
New definition of fluid exergy in a gravitational field

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Abstract: This paper discusses the concept of working fluid exergy in an atmosphere stratified by a gravitational field, particularly the hot humid air in the atmosphere. A definition of exergy, different from that recently proposed by Petela (2008), is presented. Thermodynamic diagrams illustrate its elements and its technically feasible part in a transparent way. Reference papers are given for calculation of this part as technically feasible work potential of the hot humid air. This technically feasible part of the working potential can be used in the 'solar chimney'-type power plants, in cooling towers with natural circulation and in the hypothetical plants of the solar chimney type with high vortex columns instead of solar chimney.

Keywords: thermodynamics; work potential; exergy; technically feasible part (of exergy); height potential.

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1 Introduction

In Petela (2008), the author, one of the pioneers of exergy analysis application, gives a brief review of various forms of working fluid exergy, including the exergy component related to the gravitation field in the atmosphere. The overall exergy of matter is classified as either ‘kinetic’, ‘potential’, ‘thermal’ (physical + chemical), or ‘other’. A flow system is observed to have an exergy equal to the maximum shaft work from the physical position where the ambient and engine device are immobile for the thermodynamic observer. Therefore, the kinetic component of overall exergy is equal to the kinetic energy of relative motion (relative to the ambient condition).

In this scheme, the ‘potential’ component of the overall exergy is taken as additive to the kinetic and physical part of thermal exergy, which is defined in relation to the ground-level atmosphere. It was first defined by Szargut as the buoyancy force needed to do work on ascending fluid particles. Here, we take a more generalised approach to concept of exergy, discussing the more complex concept of gravitational exergy by Petela (2008). Petela starts from the physical part of the ground-level thermal exergy, defined by the well-known relation

$$e_{x\text{ ph}} = b_{\text{ph},0} = h - h_{s,0} + T_{s,0}(s_{s,0} - s) \quad (1)$$

where h and s without index are defined for the actual condition of the working fluid in the atmosphere at the ground level; the index $s,0$ designates the values of some property of the same fluid in mechanical and thermal equilibrium with the ambient atmosphere (s) at the ground level (0); $T_{s,0}$ is the absolute temperature of the ground-level atmosphere; $b_{\text{ph},0}$ is the exergy (i.e., the available shaft work) obtained by achieving the ‘physical’ equilibrium state, with the ground-level atmosphere as the ambient condition.

In relation to the exergy $e_{x\text{ ph}}$, the additional gravitational exergy consists of two parts: ‘buoyant’ and ‘altitude’. The buoyant component is equal to the work b_b of the buoyancy force associated with ascending of an isolated working fluid ($T = \text{const}$, $p = \text{const}$) to the height H at which the buoyancy force disappears, i.e., to the height where

$$\rho(T, p) = \rho_a(H).$$

After this ascent, an equilibrium with the ambient at height H occurs. This happens in a process analogous to that related to equation (1). However, this equilibrium occurs at ambient pressure and temperature $T_{s,H}$. After the work of the buoyancy force b_b , the following work is done:

$$b_{\text{ph},H} = h - h_{s,H} + T_{s,H} (s_{s,H} - s). \quad (2)$$

The work $b_b + b_{\text{ph},H}$ can generally be transferred and done at the ground level. According to Petela, gravitational exergy is not $b_b + b_{\text{ph},H}$ but rather an addition to the ground-level physical exergy $b_{\text{ph},0}$, i.e.,

$$e_{x \text{ grav}} = b_g = b_b + b_{\text{ph},H} - b_{\text{ph},0} = b_b + b_a. \quad (3)$$

In equation (3), b_a is the difference $b_{\text{ph},H} - b_{\text{ph},0}$, named the ‘altitude exergy’.

Thus, two alternatives for ‘physically’ obtaining ‘equilibrium’ with the environment are proposed in Petela (2008): the ground-level mechanism, with work potential $b_{\text{ph},0}$, and the altitudinal mechanism, with work potential $b_b + b_{\text{ph},H}$. The exergy is taken as the higher of the two and is called the ‘mechanical exergy’, b_{mech} :

$$b_{\text{mech}} = \max[(b_b + b_{\text{ph},H}), b_{\text{ph},0}]. \quad (4)$$

The objective of this paper is to analyse the concept introduced by Petela to examine their potential for application to the adiabatic processes of humid hot air in the atmosphere. Such processes are the subject of interest in the field of solar chimney power plants, elaborated in Haaf et al. (1983) and Ninic (2006), in the field of atmospheric convection, elaborated in Renno and Ingersoll (1996) and in solar plants with a gravitational vortex column. The latest is still the subject of theoretical discussions (Michaud, 1999; Ninic, 2006; Ninic and Nizetic, 2009).

