfying the system constraints [1].

In recent years, the Model Predictive Control (MPC) methodology has garnered the attention of the power system community, mostly because: (i) it can take into account the future behavior of the system, which is very attractive for a system greatly dependent on demand and renewable generation forecasts, (ii) it can systematically handle power system constraints, and (iii) it essentially provides a feedback mechanism, which makes the system more robust against uncertainty [2].

For large microgrids, the underlying optimization problem becomes computationally demanding and often cannot be solved within one time sample (implicit/on-line MPC). The alternative approach to implementing the MPC is the off-line computation of the state feedback control law via parametric optimization (explicit/off-line MPC) [3]. Thus one obtains the optimal control input as a closed form function of the system state. The on-line implementation of the explicit MPC then becomes a simple lookup table. The downside of the explicit MPC is that the complexity of the control law often increases very fast with complexity of the original optimization problem. In practice this means that the computation of an explicit MPC is limited to systems with short prediction horizon and small number of states and/or control inputs.

Generally, large microgrids are complex systems with large number of control variables and states. Therefore, the use of the MPC approach (both explicit and implicit form) for coordinated optimal control of such systems may be prohibitive. State of the art approaches found in literature usually deal with this problem by utilizing decentralization of computation, where the main idea is to decompose the original (large) optimal control problem into a number of smaller and more tractable subproblems that can be solved in parallel. An overview of different distributed MPC approaches can be found in a recent survey paper [4], while an example of distributed microgrid control can be found in [5]. In distributed MPC schemes the convergence towards a globally optimal solution is usually attempted by employing an iterative (global) aggregation/(local) optimization procedure. Downside of the decomposition approaches is that they usually require a great number of iterations to obtain a solution [6].

To overcome this difficulty, we use the control methodology initially presented in [7]. The main idea of this centralized approach is to combine both the on-line and off-line computation when solving the overall MPC problem for coordinated control of large-scale distributed systems. A considerable amount of

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computational effort is transferred off-line allowing for more efficient on-line computation. The overall control problem is decomposed into smaller decoupled local problems related to individual subsystems and one coordination problem that describes the coupling of the subsystems. The contribution of each local subsystem to the overall cost function and its optimal control actions are determined off-line as the solution of a multi-parametric quadratic program (mp-QP). The globally optimal coordination parameters are determined on-line by solving the coordination problem based on the off-line solutions. Notice that there is no need for iterative exchange of information between the global coordinator and local subsystems to ensure the convergence to a globally optimal solution - the information (local solutions obtained off-line) is sent to the coordinator only once. Due to theoretic properties of the off-line solutions, the coordination problem becomes box-constrained separable quadratic optimization problem, which allows a design of a very efficient on-line solver [8].

In Section II the control problem is defined. Section III describes the implementation of the proposed control approach. An illustrative simulation example is provided in Section IV. Finnaly, some concluding remarks are given in Section V.

II. Problem Definition

A. System dynamics and constraints

In this paper we will consider a general microgrid concept comprising M (where M is a large number) of different controllable energy prosumers (devices that can consume and/or produce energy). Let $\mathcal{M} := \{1, 2, \ldots, M\}$. There is also a number of uncontrollable prosumers in the microgrid. However, we assume that a prediction of future power profiles of these uncontrollable prosumers is available to the microgrid coordinator. For simplicity, we also assume that all prosumers are connected to a common bus and we do not take into account reactive power flows.

The models associated with the microgrid subsystems (with some examples) are described in detail below, including the individual objectives and constraints.

1) Uncontrollable prosumers: As we already mentioned, uncontrollable prosumer is represented with a power consumption/generation profile. This profile can be predicted, e.g. using historical data, and as such can be used by the central coordinator to optimize the overall behavior of the microgrid system on a prediction horizon. Let P^{d} represent aggregated consumption/generation profile of all uncontrollable prosumers in the microgrid. Then P^{d} can be seen as a system-wide (time-variant) power reference that other prosumers in the microgrid should follow in a coordinated fashion. We denote this reference as $P^{\mathrm{MG}}_{\mathrm{ref}}$. A classical example of an uncontrollable prosumer is the critical electrical load whose demand for electrical energy must be satisfied at all times.

