Mary Leng, 2010, *Mathematics and Reality*, Oxford University Press

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The problem of proving the objective truth of mathematical statements and the objective existence of mathematical objects is one of the most compelling in the philosophy of mathematics. There are several arguments that, more or less successfully or convincingly offer reasons for believing in the (objective) existence of the mathematical realm. Of course, there are dissenting voices. A new and interesting one is the voice of Mary Leng in her thought-provoking book *Mathematics and Reality*. In the book she is concentrating her critical attention on the Quine-Putnam indispensability argument – the argument for the objective existence of mathematical items that many, friends and foes, labeled as the best argument for Platonism. Leng critically presents the argument and offers reasons for rejecting it by mainly questioning one of its premises, labeled Confirmational holism.
The book is structured in ten chapters. In the first chapters (1-4) Leng nicely introduces the subject. After presenting the Quine/Putnam argument itself, she goes on critically addressing different replies to it and to the discussions as they have been developed since. After careful and exhaustive presentation of the argument, in the remaining chapters (5-10) Leng concentrates on Confirmational holism - one of the premises in the argument, questioning it with the contrary idea to the effect that, notwithstanding the indispensability of mathematics to our best scientific theories, we are not forced to believe the literal truth of the mathematical apparatus indispensable to formulating such theory. In her own words:

> … we can account for our reasons to include mathematical hypotheses in our scientific theories without assuming that we ought to believe that these hypotheses are true. (p. 102)

In this review I shall mainly concentrate upon the contested argument. So let us have a look at it in more detail. The Indispensability argument is the argument for the acceptance of the literal truth of mathematical statements and the existence of mathematical objects. It can be formulated in the following way:

Premise 1 (P1) - Naturalim:
What we ought to believe is what ordinary scientific theories tells us is true or at least best confirmed by our ordinary practice.

Premise 2 (P2) – Confirmational holism:
The confirmation our scientific theories receive expands to their statements, both to the theoretical and to the empirical ones.

Premise 3 (P3) – Indispensability thesis
Mathematics is indispensable to science, which is to say that mathematical statements (included those that quantify over mathematical objects) are indispensable in formulating our best scientific theories.

Conclusion:
We ought to believe that there are mathematical objects or, passing to first-order, mathematics is true, i.e. mathematical objects exist.

Leng's starting view is naturalism: she endorses a naturalistic approach to ontological questions in her elaboration of the problems concerning the argument, in particular Confirmational holism (P2). And she gives reasons as to why, though we ought to maintain a naturalistic approach, we still have no need, appearances notwithstanding, to endorse the existence of mathematical objects. Even thought she follows Quine in his version of naturalism, according to which it is science that determines what there is and consequently what we ought to believe there is, she subsequently parts company with him and wants to divorce naturalism from Confirmational holism, so that her main strategy consists in

> …arguing that adopting a broadly naturalistic approach to ontology, looking to science to discover what we have reason to believe that there is, provides us with *no reason* (my italics) for believing in (abstract) mathematical objects such as numbers, functions, and sets. (p. 19)

Leng however firstly considers confronting premise P3, by claiming that indispensability argument fails even if mathematics actually *is* indispensable in formulating our scientific theories.
Anyhow, she basically denies Quine's idea that, in formulating and understanding our best scientific theories, eventually we ought to eliminate, or at least set on one side, merely practical forms of speaking and take the statements of the form "there are ϕs" ontologically at face value. And this necessarily leads to the refusal of P2: even if we accepted Field's nominalist versions of our best scientific theories, we would still have no good reason to accept the truth of all the items contained in such theories.

Leng's aim is to show, given the naturalistic background, that we do have reason to doubt the literal truth of some of the statements that are part of our best scientific theory; this would imply that it makes sense to raise the question whether the special position of the mathematical apparatus offers good reasons to resist believing in assumptions that imply the existence of mathematical objects.

Leng asks:

> if a scientist uses a sentence S in formulating his preferred scientific theory, ought we assume always that he has reason (given his own standards) to believe that S is true?

Well, as she promptly adds, sometimes scientists *cannot* believe that their theoretical hypotheses are literally true, e.g. if the truth of the hypotheses conflict with the truth of other theoretical hypotheses made elsewhere or, scientists sometimes hesitate to accept as true a theoretical component even in the absence of conflict with other theoretical assumptions. According to Maddy, and Leng agrees, both cases are standard in our scientific theorizing. Here is a worry.
Leng takes Maddy to be right, while I have some doubts concerning her proposal: it appears to me to be mixing together, not to say conflating two distinct situations, first, having endorsed contradictory theoretical statements, with second, having different

mathematical structures to choose between, depending on the application envisaged. The two are not equivalent, the former not being represented in standard scientific practice, the latter being part of the canonical way of doing science or applied mathematics in general. Indeed, it is one thing to test the applicability of a given mathematical theory (e.g. Euclidean geometry), and yet another to test mathematical rules in force within it. So, to say for example that in some context scientists believe in Euclidean geometry and that elsewhere they might go for hyperbolic geometry, is not to say that the truth of the former conflict with the truth of the latter. To hesitate to accept as truth a theoretical component is precisely to have doubts about having chosen the most appropriate mathematical structure, given the theoretical requirements.

Leng's another point against confirmational holism is offered by the fact that scientists seek for direct evidence in order to believe in the existence even of objects indispensably posited in their theories. This is to say that indispensability is not enough for them to believe the theory, and that more direct evidence is required.
Even though the point is convincing, the tendency to look in practice for what the theory confirms as being truly existing is not necessarily due to the lack of confidence in the theory itself; it could be due to a more intuitive need to have a look at what there objects are like. Or it could be a gut reaction of scientists when confronted with unintuitive or unexpected results. It does not mean that it is theory's fault, since this could be due to the fact that human nature is limited by intuition or dogmatism – a historical example could be Saccheri, who notoriously refused to accept the possibility of a system in which the Fifth Euclidean axiom did not hold.