2 Analysis and definition of the introduced concepts

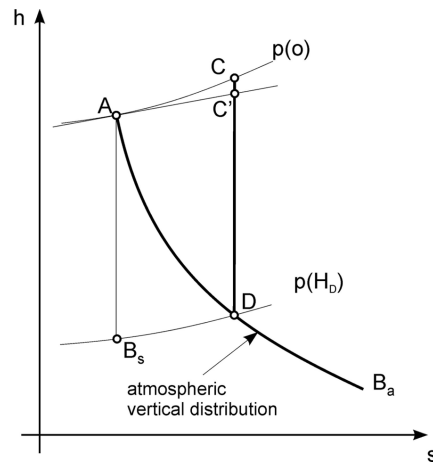
The physical exergy of matter in a gravitational field, defined as b_{mech} in equation (4), represents an extension of the concept of exergy in an atmosphere stratified by a gravitational field. However, one general remark can be made about equation (4): If the compositions of the working fluid (the substance under examination) and the atmosphere are not the same, the density of matter under the conditions $p_{s,H}$, $T_{s,H}$ does not equal to density of the atmosphere at the same height. The working fluid is, therefore, not in full mechanical equilibrium with the ambient and remains influenced by the buoyancy force even under conditions of (s, H) . Moreover, the height H at which, according to equation (2), the transfer to ambient pressure and temperature occurs is, in fact, arbitrary and achieved for an isolated working fluid at $\rho = \text{const}$. Another equally justifiable value of H could be achieved, for example, by adiabatic equilibrium ascent of the working fluid. Such an ascent would also proceed until the disappearance of buoyancy force, which is variable and depends on the density of the working fluid. The third remark is that ‘mechanical exergy’ b_{mech} in a gravitational field according to equation (4) is not single-valued. In regard to a practical implementation, exergy b_{mech} cannot be separated from its technically feasible part.

For these reasons, it is necessary to propose a different definition of exergy and its technically feasible part for its application in processes of hot humid air in the atmosphere.

3 New definition of air exergy in a gravitational field

Suppose for definiteness and simplicity, the air is considered as a working fluid, whose composition is equal to that of the atmosphere (both without humidity). In its initial state, C in Figure 1, we assume that the air is at zero height, with atmospheric pressure $p_c = p_A(0)$ and with temperature T_C higher than the ground atmospheric temperature $T_A(0)$. The vertical distribution of the atmospheric properties p, h, s is represented by the curve AB_a in Figure 1. Its relation to the drawn isentrope from the same ground-level state AB_s shows that, in this case, a stable atmosphere¹ is taken as the standard ambient.

Figure 1 State of atmosphere, hot ground-level air, and elements for defining maximum shaft work



Tangent to the isobar in the ambient state A is the ‘surroundings line’ (Bosnjakovic and Blackshear, 1965), where physical exergy, according to equation (1), is the length CC' . With notations adjusted to those in Figure 1, this relation becomes

$$e_{x,ph}(C, A) = h_C - h_A + T_A(s_A - s_C) (= CC') \tag{5}$$

where (C, A) correspond to state C , and surrounding ambient state A .

Let us now define the physical exergy of the air state C in relation to another available ambient condition under which the air is in complete mechanical equilibrium with its ambient atmosphere at the height z_D . The simplest way to do this is to imagine the adiabatic reversible ascending of the air upward until the buoyancy force disappears in point D , at which point the density of the air is equalised with the surrounding atmospheric density (process CD Figure 2). Owing to the same composition, entropy and pressure of the surrounding atmosphere to that of the current air state at point D , the air is also in full thermodynamic equilibrium with the atmosphere in the gravitational field.

Let us find an expression for this exergy $e_{x\ C(D)}$, omitting suffix ‘ph’. By definition, this exergy is the reversibly performed shaft work of the air in state C , assuming the atmospheric state D as the ambient. Such a process goes as the isentropic adiabatic CD in Figure 1, and the maximum shaft work (i.e., the exergy) is defined as

$$e_{x\ C(D)} = -\int_C^D v\ dp - g(z_D - z_C). \quad (6)$$

In relation (6), the First Law of Thermodynamics is taken into account for the flow system in the gravitational field of the Earth, according to which

$$q_{CD} = h_{D_{\text{tot}}} - h_{C_{\text{tot}}} + w_{t_{CD}} \quad (7)$$

where $w_{t_{CD}}$ is shaft work or “mechanical work of the steady-flow process” and h_{tot} (total enthalpy) is the overall specific internal energy of the mechanically extended system as a closed model of the open steady-flow system.² In this case, the system consists of flowing fluid and the Earth, with the atmosphere in the gravitational field of the Earth. The thermodynamic observer is connected with the Earth, and the overall energy of this system per 1 kg of air is

$$h_{\text{tot}} = u + pv + \frac{w^2}{2} + gz = h + \frac{w^2}{2} + gz. \quad (8)$$