2) Simple controllable prosumers: Some of the locally controllable prosumers have negligible dynamics and therefore can be described simply by their power consumption/generation profiles (similar to the uncontrollable prosumers introduced above). Let $\mathcal{S} \subset \mathcal{M}$ denote an index set of all such controllable prosumers. Then, for each prosumer $i \in \mathcal{S}$, $\widetilde{P}^{\mathrm{s}}_i$ represents the prediction of its power profile on a prediction horizon. As with the uncontrollable prosumer, the power profile of this kind of the controllable prosumer can be predicted using historical data. Moreover, its control equipment has the option to curtail its power output if needed. For example, a photovoltaic (PV) inverter has the option to curtail the power output if it cannot be immediately consumed, stored or pushed into the grid. Another example is the controllable electrical load which is curtailable to some extent, i.e. a minimum base load must be met. Therefore, the predicted power output profile $\widetilde{P}^{\mathrm{s}}_i$ can be seen as the maximum available/demanded power on a prediction horizon while the actual power output P^{s}_i can be less or equal than $\widetilde{P}^{\mathrm{s}}_i$. We model this as the following constraint that must be met at every time instant t:

$$\beta_i \widetilde{P}^{\mathrm{s}}_i(t) \leq P^{\mathrm{s}}_i(t) \leq \widetilde{P}^{\mathrm{s}}_i(t), \;\; \forall i \in \mathcal{S}, \tag{1}$$

where $\beta_i \in [0, 1]$ is the curtailment factor saying to what extent is the i-th prosumer curtailable, e.g. for a PV array $\beta = 0$ and for a curtailable load $\beta \geq 0$ is the minimal load fraction that must be met.

Since power output should not be curtailed unnecessarily, a curtailment cost is used as a local objective function for each prosumer $i \in \mathcal{S}$:

$$J_i = \sum_{t=0}^{N-1} Q^{\mathrm{s}}_i (\widetilde{P}^{\mathrm{s}}_i(t) - P^{\mathrm{s}}_i(t))^2, \tag{2}$$

where $Q^{\mathrm{s}}_i > 0$ is the i-th prosumer curtailment penalty parameter and N is the length of the prediction horizon.

3) Dynamic controllable prosumers: The rest of controllable prosumers can be described with a linear time-invariant discrete-time dynamics:

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t), \;\; \forall i \in \mathcal{C}, \tag{3a}$$

$$y_i(t) = C_i x_i(t) + D_i u_i(t), \;\; \forall i \in \mathcal{C}, \tag{3b}$$

where $\mathcal{C} = \mathcal{M} \backslash \mathcal{S}$, $t \in \mathbb{Z}$ denotes a discrete time instant, $i \in \mathcal{C}$ is the prosumer index, $x_i(t) \in \mathbb{R}^{n_{\mathrm{x},i}}$, $u_i(t) \in \mathbb{R}^{n_{\mathrm{u},i}}$ and $y_i(t) \in \mathbb{R}$ denote state, input and output of i-th prosumer at time instant t, respectively, while $A_i \in \mathbb{R}^{n_{\mathrm{x},i} \times n_{\mathrm{x},i}}$, $B_i \in \mathbb{R}^{n_{\mathrm{x},i} \times n_{\mathrm{u},i}}$, $C_i \in \mathbb{R}^{1 \times n_{\mathrm{x},i}}$ and $D_i \in \mathbb{R}^{1 \times n_{\mathrm{u},i}}$ are constant system matrices. It is assumed that $y_i(t)$ is the actual power generation/consumption of i-th prosumer at time instant t.

Some examples of dynamic prosumers are: diesel generators, combined heat and power (CHP) plants, battery storage systems, thermostatically controlled loads, wind turbines, etc.

States $x_i(t)$ and inputs $u_i(t)$ are constrained to polytopes $\mathbb{X}_i$ and $\mathbb{U}_i$, respectively, described by sets of linear inequalities, e.g. simple box constraints $\underline{x}_i \leq x_i(t) \leq \overline{x}_i$, $\underline{u}_i \leq u_i(t) \leq \overline{u}_i$. A terminal constraint can additionally be added, e.g. to the battery storage system to ensure that the storage system is not depleted at the end of the prediction horizon.

Unless indicated otherwise, for each i-th prosumer, where $i \in \mathcal{C}$, we consider the following quadratic objective function:

$$J_i = \sum_{t=0}^{N-1} Q_i |y_i(t) - y^{\mathrm{ref}}_i(t)|^2 + R_i |u_i(t) - u^{\mathrm{ref}}_i(t)|^2 + T_i |u_i(t+1) - u_i(t)|^2, \tag{4}$$

where $y_i^{\text{ref}}(t)$ is the power generation/consumption reference, $u_i^{\text{ref}}(t)$ is the input reference, and $Q_i, R_i, T_i > 0$ are cost weighting parameters. The three terms in (4) penalize the deviation from the output reference, the deviation from the input reference, and excessive input increments, respectively.

