HEATING OF OIL WELL BY HOT WATER CIRCULATION

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Abstract When highly viscous oil is produced at low temperatures, large pressure drops will significantly decrease production rate. One of possible solutions to this problem is heating of oil well by hot water recycling. We construct and analyze a mathematical model of oil-well heating composed of three linear parabolic PDE coupled with one Volterra integral equation. Further on we construct numerical method for the model and present some simulation results.

Keywords: Oil well, integro-differential equation, Volterra integral equation

Introduction

An oil well producing at low temperatures may experience large pressure drops due to high viscosity of oil and wax forming. One way to avoid these pressure drops is heating of oil by hot water recycling.

The tubing is surrounded by two annulus for water circulation. Hot water is injected into inner annulus and it flows out of the system through the outer annulus. The main technical concern is minimization of energy lost in the system while keeping oil temperature sufficiently high.

Configuration just described will be called counter flow exchange. If the hot water is injected into outer annulus and leaves the system through inner annulus, then we talk about parallel heat flow exchange.

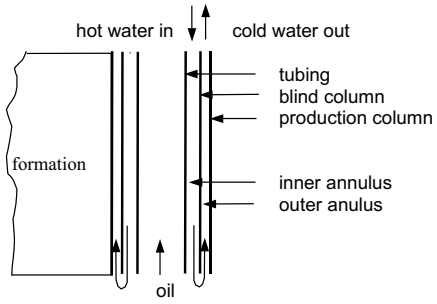

Figure 1. Counter flow heat exchange

The outline of the paper is as follows. In the first section we present a simple one-dimensional mathematical model describing the heat exchange in the system. We present only counter flow configuration since parallel flow configuration differs only in signs of water velocities. Solvability of a system of integro-differential equations describing the heat exchange is discussed in the second section. It is shown that the result of Artola [1] can be applied. In final section we discuss numerical method for solution approximation and present some numerical results for counter flow and parallel flow configurations.

A problem similar to this one was considered in engineering literature in [6].

1. Mathematical model

Cross-sectional mean velocities of oil and water in inner and outer annulus will be denoted by v_o, v_i and v_e. They are assumed to be constant, and therefore the fluids have constant pressure drops. Furthermore, to simplify the model, we neglect friction and we take mass densities ρ_o (oil), ρ_w (water) to be constant. The heat is transferred between the tubing, inner and outer annulus and the formation according to Newton's law.

With these simplifying assumptions and taking direction of z axis vertically downwards, we obtain the following three parabolic equations (see [2] for example):

$$a_o[\frac{\partial T_o}{\partial t} - v_o\frac{\partial T_o}{\partial z}] + b_o(T_o - T_i) = D_o\frac{\partial^2 T_o}{\partial z^2} \tag{1}$$

$$a_i[\frac{\partial T_i}{\partial t} + v_i\frac{\partial T_i}{\partial z}] + b_o(T_i - T_o) + b_e(T_i - T_e) = D_i\frac{\partial^2 T_i}{\partial z^2} \tag{2}$$

$$a_e[\frac{\partial T_e}{\partial t} - v_e\frac{\partial T_e}{\partial z}] + b_e(T_e - T_i) + b_f(T_e - T_f) = D_e\frac{\partial^2 T_e}{\partial z^2}, \tag{3}$$

for $z \in (0, L)$ and $t \in (0, t_{\max})$. The main variables are the temperatures of oil, water in inner annulus, water in outer annulus and the temperature of the formation, denoted respectively by T_o, T_i, T_e and T_f. All coefficients are constant and they have the following meaning: $a_o = A_o\rho_o c_o$, $a_i = A_i\rho_w c_w$, $a_e = A_e\rho_w c_w$ where A_o, A_i and A_e are cross-sectional areas and c_o, c_w are heat capacities. By b_o, b_e and b_f are denoted heat transfer coefficients from Newton's law, and by D_o, D_i and D_e thermal conductivities, multiplied by cross-section areas. In counter flow exchange all three fluid velocities are positive. From mass conservation it follows $a_i v_i = a_e v_e$.

