Imperative Logic, Moods and Sentence Radicals

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The aim of this essay is to examine two challenges that the imperative logic poses to the received view of sentence moods. According to the received view, there are three logico-semantic moods, indicative, imperative and interrogative; and there are two main components in natural language sentences, modal element and sentence radical. First we will examine whether change expression forces us to abandon one-radical-per-sentence view. Second, we will examine the doubts regarding threefold division of moods, which stem from the epistemic imperative conception of questions.

We take Wittgenstein's footnote remark to be the *locus classicus* of the received view:

> Imagine a picture representing a boxer in a particular stance. Now, this picture can be used to tell someone how he should stand, should hold himself; or how a particular man did stand in such-and-such situation; and so on. One might (using the language of chemistry) call that picture a proposition-radical. [10] §23.

Speaking in terms of the picture-metaphor: the picture, or rather - a combination of picture fragments means something, but that meaning is *unsaturated* until it has been used in a certain way. The component of the sentence that determines the *use* of the sentence-radical is called the modal element, logico-semantic mood [8] or illocutionary force indicator.

1 Change expressions and sentence radicals

It seems that the received view assumes that it is possible to extract one sentence radical from each sentence. That assumption is challenged by the change and action semantics, which in their turn seem to provide a suitable basis for imperative logic.

In a neglected paper of Lemmon [4], imperatives are treated as a kind of change expressions. Change expression is an "expression of the form (A/B) where A and B are truth functional expressions". Lemmon gives the semantics of imperatives in terms of obedience and disobedience conditions: an imperative $!(A/B)$ is obeyed if and only if the change from A to B takes place. A

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suitable reading for Lemmon-style imperative $!(A/B)$ could be: "Change initial A situation into resulting B situation!". If we take the sentence-radical to be a description of a (actual, possible, desirable,...) situation, then change-expression semantics threatens such a view, since for a change expression the use of two radicals is allowed.

Although not relying on the semantics that uses two sentence radicals, several successful action oriented approaches in imperative logic have been devised. Influential approaches include: Chellas [2], Belnap's and Perloff's [1] STIT semantics, in which the content of any imperative is an agentive, *i.e.* an agency ascribing sentence, and Segerberg's [6] approach, where imperatives are treated as prescribed actions. In this paper an update semantics for Lemmon-style imperative logic will be proposed. The proposed imperative semantics is closely related to the action logic tradition but differs in some respects. The most prominent difference in our approach is the inclusion of the information on the initial situation in the semantics of imperatives.

1.1 Twofold divisions of actions and imperatives

According to G. H. von Wright, the actions can be divided in two groups [7]. There are actions that bring about a change, *i.e.* actions of producing and destroying a state of affairs, and there are actions that prevent a change, *i.e.* actions sustaining and suppressing a state of affairs. Combining the idea of imperatives as prescribed actions with the twofold division of actions, we arrive at the twofold division of imperatives. Complementary imperatives are imperatives that command that a change is to be brought about; their form being $!(\neg A/A)$, read: "Produce A!". Symmetric imperatives command that a change is to be prevented; $!(A/A)$, read: "Maintain A!". The "right-side imperative" having the form $!(\top/A)$, or "See to it that A!" has drawn much attention in the literature. Nevertheless, the "right-side imperative" does not count as a basic on our approach. Defined in this way, the two basic imperatives bear information on the initial situation.

Following and modifying the minimal action semantics in von Wright's style [7], it seems the minimal imperative semantics requires at least three "points". The initial point is the initial situation in which the prescribed action should begin, the end-point is the situation resulting from the eventual and successful execution of the prescribed action, and the "null-point" is the situation which could occur if the course of events is not altered by the prescribed action. "Three points semantics" makes it possible to introduce three indicative modal elements,'$\cdot$' for 'in the *before*-situation it holds that...', '$\cdot_N$' for 'in the *later*-situation it is unavoidable that...', '$\cdot_P$' for 'in the *later*-situation it is possible that...'. Three point semantics can account for the following indicative entailments (where A is a contingent proposition and $A \Leftrightarrow B$ or $A \Leftrightarrow \neg B$) :

