DECISION SUPPORT SYSTEM FOR EVALUATING TAKEOVER BIDS IN PRIVATIZATION

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Abstract: *As in many transition countries, in Croatia can also be found a general tendency towards privatization of public enterprises and services. The Croatian Government is committed to the privatization of the state portfolio and has mandated Croatian Privatization Fund (CPF) to take the control of it. In order to provide potential investors with a fair and transparent transaction process CPF established tender procedures. After conducting the due diligence of the firm to be privatized the potential investors should submit their takeover bids following many criteria not only the price. Usually the potential investors are asked to propose the further investments, the time of keeping the actual employees, the intention of paying the creditors and to satisfy other requests. CPF has to choose the best bid according to the all criteria defined in the tender.*

In order to help CPF in this selection process we developed the mathematical model based on multi criteria programming and created the decision support system (DSS). We defined several criteria or objectives for the selection. The takeover bids are to be evaluated according to these objectives which may very often have different units of measurement, like in case of investment amount and employment level. In our paper CPF is called the decision maker (DM)

KEYWORDS: privatization process, investor selection, multicriteria optimization problem, multiple criteria decision making, compromise programming

1. INTRODUCTION

As we mentioned above, in Croatia, Croatian Privatization Fund (CPF) takes the control of the privatization of the public enterprises and services in the state portfolio. The success of a privatization is highly dependent on selection of a good investor. Simply looking for investor offering the highest price is not “efficient sourcing” even almost most important. Multiple criteria need to be followed and taken into account when selecting the investor. Usually, besides the price, the potential investors are asked to propose the further investments, the time of keeping the actual employees, the intention of paying the creditors and to satisfy other requests. CPF has to choose the best bid according to the criteria defined in the tender. These criteria depend on the special case of privatization. Besides the criteria mentioned above, the potential investors could be asked to announce their capital, employment level, total revenue and profit. These categories are then compared to the same categories of the public enterprise to be privatized constructing the ratio between them. For example, the ratio (capital of public enterprise)/(capital of investor) is considered. Usually, these ratios are good for taking into account the potential investor if they are less than 0.5. Total revenue is important to see the market share and profit is important to see the efficiency of the potential investors.

Considering all these criteria the best investor should be chosen. The problem is if all the criteria are equally important or we have to assign them some weights? How to assign the weights? In the literature there are mathematical models based on multi criteria optimization, data envelopment analysis, analytic hierarchical process (AHP) and other multi-attribute rating techniques. In order to help CPF we modeled this problem as a multi criteria optimization problem and created the decision support system (DSS). We followed the paper of Ng (2007) where the criteria importance are ranked in a sequence rather then specifying exact weights or exact degrees of relative preferences as by other solution techniques. In this way we almost avoid the subjective judgment incorporated in assigning the weights to the criteria. We added to Ng’s model some 0-1 variables, some new constraints specific for our problem of optimization and changed the objective function. Namely, rather than considering the weighted sum of criteria we considered the weighted sum of distances from the ideal solution, where the ideal solution is the solution that CPF would like to achieve according to all criteria.

The organization of this paper is as follows. The general mathematical model considering the general decision maker (DM) based on multi criteria optimization is presented in Section 2. The solution method is presented in Section 3, while in Section 4 there are some simulation and computational results. At the end the conclusions and the future research indications are given.

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2. MATHEMATICAL FORMULATION

For evaluating takeover bids in order to make the decision according to given criteria, we propose multiple criteria mixed integer programming model that is based on compromise programming. However, additional variables in the model are the criteria weights. Thus, the solutions of this problem will be the optimal decision and the criteria weights.

We consider a situation where the DM evaluates I takeover bids based on J criteria. The values for each criteria for every bid are given as a_{ij}, $i=1,...,I$, $j=1,...,J$. Though we assume that all criteria values are positively related to the bid evaluation score, negatively related criteria can be easily transformed by taking reciprocals. For example, if the first criterion is price, the value a_{11} is the price offered by the first bid.