Heat flow in surrounding formation is assumed to be radial with respect to the tubing and to have constant (geothermal) gradient in vertical direction. We denote by $T_z(z)$ geothermal temperature and by $T_s(r, z, t)$ the temperature in the soil. Formation temperature T_f is then given by $T_f(z, t) = T_s(r_f, z, t)$, where r_f is formation radius. The temperature T_s is the solution of the heat equation with initial temperature T_z, temperature at infinity equal to T_z, and prescribed heat flux q_f at $r = r_f$. In the other hand, q_f is given by Newton's law

$$q_f = b_f(T_e - T_f). \tag{4}$$

Then, by applying Duhamel's principle we can represent formation temperature by the formula

$$T_f(z, t) = T_z(z) + \int_0^t p(t - \tau)\frac{d}{d\tau}q_f(z, \tau)\, d\tau, \tag{5}$$

where $p(t) = P(r_f, t - \tau)/2\pi k_f$ (k_f is thermal conductivity of the soil) and $P(r, t)$ is the solution of the problem

$$\begin{cases} \dfrac{\rho_f c_f}{k_f}\dfrac{\partial P}{\partial t} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\Big(r\dfrac{\partial P}{\partial r}\Big), \quad r > r_f,\ t > 0 \\ P(r, 0) = 0, \quad r > r_f \\ P(\infty, t) = 0, \quad t > 0 \\ -2\pi k_f \dfrac{\partial P}{\partial r}\Big|_{r=r_f} = 1. \end{cases} \tag{6}$$

(ρ_f and c_f are mass density and heat capacity of the soil, respectively). It can be shown as in van Everdingen and Hurst [7] that $p(t) = O(\sqrt{t})$ and $p'(t) = O(1/\sqrt{t})$, as $t \to 0$. Therefore, $p'(t)$ is in $L^1_{\mathrm{loc}}([0, \infty))$ and we can make partial integration in (5). Taking natural assumption that $q_f = 0$ at $t = 0$ (that is $T_e = T_z$ at $t = 0$) and using (4) we obtain a

Volterra integral equation for q_f:

$$b_f(T_e(z,t) - T_z(z)) = q_f(z,t) + b_f \int_0^t p'(t-\tau) q_f(z,\tau)\, d\tau. \qquad (7)$$

This equation has the resolvent $r \in L^1_{\mathrm{loc}}([0,\infty))$ and it can be solved by formula (see Gripenberg, Londen and Staffanson [3])

$$q_f = b_f\big[T_e - T_z - r \star (T_e - T_z)\big], \qquad (8)$$

where we have introduced convolution operator

$$(r * \phi)(t) = \int_0^t r(t-\tau)\phi(\tau)\, d\tau.$$

By use of (8) and (4) we can eliminate formation temperature from (3) which is then transformed to

$$a_e[\frac{\partial T_e}{\partial t} - v_e \frac{\partial T_e}{\partial z}] + b_e(T_e - T_i) + b_f(T_e - r \star T_e) = D_e \frac{\partial^2 T_e}{\partial z^2} + F, \qquad (9)$$

where $F = b_f(T_z - r \star T_z)$ is a smooth known function. We see that equations (1), (2) and (9) represent parabolic system perturbed by the operator M given by

$$Mu(z,t) = \int_0^t r(t-\tau) u(z,\tau)\, d\tau \qquad (10)$$

The problem is to solve the system (1), (2) and (9) with suitable boundary and initial conditions. We assume given the temperatures of entering water at $z = 0$ and oil at $z = L$. At the bottom of inner and outer annulus we have equality of water temperatures and continuity of total thermal flux. Therefore, we take

$$\frac{\partial T_o}{\partial z}(0,t) = 0, \quad \frac{\partial T_e}{\partial z}(0,t) = 0, \quad T_o(L,t) = T_o^L, \quad T_i(0,t) = T_i^0, \qquad (11)$$

$$T_i(L,t) = T_e(L,t), \quad D_i \frac{\partial T_i}{\partial z}(L,t) + D_e \frac{\partial T_e}{\partial z}(L,t) = 0, \qquad (12)$$

for all $t > 0$, where $T_o^L = T_z(L)$ and T_i^0 are given. The initial conditions are

$$T_o(z,0) = T_e(z,0) = T_z(z), \quad T_i(z,0) = T_z^1(z), \qquad (13)$$

where function T_z^1 satisfies compatibility conditions $T_z^1(0) = T_i^0$, $T_z^1(L) = T_z(L)$ and it is close to geothermal temperature T_z. All functions involved are supposed to be smooth.