(Initial point: initial situation is defined) $!(A/B) \Rightarrow \cdot(A/\top)$
(End point: resulting situation is possible) $!(A/B) \Rightarrow \cdot_P(\top/B)$
(Null-point: resulting situation is avoidable) $!(A/B) \Rightarrow \cdot_P(\top/\neg B)$

The pair of a positive and a negative imperative is given by a pair of imperatives that deal with the same initial situation and which point to the alternative resulting situations. On that account, 'Don't produce A!' or '$\neg!(\neg A/A)$' is equivalent to 'Maintain $\neg A$!' or '$!(\neg A/\neg A)$'. Understood in this way, positive and negative imperatives are on an equal footing with respect to their binding force and informational layers [9].

(Imperative negation) $\neg!(A/A) \Leftrightarrow !(A/\neg A)$

Equipped with the definition for imperative negation, it seems that a variant law of contraposition holds for conditional imperatives. 'Produce B if A is the case.' is equivalent to 'Maintain $\neg B$ only if $\neg A$ is the case.' $0 \cdot(A/\top) \rightarrow !(\neg B/B) \Leftrightarrow !(\neg B/\neg B) \rightarrow \cdot(\neg A/\top)$

Given the restriction on the change expression that may occur in an imperative sentence, we are now in position to reconcile Lemmon based semantics and one-radical-per-sentence view. If A and B are contingent and logically independent propositions, it is obvious that a change expression (A/B) cannot be understood as a description of one situation.

One strategy of defending the received view could proceed as follows. It can be easily shown that any complementary or symmetric imperative can be decomposed into an indicative and a "right-side" imperative: $!(A/B) \Leftrightarrow \cdot(A/\top) \wedge !(\top/B)$. On that basis one can argue that it is a one sentence radical that is used in imperatives.

In another strategy, the two basic imperatives remain unanalyzed. In the first step, one must prove that the restriction of the formal language on the two (plus one) types of imperatives does not reduce the expressive power of Lemmon-style language. In the framework of update semantics, it can be done by showing that for each member from the family of models there is a text that generates it. After that, one may argue that it is the same descriptive content that is used both in symmetric and complementary imperatives. The case of the symmetric imperative is obvious. For complementary imperatives, one may say that the same proposition is being used in two ways: taking an intersection and a complement of the same descriptive content out of a set of valuations.

1.2 Questions: imperatives with interactively constituted semantics

The threefold division of moods is challenged by the theory of epistemic imperative. Åqvist [5] formalized questions in terms of two different modal operators: imperative operator and epistemic operator. *Yes/no* questions are interpreted as 'Let it (turn out to) be the case that either I know that A or I know that $\neg A$'. The proposed semantics of imperatives can explain why questions convey, *inter alia,* information that the interrogator does not know the answer. *Yes/no* question regarded as an epistemic imperative appears to be an instance of the complementary imperative: $!(\neg K_i A \wedge \neg K_i \neg A / K_i A \vee K_i \neg A)$. The epistemic imperatives could be used for the purpose of extending Groenendijk's [3] erotetic logic towards modelling the semantic impact of a question on the answerer's cognitive-motivational state.

It seems possible even to introduce the notion of a negative question. Using the idea that positive and negative imperatives differ only with respect to the right part of embedded change expression, the negative *yes/no* question becomes $!(\neg K_i A \wedge \neg K_i \neg A/\neg K_i A \wedge \neg K_i \neg A)$ or 'Let it remain the case that neither I know that A nor I know that $\neg A$'.

A cooperative interrogator is not certain which change she has commanded. The eventual full determination of prescribed epistemic action depends on the answerer's cognitive state. Therefore, an answerer-side semantics for epistemic imperative requires at least two distinct elements: (i) the model of interrogator's initial doxastic state and interrogator's intended doxastic state, (ii) the model of the answerer's cognitive state . The interaction of the two elements must be possible in order to enable the understanding of a question.

Epistemic imperatives may introduce doubt regarding threefold division of moods. But is it really the case that there are only two logico-semantic moods: indicatives and imperatives? The fact that the semantics of imperatives requires interaction between interrogator and the answerer in the determination of the prescribed epistemic action makes epistemic imperatives essentially different from the practical imperatives.