Remark 1. *For a battery storage system we abuse the previous notation a little bit. Namely, $y_i(t)$ here is the current battery state of charge (the amount of energy currently stored in a battery), i.e. $C_i = 1$ and $D_i = 0$, while the actual power generation/consumption is $u_i(t)$ (the rate of charge/discharge). Therefore, $y_i^{\text{ref}}(t)$ is the state of the charge reference and the input reference $u_i^{\text{ref}}(t)$ is the actual power generation/consumption reference.*

Remark 2. *For each local objective function we only require convexity and quadratic form, so the examples of objective functions given in* (2) *and* (4) *are not the only possible choices.*

B. Control problem formulation

We consider the following constrained finite-time optimal control problem for optimal coordinated control of M locally controllable prosumers in a microgrid system subject to state and input constraints:

$$\min \sum_{i\in\mathcal{S}} J_i(\widetilde{P}_i^{\text{s}}, P_i^{\text{s}}) + \sum_{i\in\mathcal{C}} J_i(x_i, u_i, y_i^{\text{ref}}, u_i^{\text{ref}}), \tag{5a}$$

$$\text{s.t.} \;\; \beta_i \widetilde{P}_i^{\text{s}}(t) \le P_i^{\text{s}}(t) \le \widetilde{P}_i^{\text{s}}(t), \;\; \forall i \in \mathcal{S}, \tag{5b}$$

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t), \;\; \forall i \in \mathcal{C}, \tag{5c}$$

$$y_i(t) = C_i x_i(t) + D_i u_i(t), \;\; \forall i \in \mathcal{C}, \tag{5d}$$

$$x_i(t) \in \mathbb{X}_i, \; u_i(t) \in \mathbb{U}_i, \;\; \forall i \in \mathcal{C}, \tag{5e}$$

$$x_i(0) = x_{i,0}, \;\; \forall i \in \mathcal{C}, \tag{5f}$$

$$\sum_{i\in\mathcal{S}} P_i^{\text{s}}(t) + \sum_{i\in\mathcal{C}} y_i^{\text{ref}}(t) = P_{\text{ref}}^{\text{MG}}(t), \tag{5g}$$

where $x_{i,0}$ denotes the initial condition for i-th prosumer. The overall objective function is a sum of M locally defined objective functions (either (2) or (4)) for each of the M controllable prosumers. Constraint (5g) provides a coupling between all subsystems. This equation essentially demands that the sum of all local power references should be equal to the overall microgrid power reference. In other words, the microgrid controller needs to optimally distribute the energy production between individual controllable prosumers in such a manner that the overall power reference is tracked, while the local objective functions are minimised and all state and input constraints are satisfied. The optimisation variables in (5) are $P_i^{\text{s}}(t)$, $x_i(t)$, $u_i(t)$, and $y_i^{\text{ref}}(t)$. Notice that when i-th prosumer is a battery storage system, $y_i^{\text{ref}}(t)$ in (5g) should be replaced with $u_i^{\text{ref}}(t)$, as discussed in Remark 1.

Assumption 1. *For simpler computation, we shall assume constant references over a prediction horizon, i.e. $P_{\text{ref}}^{\text{MG}}(t) = P_{\text{ref}}^{\text{MG}}$, $P_i^{\text{s}}(t) = P_i^{\text{s}}$, $y_i^{\text{ref}}(t) = y_i^{\text{ref}}$, and $u_i^{\text{ref}}(t) = u_i^{\text{ref}}$.*

The classical (on-line) MPC approach consists of solving (5) at every time sample and applying the first element of the optimal input sequence to each of the M subsystems (prosumers). It is clear that the optimisation problem (5) becomes computationally demanding (i.e. the number of optimisation variables and constraints grows quickly) when M is large and it might not be possible to solve it at every time sample. The complexity of (5) prevents the implementation of the explicit (off-line) MPC approach as well.

III. Controller Design

The proposed control methodology relies on a decomposition of the control problem (5) into: (i) M smaller decoupled local problems related to individual subsystems, and (ii) a centralized optimization problem that provides a coordination of individual subsystems. The local problems are parameterized and solved off-line to significantly reduce the on-line computation burden. The solution obtained after solving the coordination optimization problem is the global optimum of (5), i.e. the same solution is obtained.