2. Variational problem

We consider variational formulation of the problem (1), (2) and (9) with boundary and initial conditions (11)–(13). Without lose of generality we can consider homogeneous boundary conditions $T_o^L = T_i^0 = 0$.

We introduce Hilbert space

$$V = \{(\phi_o, \phi_i, \phi_e) \in H^1(0,L)^3 \colon \phi_0(L) = 0, \;\; \phi_i(0) = 0, \;\; \phi_i(L) = \phi_e(L)\}$$

with the norm $\|\cdot\|$ inherited from $H^1(0,L)^3$ and bilinear forms $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ over $V \times V$ defined as follows: for $\mathbf{T} = (T_o, T_i, T_e)$, $\mathbf{\Phi} = (\phi_o, \phi_i, \phi_e)$ we set

$$\begin{aligned}
\mathcal{A}(\mathbf{T}, \mathbf{\Phi}) &= \mathcal{A}_o(T_o, \phi_o) + \mathcal{A}_i(T_i, \phi_i) + \mathcal{A}_e(T_o, \phi_e) + \mathcal{B}(\mathbf{T}, \mathbf{\Phi}) \\
\mathcal{A}_o(T_o, \phi_o) &= \int_0^L (D_o \frac{\partial T_o}{\partial z} \frac{\partial \phi_o}{\partial z} - a_o v_o \frac{\partial T_o}{\partial z} \phi_o)\, dz \\
\mathcal{A}_i(T_i, \phi_i) &= \int_0^L (D_i \frac{\partial T_i}{\partial z} \frac{\partial \phi_i}{\partial z} + a_i v_i \frac{\partial T_i}{\partial z} \phi_i)\, dz \\
\mathcal{A}_e(T_e, \phi_e) &= \int_0^L (D_e \frac{\partial T_e}{\partial z} \frac{\partial \phi_e}{\partial z} - a_e v_e \frac{\partial T_e}{\partial z} \phi_e)\, dz,
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}(\mathbf{T}, \mathbf{\Phi}) &= b_o \int_0^L (T_o - T_i)(\phi_o - \phi_i)\, dz + b_e \int_0^L (T_i - T_e)(\phi_i - \phi_e)\, dz \\
&\quad + b_f \int_0^L T_e \phi_e\, dz \\
\mathcal{C}(\mathbf{T}, \mathbf{\Phi}) &= -b_f \int_0^L (r \star T_e)\phi_e\, dz.
\end{aligned}$$

Duality between V' and V will be given by the formula

$$\langle \mathbf{F}, \mathbf{\Phi} \rangle = a_o \langle F_o, \phi_o \rangle + a_i \langle F_i, \phi_i \rangle + a_e \langle F_e, \phi_e \rangle$$

where $\mathbf{F} \in V'$ is of the form $\mathbf{F} = (F_o, F_i, F_e)$, $F_o, F_i, F_e \in (H^1(0,L))'$, and brackets at the right hand side signify duality between $(H^1(0,L))'$ and $H^1(0,L)$. We set $H = L^2(0,L)^3$, with usual norm denoted by $|\cdot|$, and by identifying H with its dual we have $V \subset H \subset V'$, with dense and continuous injections. Furthermore, by $W(V,V')$ we denote the space of all functions from $L^2(0,t_{\max};V)$ with time derivative in $L^2(0,t_{\max};V')$. It is well known that $W(V,V')$ is continuously embedded in $C([0,t_{\max}];H)$.

