

# Flux Reflection Model of Ferroresonant Circuit Realized Using Matlab/Simulink

KRUNO MILIČEVIĆ, DRAGAN VULIN, DUŠAN PEJIN

Faculty of Electrical Engineering Osijek

University of Osijek

Kneza Trpimira 2b, 31000 Osijek

CROATIA

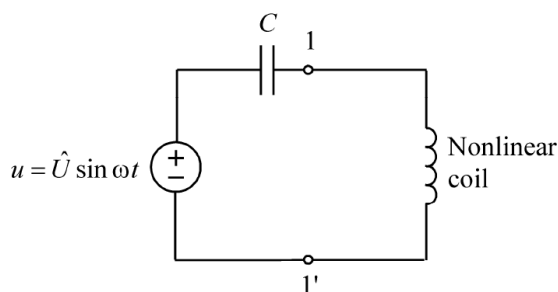
kruno.milicevic@etfos.hr <http://www.etfos.hr>

**Abstract:** The paper presents realization of flux reflection model of the ferroresonant circuit using Matlab/Simulink. Simulation is carried out employing the Dormand-Prince method with variable step of integration. During simulation the steady states are obtained by varying the amplitude of source voltage. Results of simulation are compared with results of measurements and results of simulations based on flux reflection model calculated numerically using 4th order Runge-Kutta method.

**Key-Words:** Ferroresonance, Ferroresonant circuit, Magnetization characteristic, Flux reflection model, Steady-state type, Bifurcation, Chaos

## 1 Introduction

The ferroresonant circuit is a nonlinear circuit which comprises a linear capacitor in series with a nonlinear coil, driven by a sine-wave voltage source, Fig. 1.



**Fig. 1** Ferroresonant circuit

The nonlinear coil is realized either as a winding of the unloaded iron-cored transformer or as a winding of the iron-cored reactor without air gap. The analyzed circuit is the simplest physical model of large electrical power system in which ferroresonant oscillation can occur. An example of ferroresonance in three phase transmission system is shown on Fig. 2.

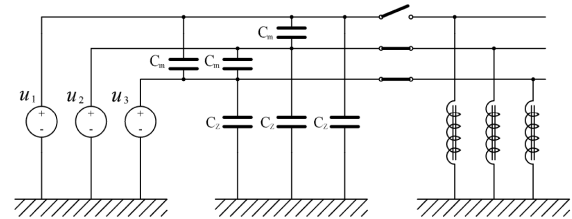
The nonlinear inductances represent iron core coils of unloaded three phase power transformer and the capacitance ( $C_m$ ,  $C_Z$ ) represents capacitance of long transmission line (or HV underground cable).

The ferroresonance occurs because of the malfunction of one pole of HV circuit breaker. The Fig. 1. shows the equivalent circuit where

$$u = (u_2 + u_3) \frac{C_m}{2C_m + C_Z} \quad (1)$$

$$C = 2C_m + C_Z$$

More practical examples are described in the literature [1].



**Fig. 2** Ferroresonance in three phase system

Although apparently simple, the ferroresonant circuit exhibits a complex steady-state behavior, e.g. bifurcations. Bifurcation is the sudden change of steady-state types caused by a small change made to the parameter values.

For instance, for a fixed set of parameters and varying the source amplitude  $\hat{U}$  only, several steady-state responses can be obtained: monoharmonic, odd higher harmonic, higher

harmonic, subharmonic, quasiperiodic and chaotic steady-state response, [2-4].

Thereby, the ferroresonance can be defined as any bifurcation from a monoharmonic response to a harmonic response (integer or non-integer) with significantly higher peak values.

From engineering point of view it is important to predict the bifurcations, because the ferroresonance can lead to transformer voltages several times the normal equipment rating. Computer simulation is an inexpensive but powerful tool for this task, [5-7]. Obviously, the prerequisite for a successful computer simulation is correct modeling of elements of ferroresonant circuit.