2 An update semantics for Lemmon-style imperative logic

(The language L_{LIL} of imperative logic) If A is a sentence in the language of classical propositional logic, then $!(A/A)$, $!(A/\neg A)$, $!(\top/A)$, $\cdot(A/\top)$, $\cdot_N(\top/A)$, $\cdot_P(\top/A)$ and their negations are sentences in L_{LIL}. If ϕ is an imperative and ψ a $\cdot$-type indicative sentence in L_{LIL}, then $\phi \to \psi$ and $\psi \to \phi$ are sentences in L_{LIL}. Nothing else is a sentence in L_{LIL}. A sequence of sentences $\phi_1; ...; \phi_n$ is a text in L_{LIL}.

Sets *Init* and *Res* are sets containing all the elements from the set *val* containing all the binary valuations for the propositional letters in the language under consideration coupled with the instants in which initial and resulting situation occur: $Init = \{\langle v, before\rangle : v \in val\}$, $Res = \{\langle v, later\rangle : v \in val\}$. The set of instants is $T = \{before, later\}$. The time designated intension of a component in a change expression is the set $|A|^t = \{\langle v, t\rangle : v \in val \wedge t \in T \wedge v(A) = \top\}$. The intension of a change expression (A/B) is the set $\|A/B\| = |A|^{before} \times |B|^{later}$. The set of cognitive-motivational states of an ideal addressee a with respect to the language under consideration is defined as $\Sigma_a = \{\langle \rho, \lambda\rangle : \rho \subseteq Init \times \lambda, \lambda \subseteq Res\}$. Its subset $\Phi = \{\langle \rho, \lambda\rangle : \langle \rho, \lambda\rangle \in \Sigma \wedge \rho = \emptyset\}$ is called the set of final states, containing state $1 = \langle \emptyset, \emptyset\rangle$.

(Basic sentences)

$$\langle \rho, \lambda\rangle[!(A/B)] = \begin{cases} \langle \|A/B\| \cap \rho, \lambda\rangle \text{ if } |B|^{later} \neq \lambda \\ 1, \text{ otherwise,,} \end{cases}$$

where $A = B$ or $A = \neg B$ or $A = \top$.

$$\langle \rho, \lambda\rangle[\cdot(B/\top)] = \langle \|B/\top\| \cap \rho, \lambda\rangle$$

$\langle\rho,\lambda\rangle[\cdot_N(\top/B)] = \langle\|\top/B\| \cap \rho,\ |B|^{later} \cap \lambda\rangle$

$\langle\rho,\lambda\rangle[\cdot_P(\top/B)] = \begin{cases} \langle\rho,\lambda\rangle \text{ if } \langle\rho,\lambda\rangle[\cdot_N(\top/B)] \notin \Phi \\ 1, \text{ otherwise.} \end{cases}$

$\langle\rho,\lambda\rangle\,[\cdot(A/\top) \rightarrow !(B/C)] = \begin{cases} \langle\rho \cap \|B/C\|\,,\lambda\rangle \text{ if } \langle\rho,\lambda\rangle\,[\cdot(A/\top)] = \langle\rho,\lambda\rangle \\ \langle(\rho \cap \|\neg A/\top\| \cap \|B/\top\|) \cup (\rho \cap \|A/\top\| \cap \|B/C\|),\lambda\rangle \\ ,otherwise, \end{cases}$

where $B = C$ or $B = \neg C$ or $B = \top$.

(Defined sentences) For $\sigma \in \Sigma_a$

$\sigma\,[\neg!(A/B)] = \sigma\,[!(A/\neg B)]$, where $A = B$ or $A = \neg B$ or $A = \top$

$\sigma\,[\neg\cdot(A/\top)] = \sigma\,[\cdot(\neg A/\top)]$

$\sigma\,[\neg\cdot_N(\top/B)] = \sigma\,[\cdot_P(\top/\neg B)]$

$\sigma\,[\neg\cdot_P(\top/B)] = \sigma\,[\cdot_N(\top/\neg B)]$

$\sigma\,[!(B/C) \rightarrow \cdot(A/\top))] = \sigma\,[\cdot(\neg A/\top) \rightarrow !(B/\neg C)]$, where $B = C$ or $B = \neg C$ or $B = \top$.