A. Parameterization and decomposition of the control problem

We define two types of parameters related to the i-th controllable prosumer: local parameters $\Phi_i \in \mathbb{R}^{n_{\Phi,i}}$ and a coordination parameter $\Theta_i \in \mathbb{R}$. The vector of local parameters Φ_i collects all (time-varying) data that are locally available (obtained from measurements, estimations, predictions from historical data, etc.) at time t. In (5) these include the initial condition $x_{i,0}$, input reference u_i^{ref}, and power profile $\widetilde{P}_i^{\text{s}}$. In other words:

$$\Phi_i = \widetilde{P}_i^{\text{s}}, \;\; \forall i \in \mathcal{S}, \tag{6}$$

$$\Phi_i = \begin{bmatrix} x_{i,0} \\ u_i^{\text{ref}} \end{bmatrix}, \;\; \forall i \in \mathcal{C}. \tag{7}$$

The coordination parameter Θ_i describes the contribution of i-th subsystem to the coupling constraint. Notice that under Assumption 1 there is only one coupling constraint (5g) and hence the coordination parameter Θ_i is indeed one-dimensional. From (5g) it is clear that Θ_i is either P_i^{s} when $i \in \mathcal{S}$ or y_i^{ref} when $i \in \mathcal{C}$. For a battery storage system u_i^{ref} and y_i^{ref} switch places, i.e. y_i^{ref} is in Φ_i and u_i^{ref} is Θ_i.

Additionally, let us define vector $z_i \in \mathbb{R}^{n_{z,i}}$ as the vector that collects all optimization variables related to the i-th prosumer (only the remaining variables that are not already collected in Φ_i or Θ_i). Then, with a straightforward algebraic manipulation, the control problem (5) can be recast as an optimization problem of the following form:

$$\min_{\substack{\Theta_1,\dots,\Theta_M \\ z_1,\dots,z_M}} \sum_{i\in\mathcal{M}} J_i(\Phi_i, \Theta_i, z_i), \tag{8a}$$

$$\text{s.t.} \quad C_i^{\text{z}} z_i \le C_i^{\text{c}} + C_i^{\Phi}\Phi_i + C_i^{\Theta}\Theta_i, \;\; \forall i \in \mathcal{M}, \tag{8b}$$

$$\sum_{i\in\mathcal{M}} \Theta_i = P_{\text{ref}}^{\text{MG}}, \tag{8c}$$

where C_i^{z}, C_i^{c}, C_i^{Φ}, and C_i^{Θ} are appropriately sized constant matrices. For $i \in \mathcal{C}$, these matrices can be obtained from $\mathbb{X}_i$ and $\mathbb{U}_i$, using the state transition equations and parameter definitions. For $i \in \mathcal{S}$, (8b) is just a reformulation of (5b).

Local objective functions J_i are the same convex quadratic functions as before, only reformulated using Θ_i, Φ_i, and z_i.

Now, we can define the local problem related to the i-th prosumer in the microgrid:

$$J_i^\star(\Phi_i, \Theta_i) = \min_{z_i} \; J_i(\Phi_i, \Theta_i, z_i), \tag{9a}$$

$$\text{s.t. } C_i^{\mathrm{z}} z_i \leq C_i^{\mathrm{c}} + C_i^{\Phi}\Phi_i + C_i^{\Theta}\Theta_i. \tag{9b}$$

The optimization problem (9) is an mp-QP with parameter vector $\left[\Phi_i^{\mathrm{T}},\ \Theta_i\right]^{\mathrm{T}}$. Notice that Θ_i is an optimization variable in (8), while in (9) it is treated as a parameter. Therefore, the local problem (9) must be solved for all admissible Θ_i, while the optimal coordination parameters $\Theta_i^\star$ are computed at the coordination level, which will be explained later in this section. Using the multi-parametric approach [3], the local problems (9) can be easily solved parameterically, to obtain the closed-form description of the optimal cost function (value function) and the optimizer. The properties of the optimizer and value function are described in the following theorem [3]:

Theorem 1. *Consider the mp-QP* (9)*. The set of feasible parameters:*

$$\mathcal{P}_i = \left\{ \Omega_i \middle| \ \exists z_i : C_i^{\mathrm{z}} z_i \leq C_i^{\mathrm{c}} + C_i^{\Omega}\Omega_i \right\}, \tag{10}$$

where $\Omega_i = \left[\Phi_i^{\mathrm{T}},\ \Theta_i\right]^{\mathrm{T}}$ *and* $C_i^{\Omega} = \left[C_i^{\Phi}\ C_i^{\Theta}\right]$*, is a polyhedral set, the value function* $J_i^\star : \mathcal{P}_i \to \mathbb{R}$ *is a convex and continuous piecewise quadratic function on polyhedra (PPWQ), and the optimizer* $z_i^\star : \mathcal{P}_i \to \mathbb{R}^{n_{\mathrm{z},i}}$ *is a continuous piecewise affine function on polyhedra (PPWA).*