The mathematical formulation is based on compromise programming which can be thought of as an effort to find a solution that comes as "close as possible" to the ideal values. Ideal solution corresponds to the best value that can be achieved for each objective, ignoring all other objectives, subject to the constraints. In this particular problem, evaluation of takeover bids, it is easy to obtain the ideal point since the number of bids is never very large. Thus, we obtain the ideal solution by taking the best value for a given criterion among the given takeover bids. For defining "closeness" to the ideal solution, we shall use the L_p (weighted) distance metric given with:

$$Lp = \left[\sum_{j=1}^{J} w_j \left[a_j^* - a_{ij} \right]^p \right]^{1/p}, i = 1,2,...,I, \quad p = 1,2,...,\infty,$$

where $a_j* = \max_i (a_{ij})$, $i = 1,2,\ldots,I$, are ideal values of criteria and $w_1, w_2,\ldots, w_J$ are criteria weights.

As mentioned before, the criteria are ranked by the DM. We can assume that criteria are arranged in descending order of importance (i.e. $w_1 \geq w_2 \geq \ldots \geq w_J$). The weights are decision variables in our model and we shall require them to be normalized and that they sum up to 1.

Moreover, we have the decision variables X_i, $i=1,2,\ldots,I$, which are binary variables denoting whether the bid is chosen or not i.e., X_i is equal to 1 if the bid i is chosen, and 0 otherwise. Since, we assume only one takeover bid can be chosen, these variables also sum up to one.

We can now give the mathematical formulation for the problem:

$$\min \left(\sum_{j=1}^{J} w_j \left(\frac{a_j^* - \sum_{i=1}^{I} a_{ij} X_i}{a_j^*} \right)^p \right)^{\frac{1}{p}}$$

$$\begin{aligned}
st. \quad & w_1 - w_2 \geq 0 \\
& w_2 - w_3 \geq 0 \\
& \quad \vdots \\
& w_{J-1} - w_J \geq 0 \\
& \sum_{j=1}^{J} w_j = 1 \\
& \sum_{i=1}^{I} X_i = 1 \\
& 0 \leq w_j \leq 1, \quad j = 1,..., J \\
& X_i \in \{0,1\}, \quad i = 1,..., I
\end{aligned}$$

The objective is to minimize the weighted relative distance form ideal solution. The solution obtained by solving this problem is called a *compromise solution*, which are always non-dominated. A solution, i.e. bid dominates all other bids if it is better than all other bids according to all criteria. Analogues, a bid is non-dominated if there does not exist a bid which is better than the given bid according to all criteria.

As p increases larger deviations get more weight. For $p = \infty$, the largest of the deviations completely dominates the distance determination. Different efficient solutions according to the multi criteria programming theory can be obtained by changing the value of parameter p.

However, sometimes the DM has some predefined weights for the criteria. For example, in many cases the price is the criterion that influences the decision with 70 percent of importance or at least 70 percent of importance. Thus, it would set the weight of the most important criterion to at least 0.7, i.e. $w_1 \geq 0.7$.

To include this possibility, we shall introduce parameters q_j, j=1,2...,J, where q_j is equal to predefined value of weight w_j , or equal to 0 if weight w_j is not predefined by the DM. This changes our model:

$$\min\left(\sum_{j=1}^{J} w_j \left(\frac{a_j^* - \sum_{i=1}^{I} a_{ij} X_i}{a_j^*}\right)^p\right)^{\frac{1}{p}}$$

$$\begin{aligned}
st.\quad & w_1 - w_2 \geq 0 \\
& w_2 - w_3 \geq 0 \\
& \quad\vdots \\
& w_{J-1} - w_J \geq 0 \\
& \sum_{j=1}^{J} w_j = 1 \\
& \sum_{i=1}^{I} X_i = 1 \\
& q_j \leq w_j \leq 1, \quad j = 1,\dots,J \\
& X_i \in \{0,1\}, \quad i = 1,\dots,I
\end{aligned}$$

Other features can also be implemented in this basic model. It can also contain upper and lower bound constraints and others. Considering relatively small number of bids, Excel solver handles this problem very quickly, without any problem. Furthermore, this problem can be reformulated in order to reduce its complexity and computational effort and time.

3. SOLUTION APPROACH

As mentioned before, the idea is to avoid DM giving the exact weights to criteria, pair wise comparisons of criteria importance or any other kind of specifying degree of relativity. Instead, the DM is only required to rank relative importance of criteria. Thus, since the model does not require pre-defined weights, decision making requires less effort from the DM. Though the ranking of criteria can be subjective to a certain degree, it is still far simpler than assigning exact values of importance to each criterion.