With this notations we can reformulate the problem (1), (2), (9), (11)–(13) in the following variational problem: find $\mathbf{T} \in W(V,V')$ such

that $\mathbf{T}(0) = \mathbf{T}^0 \in H$ and for a.e. $t \in (0, t_{\max})$

$$\langle \mathbf{T}', \mathbf{\Phi} \rangle + \mathcal{A}(\mathbf{T}, \mathbf{\Phi}) + \mathcal{C}(\mathbf{T}, \mathbf{\Phi}) = \langle \mathbf{F}, \mathbf{\Phi} \rangle, \quad \forall \mathbf{\Phi} \in V. \tag{14}$$

The linear form on the right hand side is given by

$$\langle \mathbf{F}, \mathbf{\Phi} \rangle = \int_0^L F \phi_e \, dz.$$

and it is obviously continuous.

It is easy to see that $\mathcal{A}(\cdot, \cdot)$ is continuous bilinear form on V which satisfy

$$\mathcal{A}(\mathbf{T}, \mathbf{T}) + \gamma |\mathbf{T}|^2 \geq \alpha \|\mathbf{T}\|^2, \quad \forall \mathbf{T} \in V,$$

with some constants $\alpha, \gamma > 0$. Bilinear form $\mathcal{C}(\cdot, \cdot)$ comes from perturbation operator M. It is not difficult to see that for any function $u \colon (0, t_{\max}) \to L^2(0, L)$ it holds

$$\|Mu(t)\|_{L^2(0,L)} \leq \sqrt{\mathcal{K}(t)} \Big(\int_0^t |r(t-\tau)| \|u(\tau)\|_{L^2(0,L)} d\tau \Big)^{1/2}, \tag{15}$$

where $\mathcal{K}(t) = \int_0^t |r(\tau)| \, d\tau$. From here it follows that M is linear and continuous operator from $L^\infty(0, t_{\max}; L^2(0, L))$ to itself, and it has the following continuity property: if $u_n, u \in L^\infty(0, t_{\max}; L^2(0, L))$ are such that

$$u_n(t) \to u(t) \quad \text{in } L^2(0, L) \text{ for a.e. } t \in (0, t_{\max})$$

then

$$Mu_n(t) \to Mu(t) \quad \text{in } L^2(0, L) \text{ for a.e. } t \in (0, t_{\max}).$$

Furthermore, it is easy to see that M is an operator of *local type*, as defined in Artola [1], and therefore we can apply Theorem 1 from [1] and conclude:

Theorem 1 *Variational problem (14) has a unique solution* $\mathbf{T} \in W(V, V')$ *for any* $\mathbf{T}^0 \in H$ *and* $\mathbf{F} \in L^2(0, t_{\max}; V')$.

3. Numerical approximation

In this section we discuss numerical approximation by finite difference method of the problem (1), (2), (9), (11)–(13). Instead of using equation (9) we find more convenient to apply finite difference method to the equations (1), (2), (3) and to discretize directly equations (4) and (5). We avoid numerical resolution of problem (6) by the use of Hasan and Kabir [4] approximation:

$$p(t) = p_n\Big(\frac{k_f t}{\rho_f c_f r_f^2}\Big)$$

where

$$p_n(s) = \begin{cases} \frac{2}{\sqrt{\pi}}\sqrt{s}(1 - 0.3\sqrt{s}) & \text{for } s \leq 1.5 \\ \frac{1}{2}(0.80907 + \log(s))\left(1 + \frac{0.6}{s}\right) & \text{for } s > 1.5. \end{cases}$$

Furthermore, in our problem constants D_o, D_i and D_e are very small and it is natural to consider hyperbolic system ($D_o = D_i = D_e = 0$) instead of parabolic one. Due to limited space we will not enter here into discussion of existence theory for hyperbolic system. We just note that any difference scheme adapted to hyperbolic version of the system (1)–(3) will produce certain amount of numerical dispersion that will *cover* thermal diffusion in equations (1)–(3), at least for reasonable mesh sizes. Therefore we chose to neglect thermal diffusion and consequently to drop superfluous Neumann boundary conditions for oil and water in outer annulus. This will generally change the solution just in corresponding boundary layers.