By using the value functions of local problems, $J_i^\star(\Phi_i, \Theta_i)$, $\forall i \in \mathcal{M}$, the optimization problem (8) can be stated as the following optimal coordination problem:

$$\min_{\Theta_1, \dots, \Theta_M} \sum_{i \in \mathcal{M}} J_i^\star(\Phi_i, \Theta_i), \tag{11a}$$

$$\text{s.t.} \quad \left[\Phi_i^{\mathrm{T}},\ \Theta_i\right]^{\mathrm{T}} \in \mathcal{P}_i, \ \ \forall i \in \mathcal{M}, \tag{11b}$$

$$\sum_{i \in \mathcal{M}} \Theta_i = P_{\mathrm{ref}}^{\mathrm{MG}}. \tag{11c}$$

B. Controller implementation

The controller implementation can be divided into two stages: (i) off-line precomputation, and (ii) implementation of an on-line algorithm. First stage consists of solving the local problems (9) parameterically (as mp-QPs) to obtain $z_i^\star(\Phi_i, \Theta_i)$ and $J_i^\star(\Phi_i, \Theta_i)$, which can be easily done using MPT toolbox [9].

On-line, at every time sample t, the computation is done in the following steps:

1) *Local evaluation*: Obtain measurements or estimations of local parameters $\hat{\Phi}_i$ and evaluate the local solutions for $\hat{\Phi}_i$:

$$\widetilde{z}_i(\Theta_i) = z_i^\star(\hat{\Phi}_i, \Theta_i), \ \ \widetilde{z}_i : \mathcal{I}_i \to \mathbb{R}^{n_{\mathrm{z},i}}, \tag{12a}$$

$$\widetilde{J}_i(\Theta_i) = J_i^\star(\hat{\Phi}_i, \Theta_i), \ \ \widetilde{J}_i : \mathcal{I}_i \to \mathbb{R}, \tag{12b}$$

where $\mathcal{I}_i$ is a set of intervals in $\mathbb{R}$, obtained by projecting $\mathcal{P}_i$ on $\Phi_i = \hat{\Phi}_i$.

2) *System-wide coordination*: Given the evaluated local solutions (12), solve the system-wide coordination problem to obtain the optimal value of coordination parameters $\Theta_i^\star$, $\forall i \in \mathcal{M}$:

$$\min_{\Theta_1, \dots, \Theta_M} \sum_{i \in \mathcal{M}} \widetilde{J}_i(\Theta_i), \tag{13a}$$

$$\text{s.t.} \quad \Theta_i \in \mathcal{I}_i, \ \ \forall i \in \mathcal{M}, \tag{13b}$$

$$\sum_{i \in \mathcal{M}} \Theta_i = P_{\mathrm{ref}}^{\mathrm{MG}}. \tag{13c}$$

3) *Local evaluation*: Evaluate the optimizer for the i-th prosumer:

$$z_i^\star = \widetilde{z}_i(\Theta_i^\star). \tag{13d}$$

From $z_i^\star$ one can easily obtain the optimal control sequence $u_i^\star$ (since u_i is contained in z_i). In accordance with the standard MPC receding horizon philosophy, only the first element of this sequence $u_{0,i}^\star$ is applied as a control input to the i-th prosumer.

The overview of the on-line steps is given in Fig. 1. The local evaluations are performed in parallel on the local controllers, while the system-wide coordination is performed on a centralized hardware. Notice the flow of information: the local controllers send the description of the cost function $\widetilde{J}_i : \mathcal{I}_i \to \mathbb{R}$ to the central coordinator, while the coordinator returns the value of the optimal coordination parameter $\Theta_i^\star$ (only one number). This exchange of information is performed only once per time sample.

