LOSSY IMAGE RECONSTRUCTION USING ADAPTIVE WAVELET FILTER BANK

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ABSTRACT

In this paper we have used a previously reported adaptive filter bank structure for image decomposition and lossy reconstruction. We used a robust 2D windowed LS (LSW) adaptation algorithm to change the filter parameters and to adapt them to the local image properties. To improve the coding gain of the lossy image compression scheme, quantization of the adapted filter parameters has been explored. We used a CDF-based method followed by an optimization procedure to find the best quantization values. The proposed method was applied to a number of synthetic and real world images. Reconstructed images were perceptually superior, achieving lower square error norm when compared to the well-known fixed wavelet scheme.

KEY WORDS

Lossy image reconstruction, wavelets, quincunx interpolating filters, adaptive lifting scheme

1. Introduction

Adaptive filter bank structure used in this paper enables changes of the filter bank parameters for each pixel, depending on the local properties of the analyzed image. The fixed part of the filter bank provides desired number of dual and primal vanishing moments for the corresponding limit wavelet functions. Error criterion is derived from the wavelet or approximation coefficients and the adaptation is conducted to achieve more efficient representation of the analyzed signal.

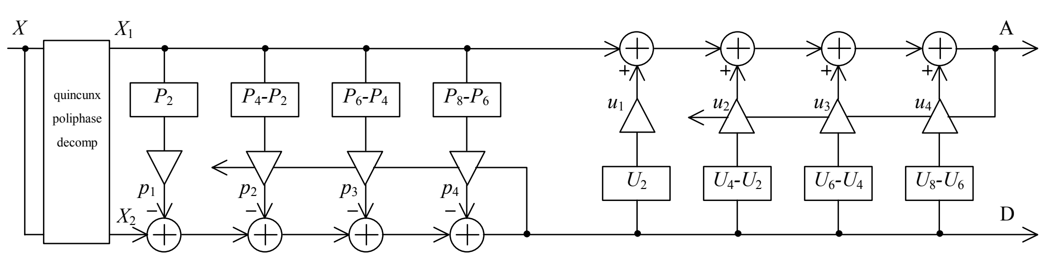

Figure 1. Structure of the adaptive wavelet filter bank. P_2 - P_8 are Neville 2D interpolating filters, U_2 - U_8 are corresponding update filters, p_i and u_i are variable parameters.

In Seršić [1] a construction of the 1D adaptive wavelet filter bank is proposed, and in Vrankić and Seršić [2] the adaptive 2D filter bank is presented. In section 2, we give a short review of the proposed 2D filter bank structure.

2. Filter Bank Structure

2.1. Lifting scheme

The lifting scheme enables construction of PR space variant and non-linear filter banks. Kovačević and Sweldens [3] proposed a construction of wavelet families of increasing order in arbitrary dimensions. The construction is based on the lifting scheme [4][5], using the interpolation of samples in multi-dimensional space [6]. Among different nonseparable 2D polyphase decomposition schemes, we have chosen the quincunx decimation. Samples from the second coset were estimated from the first coset using the 2D interpolation functions of different orders.

2.2. Adaptive lifting structure

Adaptive prediction filter (dual lifting step) used in this paper is constructed as a weighted sum of additive components:

$$P(z_1,z_2) = p_1 \cdot P_2(z_1,z_2) + p_2 \cdot [P_4(z_1,z_2) - P_2(z_1,z_2)] + p_3 \cdot [P_6(z_1,z_2) - P_4(z_1,z_2)] + p_4 \cdot [P_8(z_1,z_2) - P_6(z_1,z_2)]$$

Filters P_2, P_4, P_6 and P_8 are Neville interpolating filters described in Kovačević and Sweldens [3]. FIR filter coefficients selected from symmetric interpolation neighborhoods (rings shown in **Figure 2**) are given in **Table 1**.

If the multiplying parameters $\{p_1, p_2, p_3, p_4\}$ are constant, chosen from sets $\{1,0,0,0\}_{\mathrm{I}}$, $\{1,1,0,0\}_{\mathrm{II}}$, $\{1,1,1,0\}_{\mathrm{III}}$ and $\{1,1,1,1\}_{\mathrm{IV}}$, lifting steps $P_2(z_1, z_2)$, $P_4(z_1, z_2)$, $P_6(z_1, z_2)$ and $P_8(z_1, z_2)$ respectively are obtained. They correspond to 2, 4, 6 or 8 vanishing moments of the corresponding 2D limit wavelet function.

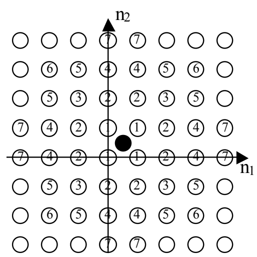

Figure 2. The quincunx lattice in the sampled domain containing X_1 coset samples. Ring numbers are marked. Black circle represents position of the predicted sample in the X_2 coset.

$2n$	ring 1	ring 2	ring 3	ring 4	ring 5	ring 6	ring 7	
2	1							$\times 2^{-2}$
4	10	-1						$\times 2^{-5}$
6	174	-27	2	3				$\times 2^{-9}$
8	23300	-4470	625	850	-75	9	-80	$\times 2^{-16}$

Table 1. Quincunx Neville filters coefficients. $2n$ is the number of vanishing moments.