Since there is no kinetic energy in states C and D , and the CD process is adiabatic, equation (7) is transferred to

$$w_{t_{CD}} = h_C - h_D - g(z_D - z_C). \quad (9)$$

By definition, the exergy is equal to this shaft work. To another thermodynamic observer who is connected with the parcel of air in motion, the First Law for an adiabatic equilibrium process gives

$$v\ dp = dh. \quad (10)$$

Thus, equation (6) arises from equations (9) and (10).

The derived expression for the exergy of hot air in the atmosphere (6) is also graphically represented in Figure 2. If it is taken into account hydrostatic law, $1/v_a \cdot g \cdot dz = -dp$, then follows,

$$g \cdot (z_D - z_C) = -\int_A^D v_a(p)\ dp \quad (11)$$

where $v_a(p)$ is the vertical dependence of the specific volume on pressure in the atmosphere. According to relations (9), (10) and (11), the exergy defined by equation (6) is represented by the surface ACD in Figure 2, with state D as the referent ambient state.

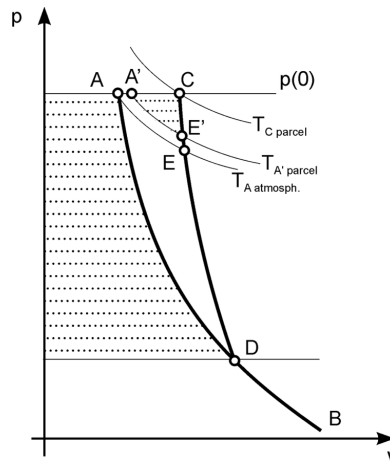
In the same figure, the surface ACE is the exergy of the same air with the level state A used as the ambient state. According to equation (5), this means that the surface ACE is $e_{x\ C(A)}$. The proof for this is that the process CEA is a reversible transition from C to A , and CEA is the shaft work obtained. The exergies defined for ambient conditions A and D are related by the following

$$e_{x\ C(D)} = e_{x\ C(A)} + A_{AED} > e_{x\ C(A)} \quad (12)$$

4 Extension of equation (6) to the general case

In general, the observed air and ambient atmosphere are not of the same composition. Once again, for the sake of simplicity and applicability, let the difference in the composition be the difference in the humidity content x (kg/kg_{dry.air}) of the observed air particle $x(C)$ and the atmosphere $x_a(A) \neq x(C)$ in Figure 3. Let $T(C) > T(A)$ and $x(C) > x_a(A)$. Unlike in the case of systems with the same composition given in the previous section, this time a common $h - s$ or $p - v$ diagram cannot be used for the hot air parcel and atmosphere. However, pressure and specific volume have important roles, so the atmosphere and observed air parcel states can be shown in the same $p - v$ diagram (Figure 3) when drawn without common isotherms.

Figure 3 Exergy of air $e_{x_{C(A)}}$ and $w_{t_{CD}}$ for $x_c \neq x_a(z)$



For free motion of discrete air parcels through the atmosphere without mixing, the pressures in the upper part of this parcel and in the surrounding atmosphere are equal, and the buoyancy force from below depends on density differences. In Figure 3, as well as Figure 2, the state C of the air can be reversibly brought to a state with pressure and temperature equal to the ground-level atmosphere, where $T_{A'}(\text{air parcel}) = T_A(\text{atmosphere})$ and $p_{A'} = p_A$. The reversible maximum obtained ground-level shaft work is equal to physical exergy denoted by $e_{x_{C(A)}}$. According to equation (5), it is equal to the hatched surface $A_{CE'A'}$