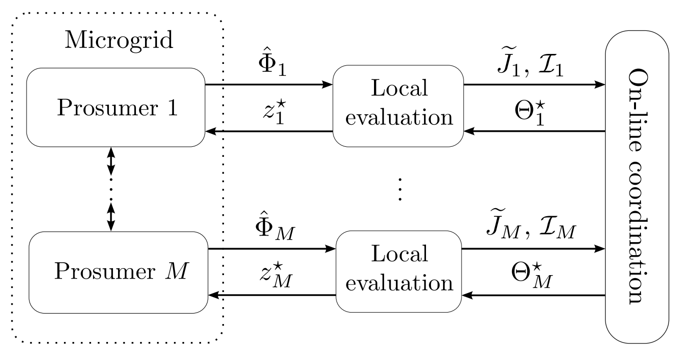

Fig. 1. Coordinated control of a microgrid with M controllable prosumers.

The local computation boils down to evaluation of PPWA and PPWQ functions, a relatively simple task that can be performed efficiently and thus introduces a negligible computational overhead. A real computational burden lies in solving (13) so an efficient algorithm for solving (13) is needed.

In [7] it was shown that an extremely efficient way of solving the coordination problem can be devised by utilizing a specific structure of the problem at hand. Namely, we can use the fact that $\widetilde{J}_i$ is a convex and continuous one-dimensional function defined on intervals (if Assumption 1 is fulfilled):

$$\widetilde{J}_i : \mathcal{I}_i \to \mathbb{R}, \ \mathcal{I}_i = \left\{ I_{i,j-1} \leq \Theta_i \leq I_{i,j} \right\}_{j=1}^{n_i^{\mathrm{r}}}, \tag{14}$$

$$\forall \Theta_i \in [I_{i,j-1}, I_{i,j}] : \widetilde{J}_i = \frac{1}{2} a_{i,j}\Theta_i^2 + b_{i,j}\Theta_i + c_{i,j}, \tag{15}$$

where n_i^{r} is a number of interval partitions for i-th subsystem, $a_{i,j}$, $b_{i,j}$, $c_{i,j} \in \mathbb{R}$ are parameters of the quadratic function of

i-th subsystem in j-th interval partition, while $I_{i,j} \in \mathbb{R}$ are the endpoints of those intervals.

After a simple affine transformation of variables, (13) can be recast in terms of new variables $\Theta_{i,j}$ as:

$$\min_{\substack{\Theta_{1,1},\ldots,\Theta_{1,n_1^{\mathrm{r}}},\ldots,\\ \ldots,\Theta_{M,1},\ldots,\Theta_{M,n_M^{\mathrm{r}}}}} \quad \sum_{i=1}^{M}\sum_{j=1}^{n_i^{\mathrm{r}}} \frac{1}{2} a_{i,j}\Theta_{i,j}^2 + (a_{i,j}I_{i,j-1} + b_{i,j})\Theta_{i,j}, \tag{16a}$$

$$\text{s.t.} \qquad 0 \leq \Theta_{i,j} \leq I_{i,j} - I_{i,j-1}, \tag{16b}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{n_i^{\mathrm{r}}} \Theta_{i,j} + I_{i,0} = P_{\mathrm{ref}}^{\mathrm{MG}}, \tag{16c}$$

where in (16b) $i = 1, \ldots, M$ and $j = 1, \ldots, n_i^r$. The optimizers $\Theta_{i,j}^\star$ of (16) can be used to compute the optimizers $\Theta_i^\star$ of (13) in a following way:

$$\Theta_i^\star = I_{i,0} + \sum_{j=1}^{n_i^r} \Theta_{i,j}^\star. \tag{17}$$

For a proof and more details, see [7].

The optimization problem (16) has a separable quadratic cost function, separable box constraints and one linear equality constraint. This type of quadratic optimization problem is recognised in literature as a continuous quadratic knapsack problem. There is a very efficient approach for solving this type of problems known in literature: a variable fixing algorithm [8]. On average the algorithm requires $\mathcal{O}(n)$ steps to solve (16) and the worst case performance is $\mathcal{O}(n^2)$. In terms of average run time it outperforms other algorithms for the continuous quadratic knapsack problem [10].

IV. Simulation Example

In this Section we consider a simple example of a microgrid system on which we illustrate the proposed coordination strategy. The considered microgrid comprises three components: (i) a small biomass combustion turbine generator (CTG), (ii) a battery storage unit, and (iii) a photovoltaic (PV) generator. We assign indexes to each component: $i = 1$ for the CTG, $i = 2$ for the battery storage, and $i = 3$ for the PV generator. Each component optimizes its local operation according to its own interests and also contributes to the overall coordination requirement which is in this case tracking of the power generation reference. In the following we briefly describe dynamic models of these components and their local optimization functions and constraints.