Structure in **Figure 1** enables splitting of the prediction filter in a fixed and variable part. Desired number of vanishing moments $2n$ is achieved by fixing factors p_1 to p_n to value 1. The residual parameters are used as variables that can be changed at each point of the decomposition.

Update filters U_i are constructed as half the adjoint of the corresponding predict filters, $U_i = P^*_i / 2$. In order to provide vanishing moments to the limit scale function, we set the gains $u_1 - u_4$:

$p_1,\ldots, p_n = 1$	n=1 (I)	n=2 (II)	n=3 (III)	n=4 (IV)
u_1	**1**	**1**	**1**	**1**
u_2	3/2	**1**	**1**	**1**
u_3	–	3/2	**1**	**1**
u_4	–	3/2	3/2	**1**

Table 2. Gain u_i depends on the actual number of zeros of the high-pass filter, unless we fix less or equal number of zeros of the low-pass filter.

If the number of zeros of the LP filter (f=Nyquist) is less or equal to the number of zeros of the HP filter (f=0) we have “independent” vanishing moments. They do not depend on the remaining free parameters, too.

The construction of the adaptive filter bank is presented in Vrankić and Seršić [2].

3. Lossy Image Reconstruction Results

3.1. Adaptation and quantization of filter parameters

Adaptation results discussed in this paper are based on the filter bank structure with fixed $p_1 = u_1 = 1$. Only p_2 is being adapted and other parameters were set to zero. At first, we applied the adaptive wavelet filter bank to a synthetic image X composed of 2 horizontal sine waves of different frequencies and different spatial angles.

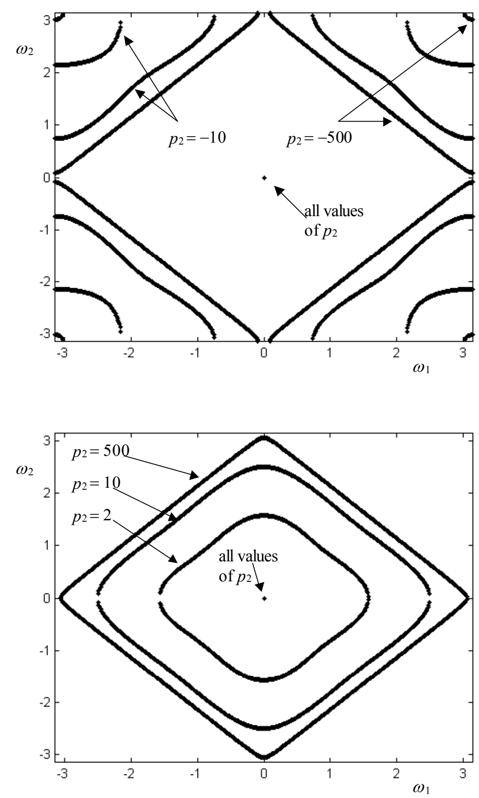

Figure 3. Zero locations in the frequency responses of the adaptive two-dimensional HP filter for different values of the parameter p_2. Top: negative values of p_2. Bottom: positive values of p_2.

2D windowed least squares (LSW) adaptation was computed on the finite rectangular $m \times m$ neighborhood of the observed pixel. Adaptation algorithm chooses the value of the variable filter parameter that gives the minimum square norm of the wavelet coefficients for the whole neighborhood. The wavelet coefficients are treated as the prediction error. We used a modification of the 2D LSW algorithm to make it more robust, i.e. to trace prevalent image characteristics. Instead of using the whole adaptation window, adapted values of p_2 were based on $M < (m \times m)$ pixels that generate lower quadratic prediction error (outliers excluded). For details see Vrankić and Seršić [2].

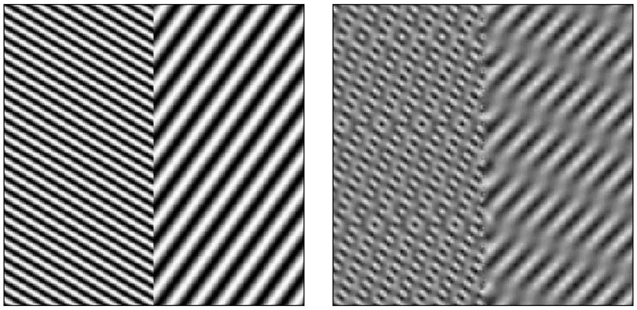

Figure 4. Left: original image. Right: reconstructed image after five decomposition levels with fixed P_4 and U_2 (no adaptation used). 95% of D coefficients are set to zero. Annoying artifacts caused by the lossy reconstruction are clearly visible.

Parameter p_2 tends to adapt into one of the two values in areas corresponding to different sine frequencies (**Figure 5**). Zero lines from **Figure 3** adjust to cancel the sine waves. Prediction becomes near optimal, turning majority of the wavelet coefficients to zero. Lossy reconstruction (percentage of D coefficients being set to zero) now gives significantly better results (**Figure 5**) when compared to the fixed filter banks (**Figure 4**).