## 2 Modeling of ferroresonant circuit

The winding resistance of the coil is neglected, and the resistor, that represents the core losses of the coil, is assumed to be linear. State equations of the ferroresonant circuit, Fig. 1., are

$$\begin{aligned} \frac{d\bar{\varphi}}{dt} &= \bar{u}_L = \bar{U} \sin \bar{\omega}t - \bar{u}_C \\ \frac{d\bar{u}_C}{dt} &= \frac{1}{C} \left[ \frac{1}{R} (\bar{U} \sin \bar{\omega}t - \bar{u}_C) + \bar{i}_L(\bar{\varphi}) \right] \end{aligned} \quad (2)$$

where overline denotes per-unit values.

The magnetization nonlinearity  $\bar{i}_L(\bar{\varphi})$  is odd-symmetric and monotonically increasing. The magnetization nonlinearity is based on measurements carried out on a nonlinear coil regarding the above-mentioned assumptions made about properties of the coil. For example, one ought to rely on the results of standard measurements carried out on the nonlinear coil by the manufacturer [8].

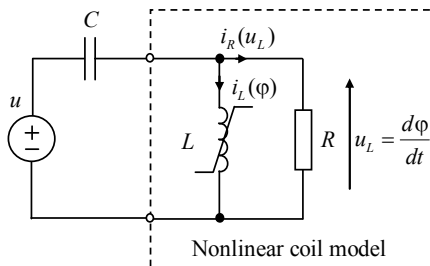


Fig. 3 Ferroresonant circuit

However, there are several problems regarding the modeling [9]:

- because of a limited thermal endurance of the coil, measurements cannot be carried out by applying the sine-wave voltage for the full range of flux and voltage peak values which occur in some operation modes of ferroresonant circuit.

- the parameters of model depend on the number of measurements, that is, on the number of measured values on  $U$ - $I$  characteristic. Namely, a piecewise linear form of the magnetization nonlinearity, that is obtained by a small number of measurements, would not be as smooth as necessary. In the case of polynomial form of the magnetization nonlinearity, a small number of measurements could result with a significant interpolation error.

## 3 Flux reflection model

In order to avoid above mentioned modeling problems, the flux reflection model of the ferroresonant circuit is defined using characteristic behavior that nonlinear coil exhibits in a ferroresonant circuit [10]. The behavior was identified from the flux waveforms obtained by measurements and by computer simulation.

The model comprises linearized state-equations of the ferroresonant circuit:

$$\begin{aligned} \frac{d\bar{\varphi}}{dt} &= \bar{u}_L = \bar{U} \sin \bar{\omega}t - \bar{u}_C \\ \frac{d\bar{u}_C}{dt} &= \frac{1}{C} \left[ \frac{1}{R} (\bar{U} \sin \bar{\omega}t - \bar{u}_C) + \frac{1}{L} \bar{\varphi} \right] \end{aligned} \quad (3)$$

with the limitation of flux values and the flux reflection realized using the following pseudocode:

FOR each integration step  $j$

calculate  $\left. \frac{d\bar{\varphi}}{dt} \right|_{j+1}$  as defined by used iterative method

IF  $|\bar{\varphi}(j)| > \bar{\Phi}_{sat}$  THEN

set  $\left. \frac{d\bar{\varphi}}{dt} \right|_{j+1}$  to be equal to  $-\left. \frac{d\bar{\varphi}}{dt} \right|_j$

END IF

END FOR

Parameter values are obtained for laboratory ferroresonant circuit, which is based

on a ferroresonant part of real electrical network [11]:

$$\bar{\omega} = 1 ; \bar{C} = 1 ; \bar{R} = 2 ; \bar{L} = 5.4 \quad (4)$$

Thereby, the inductance value  $\bar{L} = 5.4$  is obtained from linear part of magnetization characteristic.

In this paper the flux reflection model will be realized using Matlab/Simulink and the results of simulation will be compared with results of measurements and results of simulations based on flux reflection model calculated numerically using 4th order Runge-Kutta method with fixed step of integration (10000 integration steps per period) [10].

#### 4 Realization in Matlab/Simulink

The model realized using Matlab/Simulink is shown on Fig. 4. Simulation is carried out employing the Dormand-Prince method with variable step of integration [12].