A. Biomass CTG

The following continuous-time state-space model describes a simple dynamics model of the CTG:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{\delta}{\alpha} & -\frac{\beta}{\alpha} & \frac{a}{\alpha} \\ 0 & 0 & -\frac{1}{b} \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{b} \end{bmatrix} u_1, \tag{18a}$$

$$y_1 = \begin{bmatrix} c & 0 & 0 \end{bmatrix} x_1, \tag{18b}$$

where $a = 1$, $b = 8$, $c = 0.1$, $\alpha = 0.45$, $\beta = 8.04$, and $\delta = 0.1$. The model (18) is taken from [11]. The input u_1 is a fuel reference and the output y_1 is the generated electrical power. A discretization of (18) with a sample time $T_{\mathrm{s}} = 10$ s was performed to obtain a discrete-time state-space model of the CTG (i.e. matrices A_1, B_1, C_1, and D_1 in (3)). States and inputs are constrained with simple box constraints $\underline{x}_1 \leq x_1(t) \leq \overline{x}_1$, $\underline{u}_1 \leq u_1(t) \leq \overline{u}_1$, where $\underline{x}_1 = [0, -8, -1]^{\mathrm{T}}$, $\underline{x}_1 = [10, 8, 1]^{\mathrm{T}}$, and $\overline{u}_1 = -\underline{u}_1 = 1$. The local objective function is defined as in (4), with $u_1^{\mathrm{ref}}(t) = 0$, $Q_1 = 250$, $R_1 = 2$, $T_1 = 0$.

B. Battery storage system

The following discrete-time dynamic model of the battery storage is used:

$$x_2(t+1) = x_2(t) - u_2(t),\ y_2(t) = x_2(t), \tag{19}$$

where $x_2(t)$ denotes the amount of energy stored in the battery (i.e. the state of charge, SOC) at time instant t and $u_2(t)$ denotes the rate of charge and discharge ($u_2(t) > 0$ means discharging and $u_2(t) < 0$ means charging). For simplicity, we do not model losses during charging/discharging. The amount of energy $x_2(t)$ in the battery and its rate of charge/discharge are constrained as follows: $0 \leq x_2(t) \leq 1$, $-0.1 \leq u_2(t) \leq 0.1$. The local objective function is defined as in (4) (note the Remark (1)), with $Q_2 = 10$, $R_2 = 1$, $T_2 = 0$.

C. Photovoltaic generator

A photovoltaic generator is modeled as discussed in Section II-A2. Its local constraint is defined as in (1) with $\beta_3 = 0$. The local objective function is defined as in (2) with $Q_3^{\mathrm{s}} = 250$.

D. Coordination problem

Local parameters are denoted by Φ_i and coordination parameters by Θ_i:

$$\begin{aligned} \Phi_1 &= x_{1,0}, & \Theta_1 &= y_1^{\mathrm{ref}}, \\ \Phi_2 &= \begin{bmatrix} x_{2,0}, & y_2^{\mathrm{ref}} \end{bmatrix}^{\mathrm{T}}, & \Theta_2 &= u_2^{\mathrm{ref}}, \\ \Phi_3 &= \widetilde{P}_3^{\mathrm{s}}, & \Theta_3 &= P_3^{\mathrm{s}}. \end{aligned} \tag{20}$$

Local control problems (which are of the form (9)) are solved parametrically (as mp-QPs), and local value functions are obtained as functions of local and global (coordination) parameters. For every local problem we use the same prediction horizon $N = 6$. At every time instant new measurements/estimations/predictions of local parameters $\hat{\Phi}_i$ are obtained, local solutions are evaluated, and the coordination problem (16) is solved. Optimal control actions for each subsystem are then determined by evaluating local optimizers at globally optimal coordination parameters obtained by solving the coordination problem.

E. Simulation

We simulate the microgrid operation in closed-loop for one hour, i.e. for 360 time steps. In the simulated scenario the battery storage was tasked with tracking the state of charge reference $y_2^{\mathrm{ref}} = 0.8$.