We apply explicit finite difference scheme of first order with convective terms treated by *upwinding*. In all the experiments we have used a uniform grid in space and time. The spatial step h and time step τ are related by the fixed positive number λ through relation $\lambda = \tau/h$.

In discretization of integral equation (5) we use composite trapezoidal rule which gives the following procedure for calculation of formation temperature at $t = n\tau$ and $z = ih$:

$$T^n_{F,i} = \frac{1}{1 + P_1}\left(T_{Z,i} + \sum_{k=1}^{n-1}(T^k_{V,i} - T^k_{F,i})(P_{n+1-k} - P_{n-1-k}) + P_1 T^n_{V,i}\right).$$

As a consequence of the convolution in formula (5) we see that the solution on next time level includes the solutions on all previous time levels.

It can be shown that that described explicit scheme is TVB (total variation bounded) and L^∞-stable if the following CFL condition is satisfied:

$$\lambda \leq \frac{1}{\max\{v_o, v_i, v_e\} + Ch},$$

where $C > 0$ is certain constant that can be calculated from the coefficients in (1)–(3).

We now proceed with some numerical results. To evaluate the merits of one flow arrangement over another (counter flow and parallel flow), some conditions must be equal. The interval of time during which the water is cooled is not equal to the interval of time during which the water is heated. The sum of these time intervals we call circulating period or

cycle. Both method can now be compared using the same circulating period.

Results of our simulations after four cycles are presented in the figure Fig. 2. Counter flow heat exchange temperature calculations are shown on the left figure. The tubing temperature is almost always less then inner and greater than outer annulus temperature.

Parallel flow heat exchange temperature calculations are shown on the right figure. The tubing temperature lies between the inner annulus temperature and formation temperature. In this case oil temperature is lower than any annulus temperature. Besides, formation temperature is higher than in the previous case.

Tubing temperature as well as outer annulus temperature reach very soon almost constant level. The important thing to note with respect to the bottom-hole fluid temperature is that this temperature continually changes with time. A steady-state condition is never attained. Hence the stabilization of both outlet temperatures does not mean that all of the temperatures in the circulating system are constant.

Under the same conditions we found that in parallel-flow arrangement temperature drop is smaller. Therefore, we may conclude that parallel flow seems to be better.

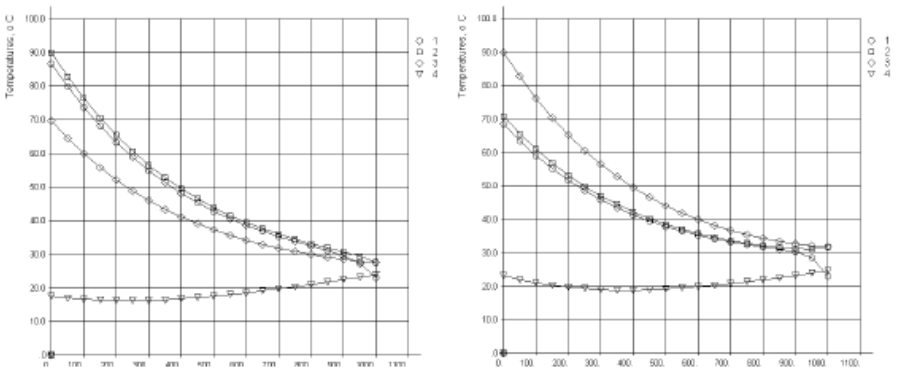

Figure 2. Temperature calculation, left for counter flow heat exchange and right for parallel flow heat exchange.
Legend: 1=□ inner annulus, 2=◊ outer annulus, 3=⊙ tubing, 4=▽ earth

To conclude we point out that the linear model presented in this article has simplicity as its main advantage. It is not difficult to implement it in a computer code and it gives certain *initial* estimate of heat exchange in the system. Yet, important physical processes, such as dissipation due to friction and variations of viscosities and mass densities with the

temperature, are not taken into account. They lead to nonlinear model that will be considered in our forthcoming publication.

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