During simulation the steady states are obtained by varying the amplitude of source voltage within the range:

$$0 < \hat{U} \leq 3 \quad (5)$$

with the step of the amplitude variation  $\Delta \hat{U} = 0.05$ . The simulation parameters are the same as they were in measurements and in simulation using 4th order Runge-Kutta method.

Characteristic steady-states obtained by simulations and measurements are shown in Table 1. During the experiments and the simulations substantially long delay of 50 periods was introduced for the capture of waveforms such that the system had enough time to settle down to a steady-state response, eliminating any transitory effects. Afterwards,

the steady-state types were identified using harmonic analysis.

There is no significant disagreement between results of Matlab/Simulink simulation and measurements and results obtained using 4th order Runge-Kutta method, i.e. Matlab/Simulink simulation results with characteristic steady-state types, including the periodic windows as well.

Furthermore, Matlab/Simulink simulation has confirmed the impact of the initial conditions on the initiation of ferroresonance. Namely, the odd higher harmonic steady-state appears and disappears at different values of increasing and decreasing  $\hat{U}$ ; i.e. at  $\hat{U} = 1.45$  and  $\hat{U} = 0.75$ , respectively.

#### 5 Conclusion

Model realized in Matlab/Simulink enables a simple variation of parameter values and a comprehensive analysis of ferroresonant circuit in a relatively short time.

Results obtained using Matlab/Simulink simulation validate realized model as a model usable for simulating ferroresonant circuit, i.e. the simulation resulted with properties characteristic for ferroresonant circuit:

- steady-state types,
- impact of initial conditions,
- appearance of periodic windows.

Between two flux reflections, that is, for flux values  $|\bar{\varphi}| < \bar{\Phi}_{sat}$  the flux reflection model is linear. Hence, the future work will address the analytical approach for analysis of the model, in order to determine explicitly the ranges of parameter values and values of initial conditions at which a particular steady state would arise.