Figure 2 shows various time responses obtained in simulation. We can see that each local subsystem is operated successfully according to its own local interest, e.g. the battery

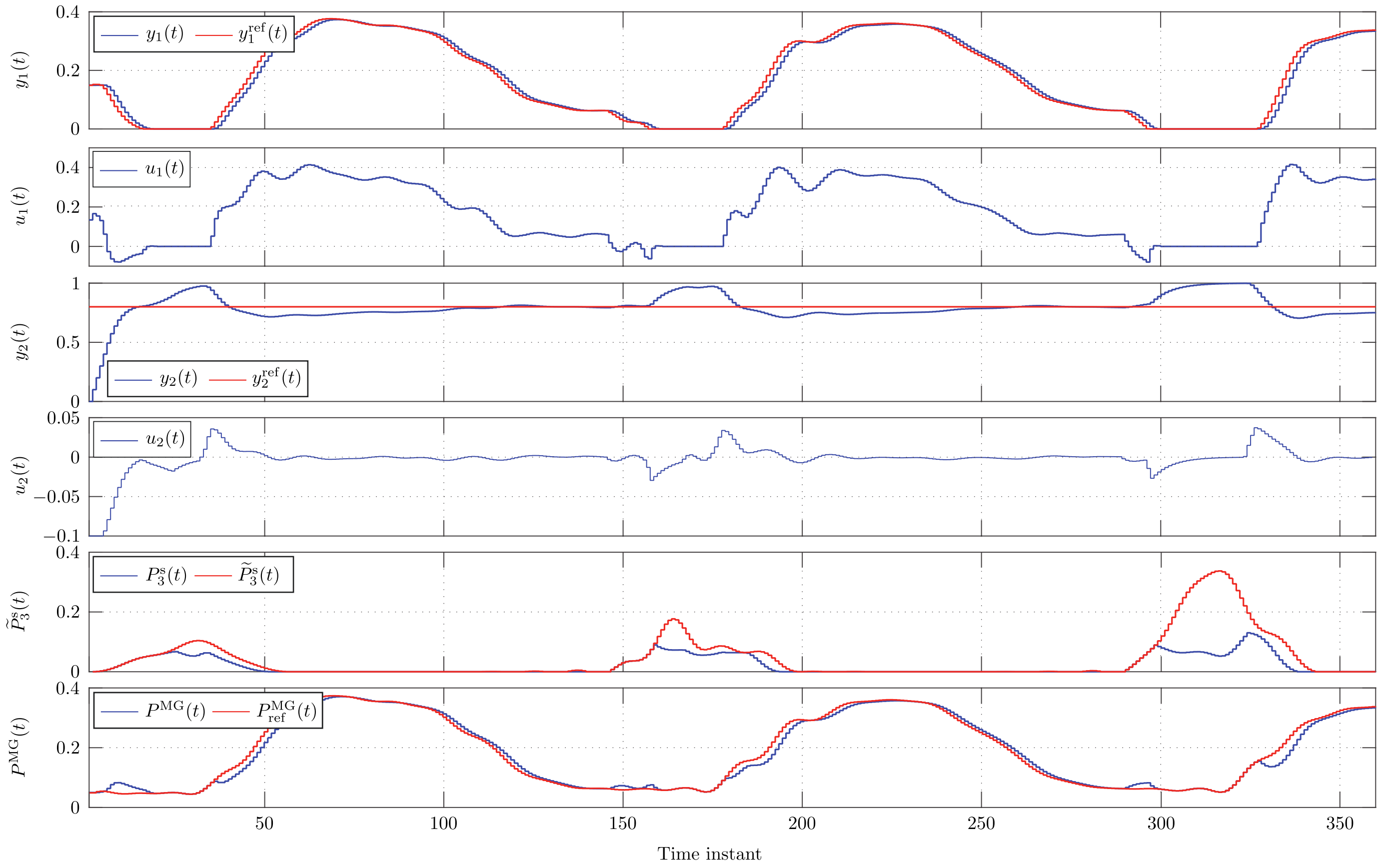

Fig. 2. Time responses of the microgrid subsystems in simulation: (i) actual CTG power production (blue) and its power production reference (red), (ii) CTG control action, (iii) battery SOC (red) and battery SOC reference (blue), (iv) battery rate of charge/discharge, (v) available PV power output (red) and actual PV power output (blue), and (vi) microgrid power reference (red) and actual microgrid total power production (blue).

SOC is kept close to the desired reference and it deviates from this reference only when it is in the interest of the overall coordination of the microgrid. The bottom figure shows that the microgrid coordination was performed in a satisfactory manner as we can see that the actual microgrid power production follows the desired reference very closely.

V. Conclusion

In this paper we presented a method for coordinated microgrid control based on parameterized costs of local operation declared by microgrid's constituent subsystems. The method is based on a decomposition and parameterization of the overall control problem which is then solved by combination of off-line and on-line computation. Performance of the proposed control strategy was tested on a simulation case study of a simple microgrid system.

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