Moreover, this DSS does not require optimization background. The DM is required only to import criteria list, rank the criteria and give the bid list with criteria values to get to decision. For example, the DM can import the following criteria: the price, the further investments, the time of keeping the actual employees, the intention of paying the creditors, the potential investor’s capital, employment level, total revenue and profit. The DM can import any criteria he considers important for a privatization of a certain public enterprise. This means that DSS is created generally and can be adapted to any specific privatization case.

Firstly, the DSS eliminates the bids that do not satisfy lower or upper bound of the criteria, if there are such bounds. As the next step, the DSS checks for takeover bids that are dominated by all other bids and rejects them. In case of existence of a takeover bid that dominates all other bids, there is no need for further investigations because the DMs choice should be obvious. Dominated bids are the bids where the values of all criteria are less than the values of the criteria from at least one of other bids and will not be taken into account for sure. If there are bids left for evaluation, the decision making process comes to the third stage where it will require mathematical model for solving the problem.

Sensitivity analysis is offered in a „close to common-sense“ way. After getting the decision proposal, the DM can look and specify the value of criteria he would like to improve in the given bid.

4. SIMULATION AND COMPUTATIONAL RESULTS

There are four stages of processing the criteria and evaluating bids for the DM using this DSS. In order to describe those stages we now give an example of the process.

Firstly, the DM specifies the criteria. For example, criteria could be the price, the further investments, the time of keeping the actual employees, the intention of paying the creditors, the potential investor's capital, employment level, total revenue and profit. However, here we will use general notation: criterium1, criterium2, and so on. The DSS has a list of predefined criteria and a list of criteria which the DM can just use like it is given by default. However, criteria, lower and upper bounds and especially criteria rank may differ from case to case. Thus, the DM is given a possibility to add or remove criteria, change criteria list by choosing which criteria to follow from the list of given criteria and change all of its parameters. Finally, the DM ranks the criteria by simply assigning the numbers where larger number stands for bigger importance of the criterion.

Considering the upper and lower bounds, the DM can set the lower bound in case of maximizing an objective, i.e. positive correlation to bidder score or the upper bound in case of minimizing the objective.

Moreover, aside from quantitative criteria in sense that the DM specifies the quantity, for example selling price or guaranteed employment, there could be qualitative criteria meaning that there are some predefined values the criteria may have. This is the case when considering the criteria of respecting the contract where there could be two options, yes or no. We will assume answer "yes" to be desired answer. Thus, if the bound for this criterion is set to "yes", it means that all the bidders with "no" as an answer will be eliminated which is the case when respecting the contract is obligatory for bidders.

CRITERIA LIST

	Criterium	Objective	Lower/Upper bound	Unit	Rank
☑	Criterium1	MAX			1
☑	Criterium2	MAX	>= 5		2
☑	Criterium3	MAX			5
☑	Criterium4	MAX			4
☑	Criterium5	MAX			3
☑	Criterium6	MAX			6
☑	Criterium7	MAX			7
☑	Criterium8	MAX	>= 5		8
☑	Criterium9	MIN	<= 10		9
☑	Criterium10	- (MAX / MIN / -)	yes		10

Afterwards, the DM imports the data about the bids. Again, the type of data is here taken in consideration as shown below:

BIDS LIST

Bids	Criterium1	Criterium2	Criterium3	Criterium4	Criterium5	Criterium6	Criterium7	Criterium8	Criterium9	Criterium10
Bidder1	10	10	5	10	2	2	5	5	5	yes
Bidder2	10	6	11	10	2	2	5	5	4	yes
Bidder3	5	6	7	10	8	7	5	5	5	yes
Bidder4	10	6	11	10	2	2	5	5	6	yes (yes / no)
Bidder5	5	10	5	10	2	2	5	5	5	

The data is then transformed corresponding to correlation with score i.e. objective functions specified, the upper and lower bounds are checked and the ideal solution is calculated. Afterwards, relative deviations from criteria values of each bid to ideal value are calculated in the following way:

$$\frac{a_j^* - a_{ij}}{a_j^*},\ j = 1,\ldots,J,\ i = 1,\ldots,I.$$

It is not necessary to show the process of coming to the solution, these steps can be hidden for the DM. However, in order to follow the mathematical formulation we show them on the following picture:

BIDS LIST

Bids	Criterium1	Criterium2	Criterium3	Criterium4	Criterium5	Criterium6	Criterium7	Criterium8	Criterium9	Criterium10
Bidder1	10	10	5	10	2	2	5	5	5	1
Bidder2	10	6	11	10	2	2	5	5	4	1
Bidder3	5	6	7	10	8	7	5	5	5	1
Bidder4	10	6	11	10	2	2	5	5	6	1
Bidder5	5	10	5	10	2	2	5	5	5	0

TRANSFORMED NEGATIVE CORRELATION TO SCORE

Bids	Criterium1	Criterium2	Criterium3	Criterium4	Criterium5	Criterium6	Criterium7	Criterium8	Criterium9	Criterium10
Bidder1	10	10	5	10	2	2	5	5	0,20	1
Bidder2	10	6	11	10	2	2	5	5	0,25	1
Bidder3	5	6	7	10	8	7	5	5	0,20	1
Bidder4	10	6	11	10	2	2	5	5	0,17	1
Bidder5	5	10	5	10	2	2	5	5	0,20	0

IDEAL SOLUTION

Ideal:	**10**	**10**	**11**	**10**	**8**	**7**	**5**	**5**	**0,25**	**1**

RELATIVE DISTANCE FROM BEST CRITERIA VALUE

Bids	Criterium1	Criterium2	Criterium3	Criterium4	Criterium5	Criterium6	Criterium7	Criterium8	Criterium9	Criterium10
Bidder1	0,00	0,00	0,55	0,00	0,75	0,71	0,00	0,00	0,20	0,00
Bidder2	0,00	0,40	0,00	0,00	0,75	0,71	0,00	0,00	0,00	0,00
Bidder3	0,50	0,40	0,36	0,00	0,00	0,00	0,00	0,00	0,20	0,00
Bidder4	0,00	0,40	0,00	0,00	0,75	0,71	0,00	0,00	0,33	0,00
Bidder5	0,50	0,00	0,55	0,00	0,75	0,71	0,00	0,00	0,20	1,00

Before making the decision, the DM can specify some of the weights if necessary. In the picture shown bellow, the DM has specified weight of at least or exactly 70% for the second criteria. However, he can leave the weights blank. That way, the rank of the criteria will be respected but without any other assumptions on the weights. Afterwards, he gets the decision proposal which in this example is the bidder1:

BIDS LIST

Bids	Criterium1	Criterium2	Criterium3	Criterium4	Criterium5	Criterium6	Criterium7	Criterium8	Criterium9	Criterium10
Bidder1	**10**	**10**	**5**	**10**	**2**	**2**	**5**	**5**	**5**	**1**
Bidder2	10	6	11	10	2	2	5	5	4	1
Bidder3	5	6	7	10	8	7	5	5	5	1
Bidder4	10	6	11	10	2	2	5	5	6	1
Bidder5	5	10	5	10	2	2	5	5	5	0

WEIGHTS

Criterium1	Criterium2	Criterium3	Criterium4	Criterium5	Criterium6	Criterium7	Criterium8	Criterium9	Criterium10
	0.7								

DECISION

Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
yes	no	no	no	no

Apart from the form of sensitivity analysis mentioned in the previous section, this model can offer a way of analyzing the effects on changing the ranking and changing of bids criteria values which could be useful in negotiations. However, we leave it for future work.

5. CONCLUSIONS

In this paper we constructed the decision support system (DSS) in order to help Croatian Privatization Fund (CPF) to make a decision of choosing the investor in a process of privatization. When CPF announces the tender for privatizing a public enterprise it should give the criteria which will be evaluated during the process. The potential investors should apply giving their bids. The investor is chosen according to all criteria not only to one of them. For evaluating takeover bids in order to make the decision according to given criteria, we proposed mathematical model based on multiple criteria mixed integer programming and compromise programming. We presented an example with general criteria. In order to make a decision, for the decision maker (in this case CPF) it is not necessary to know any optimization technique. The only thing he has to do is to specify the criteria, rank them and input the data about the bids.

The advantage of this solution method regarding the others known in the literature is the fact that the DM does not have to specify the weights for the criteria. The weights are here the variables not the parameters. In this way we avoided the subjective judgment incorporated in assigning the weights to the criteria.

The DSS could be improved in giving the sensitivity analysis what will be done in future work. However, in the case of problems of large dimensions a heuristic could be constructed or some theoretical results obtained in order to solve the problem in an efficient way.

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