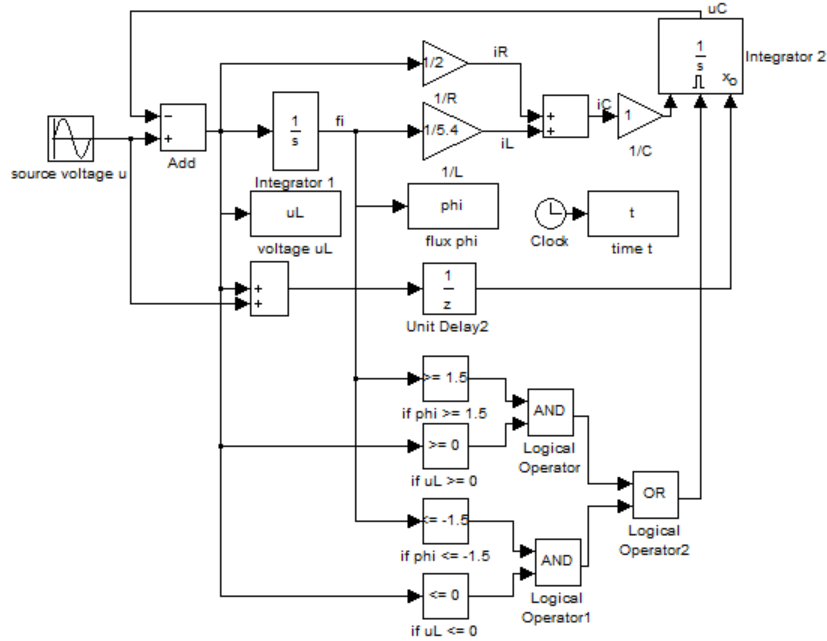
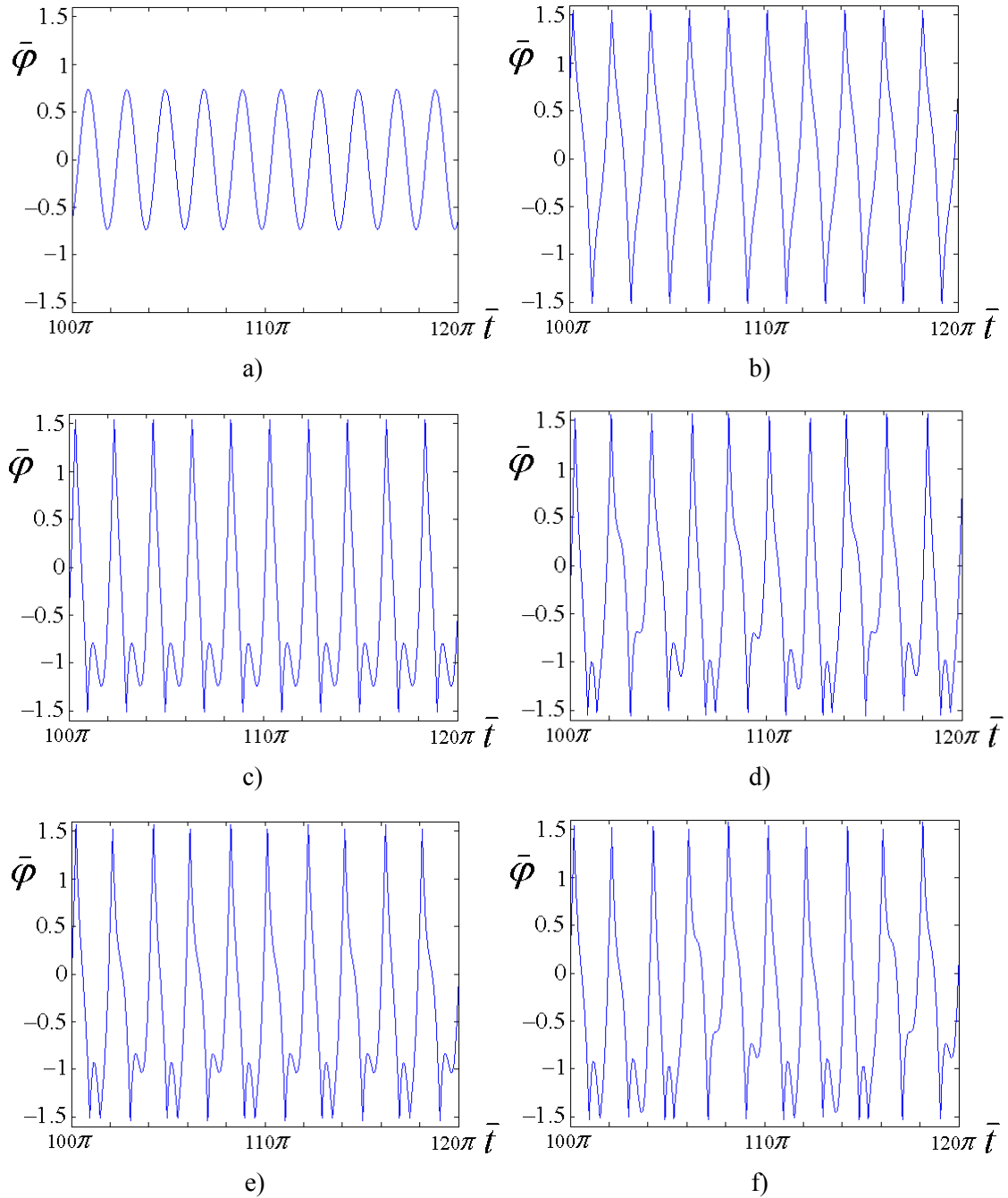


Fig. 4 Simulink model

Table 1: Steady-states of the flux reflection model obtained by measurements and simulations

