

Dynamic Analysis of Loads Moving Over Structures

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1. Introduction

The increase in traffic intensity and speed nowadays requires a more complex analysis of structures than in the past; today it is necessary to consider dynamic behaviour of structures induced by the loads moving over a structure. The simplest case of a moving-load (dynamic) analysis is the case of a simple beam over which a concentrated load is moving, which is represented with a 4th order partial differential equation (PDE).

PDE for moving load has been solved numerically, which has several benefits in comparison to a closed-form solution (application of boundary conditions, introduction of damping and discrete structural elements like springs and dashpots and many more). The average acceleration method has been employed since the direct use of finite differences had turned out to be practically unusable.

2. Description of the problem

Problem of a massless load moving on a beam is described by the well-known partial differential equation [1]

$$EI \frac{\partial^4 u}{\partial x^4} + m \cdot \frac{\partial^2 u}{\partial t^2} - P(t) = 0 \quad (1)$$

with the analytical solution obtained most easily through the use of harmonic series:

$$u(x,t) = \frac{2 \cdot P}{\rho \cdot L \cdot \omega(1)^2} \left[\sum_{k=1}^{50} \left[\frac{1}{k^2(k^2 - \alpha^2)} \cdot \left(\sin\left(k \cdot \pi \cdot v \cdot \frac{t}{L}\right) - \frac{\alpha}{k} \cdot \sin(\omega(k) \cdot t) \right) \cdot \sin\left(k \cdot \pi \cdot \frac{x}{L}\right) \right] \right] \quad (2)$$

3. Numerical solution of the discretised problem

One approach to the above problem is to consider the discrete version of the problem (one with finite number of masses). Discrete form of the above equation in matricial notation is

$$M\ddot{D} + C\dot{D} + SD = A(t) \quad (3)$$

Assuming that the acceleration is constant within the time interval (average acceleration method [2]), from the above equation we obtain the following incremental form:

$$M\Delta\ddot{D}_j + C\Delta\dot{D}_j + S\Delta D_j = \Delta A_j \quad (4)$$

In this equation, we have unknown incremental accelerations, incremental velocities and incremental displacements. Introducing the assumption that the acceleration is constant within a time interval, we obtain the final equation to be solved:

$$M \left[\frac{4}{(\Delta t_j)^2} \Delta D_j - \bar{Q} \right] + C \left[\frac{2}{(\Delta t_j)^2} \Delta D_j - \bar{R}_j \right] + S \Delta D_j = \Delta A_j \quad (5)$$

The solution procedure together with the values of various components has been coded in MathCAD program (after we solve for the incremental accelerations, velocities and displacements, their total values are easily obtained through addition.) [3]

4. Comparison between analysis and measurement

Based on the above equations, an existing 2D finite element computer program OKVIRW has been extended to accommodate the moving-load analysis. [3]

It is instructive to compare the numerical results with the measurements that have been done on the overbridges "Čakovec" and "Paka" on the highway "Zagreb - Čakovec". We can see that the numerical solution is more than satisfactory.

Measurement	Numerical solution	Difference
4,38 Hz	4,54 Hz	3,65 %
15,80 Hz	17,24 Hz	9,11 %

Table 1. Comparison of the first two vertical frequencies for the overbridge "Paka"

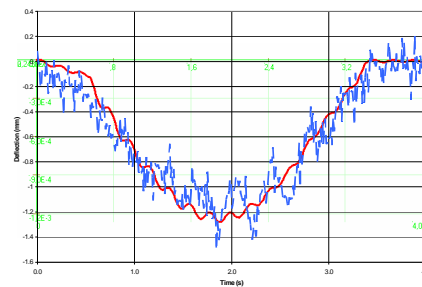


Figure 1. Mid-point displacement history for the overbridge "Paka"
(--- measurement, — numeric solution)

Measurement	Numerical solution	Difference
5,628 Hz	5,376 Hz	4,48 %

Table 1. Comparison of the first vertical frequency for the overbridge "Čakovec"

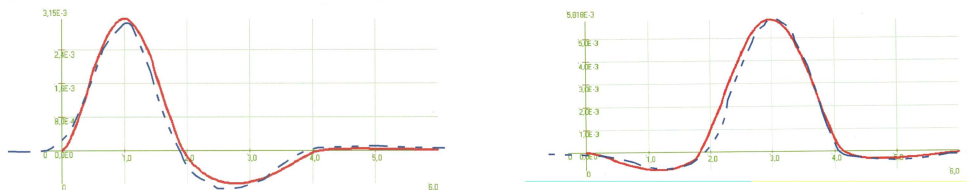


Figure 2. Mid-span displacement history for the first and the second span
(--- measurement — numeric solution)

5. Conclusion

As it can be seen from the examples the numerical approach to the problem of a moving load is quite suitable for engineering purposes: the solutions are accurate and the procedure based on average acceleration is robust. Further benefit of the numerical formulation is that various boundary conditions and damping can all be easily taken into analysis.

Acknowledgement

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References

- [1] Inglis, C.E., "Mathematical Treatise on Vibration in Railway Bridges", Cambridge University Press, London UK, 1934.
- [2] Weaver, W., Johnston, P. R.: Structural Dynamics by Finite Elements, Prentice-Hall, Inc., Englewood Cliffs, New Jersey 1987.
- [3] Štimac, I., "Analiza mostovskih konstrukcija pobuđenih pokretnom masom", Magistarska radnja, Zagreb, 2003.