$$e_{x_{C(A)}} = A_{CE'A'} \quad (13)$$

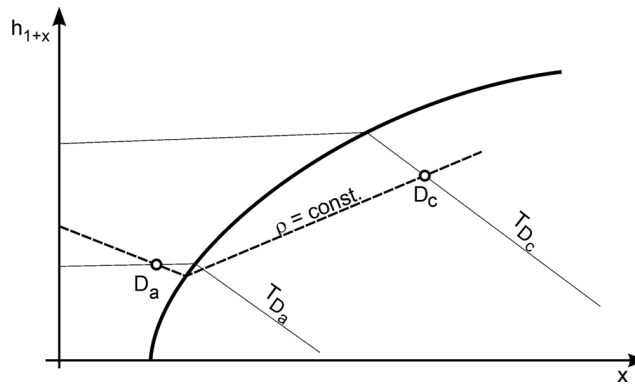
Let us now define the exergy in regard to the given vertical distribution of the atmospheric states in the same way as in the case of the equal compositions of the air in state C and the atmosphere. Let us define the shaft work during reversible air ascending from state C to height z_D , where buoyancy force disappears, as the first step in determining this exergy value. If this atmosphere is high enough, exergy value takes place in the point of intersection of the isentrope through C and the atmospheric characteristic AB , Figure 3, point D . The isentrope inclination and the level of intersection with the characteristic AB depend on whether there is water vapour

condensation during the process CD . The maximum shaft work gained during the process CD is, according to the preceding, equal to the deformed triangle ACD surface area. This area is also defined by equation (6). The (total) exergy of the air in state C is, however, somewhat higher in this case, because the states of the atmosphere and air observed in point D of the diagram are not identical. After this point in Figure 3, there follow different states of atmosphere (D_a) and the observed air of initial state $C_{(D_c)}$, defined by

$$p_{D_a} = p_{D_c} = p_D, \quad \rho_{D_a} = \rho_{D_c}, \quad T_{D_a} < T_{D_c}, \quad x_{D_a} < x_{D_c}. \quad (14)$$

These two states can be shown as such in $h-x$ diagram of the humid air (for pressure p_D), Figure 4.

Figure 4 $h-x$ chart of the moist air on the height level $z = z_D$



Ascending reversibly up to the height z_D , the observed air has already given the shaft work equal to the value $e_{x_{C(D)}}$. According to equation (6), it is equal

$$w_{1_{CD}} = A_{ACD}. \quad (15)$$

At the height z_D , in the state D_C , this air in relation to the local atmosphere of state D_a also has an additional working capacity. Owing to the lack of buoyancy force, it can be calculated by the standard formula (1), however applied to the mixture of ideal gases. They are in total equilibrium with the local atmosphere only at its temperature and in partial pressures the mixture components have in the atmosphere. The condensate in the state D_c contributes by its additional exergy. If we denote the additional working capacity by $e_{x_{D_c(D_a)}}$, the total exergy of the air of state C in the stratified atmosphere of different composition is

$$e_{x_{C(D)}}^{\text{diff.comp.}} = e_{x_{C(D)}} + e_{x_{D_c(D_a)}}. \quad (16)$$

In equation (16), $e_{x_{C(D)}}$ is exergy according to equation (12), and $e_{x_{D_c(D_a)}}$ is additional exergy, related only to the temperature difference $T_{D_c} - T_{D_a}$ and humidity content difference $x_{D_c} - x_{D_a}$. These differences are, however, limited by the obligatory relation $\rho_{D_c} = \rho_{D_a} = \rho_D$. On the other hand, the working potential $e_{x_{D_c(D_a)}}$ is difficult to obtain technically, particularly at the height z_D . On the contrary, the adiabatic process CD , by which $e_{x_{C(D)}}$ is defined, is technically obtainable in its technical realisation in the

solar-chimney-type power plants according to Ninic (2006), in cooling towers with natural circulation, in meteorological tornado phenomena, and, for now, only theoretical GVC power plants, according to Ninic (2006), Ninic and Nizetic (2009), and Nizetic (2010). Owing to all these applications, we consider the presentation of total exergy of hot humid air in the form of two members in equation (16) additionally justified.

In both discussed cases of defining the exergy of the air in stratified atmosphere, both by relation (6) and relation (16), it was assumed that the height at which the buoyancy force disappears (state D in figures) is generally reachable. If, for any reason, this was not the case, the highest reachable atmospheric state would result in maximum work. Let us denote such atmospheric state by D_a , and by D_c the corresponding state of the observed air the same height denoted by $z_{D^*} < z_D$. Reasoning in the way shown, the same formulae (6) and (16) would remain in force, however the higher limit of the integral in $e_{\chi C(D)}$ should be replaced by D_c , and z_D by z_{D^*} .

5 Possibility of hot humid air exergy exploitation

The present-day and the projected processes using hot humid air exergy include the air flow processes in wet cooling towers with natural circulation and solar chimney power plants. In cooling towers, the obtained work of buoyancy force along the cooling tower height is used for maintaining air circulation. In this way, the ground-level atmospheric temperature is given the role of the heat sink temperature in power plant thermodynamic cycle. Technically feasible hot humid air exergy obtained, inside the cooling tower fill, is expressed by the first member in formula (16). In this case, it is $e_{\chi C(D^*)}$ with integration limits in equation (6) from the upper fill edge as z_c , to the top of the shell, which, according to Section 4, should be denoted by z_{D^*} . This work potential is used for internal friction overcoming and circulation maintenance.