Measurements [16]		4th order Runge-Kutta method [16]		Matlab Simulink		Steady-states
Increasing $\hat{U}$	Decreasing $\hat{U}$	Increasing $\hat{U}$	Decreasing $\hat{U}$	Increasing $\hat{U}$	Decreasing $\hat{U}$	
$0 < \hat{U} < 1$	$0 < \hat{U} < 0.7$	$0 < \hat{U} < 1.15$	$0 < \hat{U} < 0.8$	$0 < \hat{U} < 1.45$	$0 < \hat{U} < 0.75$	Monoharmonic steady-state, Fig. a
$1 \leq \hat{U} < 1.4$	$0.7 \leq \hat{U} < 1.4$	$1.15 \leq \hat{U} < 1.7$	$0.8 \leq \hat{U} < 1.7$	$1.45 \leq \hat{U} < 1.7$	$0.75 \leq \hat{U} < 1.7$	Odd higher harmonic steady-state, Fig. b
$1.4 \leq \hat{U} < 2.2$		$1.7 \leq \hat{U} < 2.25$		$1.7 \leq \hat{U} < 2.15$		Even and odd higher harmonic steady-state, Fig. c
$2.2 \leq \hat{U} < 3$		$2.25 \leq \hat{U} < 3$ (no periodic windows)		$2.15 < \hat{U} < 3$		Chaotic steady-state, Fig. d; including periodic windows
				Period-two steady-state: $\hat{U} = 2.4$		
Period-two steady-state, Fig. 5e: $2.2 \leq \hat{U} < 2.6$				Period-five steady-state, Fig. 5f: $\hat{U} = 2.55$		
				Period-five steady-state: $\hat{U} = 2.75$		



**Fig. 5** Results of simulation obtained by Matlab Simulink:

- a) Monoharmonic steady-state
- b) Odd higher harmonic steady-state
- c) Even and odd higher harmonic steady-state
- d) Chaotic steady-state
- e) Period-two steady-state
- f) Period-five steady-state

## References:

- [1] IEEE Working Group, "Modelling and analysis guidelines for slow transients – part III: The study of ferroresonance", *IEEE Transactions on Power Delivery*, Vol.15, No. 1, 2001, pp. 255-265.
- [2] P. S. Bodger, G. D. Irwin, D. A. Woodford, A. M. Gole, "Bifurcation route to chaos for a ferroresonant circuit using an electromagnetic transients program", *IEE Proceedings-Gener. Transm. Distrib.* Vol.143, No.3, 1996, pp. 238-242.
- [3] S. R. Naidu, B. A. de Souza, "Newton-Raphson approach for the analysis of ferroresonant circuit", *IEE Proceedings Gener. Transm. Distrib.* Vol.144, No.5, 1997, pp. 489-494.
- [4] A. Ben-Tal, Vivien Kirk, G. Wake, "Banded chaos in power systems", *IEEE Transactions on Power Delivery*, Vol.16, No.1, 2001, pp. 105-110.
- [5] P. Moses, M. Masoum, "Modeling subharmonic and chaotic ferroresonance with transformer core model including magnetic hysteresis effects", *WSEAS Transactions on Power Systems*, Vol.4, 2009, pp. 361-371
- [6] K. Milicevic, I. Rutnik, and I. Lukacevic, "Impact of voltage source and initial conditions on the initiation of ferroresonance," *WSEAS Transactions on Circuits and Systems*, vol. 7, 2008, pp. 800–810
- [7] J. Drápela, P. Toman, J. Orságová, M. Krátký, "Simulation of Ferroresonance Phenomena in Power Systems", *Proc. of the 5th WSEAS/IASME Int. Conf. on Electric Power Systems, High Voltages, Electric Machines*, Tenerife, Spain, December 16-18, 2005 (pp. 373-377)
- [8] W. L. A. Neves, H. W. Dommel, "On Modelling Iron Core Nonlinearities", *IEEE Transactions on Power Systems*, Vol.8, No.2, 1993, pp. 417-425
- [9] K. Miličević, I. Lukačević, I. Flegar, "Modeling of Nonlinear Coil in a Ferroresonant Circuit", *Electrical engineering (Archiv fur Elektrotechnik)*, Vol. 91, 2009, pp. 51-59
- [10] K. Miličević, I. Flegar, D. Pelin, "Flux Reflection Model of the Ferroresonant Circuit", *Mathematical Problems in Engineering*, Vol. 2009, 2009, pp. 1-13
- [11] K. Miličević, D. Pelin, "Experimental Ferroresonant Circuit as a Physical Model of a Ferroresonant Part of the Electrical Power Network", *Energija*, Vol. 58, 2009, pp. 270-289
- [12] The MathWorks – MATLAB and Simulink for Technical Computing, 31st of January 2011, <http://www.mathworks.com/help/toolbox/simulink/gui/bq7cmisp-1.html>