How can we then explain the success of our scientific practice without accepting the truth of the background theory? How can we have good and indispensable explanations that are not true? - as many philosophers asked.[1] In order to answer this question, Leng defends a version of the instrumentalist approach to empirical science, the so-called fictionalism. The basic idea consists in the possibility of idealized theories in which the idealized objects are sufficiently well-related to the real ones for the theories to be useful in practice. Examples of such use of idealization are legion. A nice one is offered by the theory of fluids, detailed in the book, in which we assume real fluids to be continuous so that, even though such an idealized theory turns out to be useful, the fact remains that we accept literally false assumptions about real things. The continuous-fluid theory is then successful not because being true but because it is a sufficiently good approximation of the real state of affairs for all the results and predictions we need.
The mathematical apparatus used in our scientific theories, on the other hand, has a special place, it is "not…enough by itself to force us to abandon Quine's confirmational picture, for their treatment might tell us nothing about their ultimate confirmational status." (p.110) Leng then presents independent grounds against Confirmational holism, taking the starting point to be the need to present good reasons – independent of the special treatment mathematical assumptions enjoy - for not being forced to accept *all* the assumptions of our most successful scientific theories; and she amply caters to this need in the book.

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[1] For instance and prominently Shapiro in his 'Modality and Ontology' (1983, Mind, New Series, Vol. 102, No. 407, pp. 455-481)

Furthermore, Leng gives reasons for sustaining the analogy between models in empirical science and fictional characters. The use of mathematics in scientific theories can be seen as analogous to the process of fiction-making that does not force us to accept an ontology of fictional characters:

> utterances within and about fiction, and even the use of such utterances indirectly to express contents that we take to be true, need not commit us to belief in the existence of fictional characters. (p. 171)

According to such a fictionalist theory, mathematical statements within scientific theories can be treated as "merely useful theoretical fictions". So we need not to believe the literal truth of the mathematical statements we use even though by using them we do get the right results concerning how things are within the domain of non-mathematical objects. Leng's proposal is hence to accept our extant scientific theories merely as being nominalistically adequate instead of being true.

Leng is right in saying that *if* the idealizations, as the theory of ideal(ized) fluids, are indispensable, we clearly cannot be committed to believing the truth of literally false theories.

What might be the problem is that it is not clear that and how such idealized theories are, as a matter of fact, indispensable. Even if the fact is that we find it more preferable to use ideal description in developing the theory of X than some more literal account, it is still an open question how a literally false part of the theory of X can be indispensable for the theory of X. The theoretical entities that we take to be idealizations of physical objects can hardly be said to be indispensable, for how is something that is literally false of certain physical objects to be indispensable in order to develop a theory about the very same objects? Many authors (see again Shapiro (1983)) find this point very much open to discussion.

The fictionalist status of mathematical objects within scientific theories also generates difficulties of its own. Let us take as an example the discovery of the planet Uranus. Was mathematics not used for the prediction of the existence of Neptune, based on the investigation of the orbit of Uranus and the mathematical model not fitting into the real situation? The astronomers were looking for a theory that would match precisely that real situation. Mathematics was used but the results did not match with the observations of the orbit of Uranus, which led them to the assumptions that a (still to be discovered) planet might be the cause of the discrepancy – Why so? Well, it was recognized that the observation went wrong in relation to the theory, i.e. it was to a large extend mathematics that told us that there must be another object in the universe, does securing the route to empirical success. Would the astronomers, in the counterfactual situation, have bothered to make what they, following Leng's advice, took to be merely *fictional* mathematical results fit reality itself? Their underlying hypothesis obviously was mathematics as being literally true. (To use an analogy, would Schliemann have excavated the site had he believed that Homer's Troy had been a merely fictional entity? The success here, as in the case of Neptun, was to take the relevant portion of the discourse literally, loaded with heavy ontological commitments that accompany its literal reading. If Leng replies that success does not support assumptions of reality, a slippery slope opens; the ultimate price to pay might be a thoroughgoing anti-realism encompassing molecules, electrons and ions as well.

Another difficulty concerns the use of ideal objects, e.g. point masses, as representations of real objects that is, taken literally, false even though useful. The results we get out of such theories are useful and lead us to true predictions, and even so we do not take them to be literally true so that our approach can be depicted as being fictionalist. But with e.g. natural numbers used in those theories the situation does not seem to be analogous, for there is no clear general answer to the question: what are the natural numbers an approximation of in the empirical world?

At this point someone might object that even if we accepted Leng's fictionalism and the battle for the Indispensability argument were lost, there would still be no conclusive argument that there are no mathematical objects. Leng's response is along the lines of Field's: the indispensability argument is the best one available so we are right in being persuaded that mathematical objects do not exist once this argument of Quine and Putnam has been successfully rejected:

> If we account for our successful scientific practices *without* assuming that our mathematically stated empirical theories assert truths about mathematical objects, then this provide with a positive reason to *reject* the claim that there are any mathematical objects. (p.259)

Let me leave the discussion of this provocative issues at that, for the lack of space.
In conclusion, I would recommend Leng's *Mathematics and Reality* as philosophically inspiring, while at the same time quite enjoyable-to-read book for everyone interested in the philosophy of mathematics, as well as a useful and nice reference book to be used in philosophy-of-mathematics courses.