(Text)

$\sigma[\phi_1; ...; \phi_n] = \sigma\,[\phi_1]\,...[\phi_n]$.

2.1 Some examples

On this account it is obvious that the Imperator cannot coherently command (1) changing of the situation that does not obtain. Imperator cannot command an action that should bring about a historically impossible (2) or a historically inevitable situation (3 with analytical consequence added).

(1) The door is closed. Close it!

$\forall\sigma : \sigma[\cdot(C/\top); !(\neg C/C)] \in \Phi$

(2) Post the letter! But you will not post it.

$\forall\sigma : \sigma[!(\neg P/P); \cdot_N(\top/\neg P)] \in \Phi$

(3) Stay tall.

$\forall\sigma : \sigma[!(T/T); \cdot_N(\top/T)] \in \Phi$.

2.2 A sketch of semantics for epistemic imperatives

Using the idea from simple dynamic semantics, the interrogator's cognitive state may be modeled as a set of doxastically possible situations, and situations are modelled as sets of propositional letters. Thus K_iA becomes identified with the assertion that there is in an information state σ such that A is accepted in it, $[A]\,\sigma = \sigma$ and such that agent i is in σ, plus the assertion that A is the case. The modelling must be modified accordingly. We have to change the points in the model in such a way that the sets of valuations take the role previously assigned to (time indexed) valuations.

We will restrict modelling of the interrogator's cognitive state to the set l of propositional letters appearing in the *desideratum* of the question. Let $\sigma \subseteq \wp(l)$ where l is set of propositional letters occurring in the desideratum. We will need these sets of possible interrogator's doxastic states with respect to A: $|\neg B_iA| = \{\sigma \mid \sigma\,[A] \neq \sigma\}$, $|B_iA| = \{\sigma \mid \sigma\,[A] = \sigma \wedge \sigma \neq 1\}$, $|\neg B_i\neg A|$, $|B_i\neg A|$.

For doxastic assertions without iterated operators we set $|\phi \wedge \psi| = |\phi| \cap |\psi|$, $|\phi \vee \psi| = |\phi| \cup |\psi|$ and $\|\phi/\psi\| = |\phi| \times |\psi|$. We define the function *block* that delivers partitions of information states for disjunctive epistemic assertions as

$$block(E) = \begin{cases} \{|B_i\phi_1|, ..., |B_i\phi_n|\}, \\ \quad \text{if } E = \ K_i\phi_1 \vee ... \vee K_i\phi_n \text{ and } n \geqq 2 \\ \emptyset, \text{ otherwise.} \end{cases}$$

The determination of the desirable partition depends on the answerer's cognitive state. In order to enable interaction of the two elements, the set of relevant cognitive-motivational states of the answerer must at least combine the model ρ of interrogator's cognitive-motivational state (combining a set of possible ignorant states and set of possible desired states) with the answerer's cognitive state λ: $\Sigma = \{\langle\rho, \lambda\rangle \mid \rho \subseteq \wp(l) \times \wp(l), \lambda \subseteq \wp(l)\}$. The proposed semantics of the *yes/no* question shows that the epistemic action remains undefined if the answerer does not know the answer:

$$\langle\rho, \lambda\rangle[!(\neg K_iA \wedge \neg K_i\neg A/K_iA \vee K_i\neg A)] =$$

$$= \begin{cases} \langle||\neg B_iA \wedge \neg B_i\neg A/B_iA \vee B_i\neg A|| \cap \rho \cap \lambda, \lambda\rangle \\ \quad \text{if } \exists\lambda' : \lambda' \in block(K_iA \vee K_i\neg A) \wedge \lambda \subseteq \lambda' \\ \langle||\neg B_iA \wedge \neg B_i\neg A/B_iA \vee B_i\neg A|| \cap \rho, \lambda\rangle, \text{ otherwise.} \end{cases}$$

The inability of Interrogator to completely determine the desired state shows the difference between epistemic and practical imperatives. On the other hand, epistemic imperatives bring information on the state that is to be changed and that makes them similar to practical imperatives.

References

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