The same refers to operation of solar chimney power plants, as well as of yet only theoretical GVC power plants, as special extension of ‘classical’ solar chimneys (Ninic, 2006; Ninic and Nizetic, 2009). Such applications are indeed one of the justifications for given definition of hot humid air exergy in the form of two members in equation (16).

The process in which the air exergy is created is heat transfer in ground-level solar collector in the case of classical solar chimneys, and heat and mass transfer in the case of GVC power plants. The same happens in the fill of wet cooling towers with natural circulation. In all three cases of the application, the process is a part of atmospheric humid air cycle. In these cycles, the heat is added at the temperature higher than the ground-level atmospheric temperature, and rejected out of the plant at greater or lower height from the ground. At these heights, the temperature of the atmosphere as the heat sink is lower than that of the heat source for 10–30 K. In the case of the GVC cycles with the height z_{D^*} approx. 10,000 m, this temperature difference would be up to 80 K. Intense storm mechanisms are based on similar power cycles in the atmosphere (Renno and Ingersoll, 1996; Ninic et al., 2006).

There is the possibility that all of the work obtained by the buoyancy force effect along $0 < z < z_{D^*}$ can be concentrated to the ground level. Hence, this is the only place where turbines for shaft work production can be posted. A possible way to realise such concentration without a solid ‘chimney’ is described in Ninic (2006) and partially elaborated in Ninic and Nizetic (2009).

6 Conclusion

There is great need for defining the concepts of hot humid air exergy in an atmosphere stratified by gravitational field. These concepts are related to solar energetics and, to a certain extent, meteorology. R. Petela's systematic approach in Petela (2008) was a welcome contribution in this field. However, the improvement in the definition of exergy proposed in our paper gives, among other things, a better basis for the separation of the technically and meteorologically 'usable part' of the hot humid air exergy in the stratified atmosphere. The contribution of this paper is in the formulation of the technically feasible part $e_{xC(D)}$ of the total exergy in equation (16) and justification of its application, from meteorology to cooling towers with natural circulation, solar chimney power plants and power plants with gravitational vortex columns.

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Notes

¹According to the stability criterion

$$\left(\frac{\partial \rho}{\partial p}\right)_s < \left(\frac{\partial \rho}{\partial p}\right)_a,$$

where index s denotes the isentrope and index a denotes the atmospheric vertical profile.

²This topic is discussed more in detail in Ninic (2008).

Nomenclature

b_a	Specific ‘altitude exergy’, J/kg
b_b	Specific work of the buoyancy force, J/kg
b_g	Specific shaft work defining gravity exergy, J/kg
$b_{ph,H}$	Available shaft work at H level, J/kg
$b_{ph,0}$	Available shaft work at zero level, J/kg
b_{mech}	Mechanical exergy, J/kg
e_x	Specific exergy, J/kg
$e_{x\ grav}$	Specific ‘gravity’ exergy, J/kg
$e_{x\ ph}$	Specific physical exergy, J/kg
$e_x^{diff.comp.}$	Specific exergy for air in atmosphere of different composition, J/kg
$e_{x\ C(A)}$	Specific exergy of the air in state C for atmosphere in the state A , J/kg
g	Gravitational acceleration, m ² /s
H	Height, m
h	Specific enthalpy, J/kg
h_{tot}	Specific total enthalpy, J/kg
$h_{s,H}$	Specific enthalpy in the state of equilibrium with the ambient atmosphere of level H , J/kg
$h_{s,0}$	Specific enthalpy in the state of equilibrium with the ambient atmosphere at zero level, J/kg
p	Pressure, Pa
q	Quantity of heat, J/kg
s	Specific entropy, J/kgK
$s_{s,H}$	Specific entropy in the state of equilibrium with the ambient atmosphere of level H , J/kgK
$s_{s,0}$	Specific entropy in the state of equilibrium with the ambient atmosphere at zero level, J/kgK
T	Temperature, K
$T_{s,H}$	Ambient temperature at H level, K
$T_{s,0}$	Ambient temperature at zero level, K
v	Specific volume, m ³ /kg
v_a	Specific volume of the atmosphere, m ³ /kg
w	Velocity relative to ambient, m/s
w_t	Shaft work, J/kg
z	Level from the ground, m
<i>Greek</i>	
ρ	Density of working fluid, kg/m ³
ρ_a	Density of the atmosphere, kg/m ³