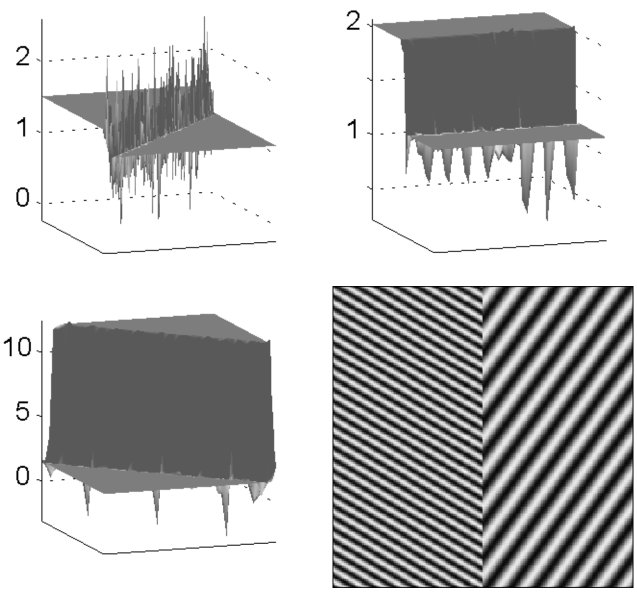

Figure 5. Adapted filter parameter p_2 computed for the image composed of two sine waves. Robust 2D LSW adaptation was used on a 3x3 window with M=6. Results are given for the first three decomposition levels (corresponding central squares are shown). Except for the transition area, the parameters are two-valued. Bottom right: Resulting reconstruction after 5 decomposition levels (95% D were set to zero). There are no visible annoying artifacts!

Of course, information is now partially transferred to p_2 filter parameters. Yet, the parameters are almost everywhere set to two discrete values and coding gain is still very high.

To improve the coding gain of real images we have considered quantization of filter parameters. Filter parameters were quantized in each decomposition level with a chosen set of quantization values. Detail (and approximation) coefficients for a given decomposition level were recomputed based on the new, quantized values of the filter parameters. The new wavelet coefficients are generally slightly worse in the entropy sense, when compared to those obtained by using the non-quantized filter parameters. Fourth row in **Figure 7** shows that improper selection of quantization vector can result in significant reconstruction errors after discarding certain percentage of D coefficients. To find a set of quantization values that are close to optimal we have created an algorithm based on the cumulative distribution function (CDF). It calculates the CDF of p_2 parameters (for every decomposition level separately) and finds N values of p_2 that are equidistant in terms of the CDF values (see **Figure 6**). These values are chosen as quanta for a given decomposition level. Example of the CDF-based quantization values for 5 decomposition levels is given in **Table 3**. Now, reconstruction gives much better results (last row in **Figure 7**).

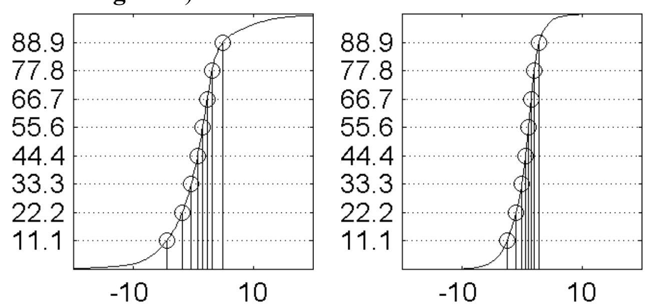

Figure 6. Cumulative distribution functions of p_2 parameters in first decomposition level for images Barbara (left) and Lena (right). Eight equally spaced CDF values are marked with circles. Their corresponding values on the x-axis are a good choice for quantization of p_2 parameters.

1. level	2. level	3. level	4. level	5. level
-4.4599	-9.2365	-8.0724	-5.3906	-4.1480
-1.8431	-6.3393	-5.5690	-2.1014	-1.3884
-0.4039	-3.4420	-2.5648	-0.7099	-0.2478
0.6428	-1.6485	-0.8124	0.0491	0.2673
1.5587	-0.5448	0.0638	0.6184	0.7089
2.3437	0.4209	0.6896	1.1244	1.1872
3.1287	1.3867	1.3155	1.7570	1.7391
4.9604	3.3182	2.1917	2.7690	2.5854

Table 3. Quantization values of p_2 parameters for image Barbara obtained using the CDF-based algorithm. For every decoposition level the eight quantization values were calculated.

In order to minimize the reconstruction error norm we have used MATLAB's optimization function *fminsearch*. The CDF-based quantization vectors were used as the initial points for the optimization procedure. Optimization was done level by level, i.e. filter parameters in the next decomposition level were optimized based on the outputs of the previous, already optimized level. Results for some images are shown in **Table 4**.

Figure 7. First row: the original image. Following rows show the 5-level lossy reconstruction (85% coeffs set zero). Second row: fixed P_4. Third row: adapted p_2 (window 3x3, M=6). Notice the high-valued artifact on the left leg caused by high values of p_2. It causes very high overall error (see **Table 4**). Fourth row: adapted p_2 quantized with ill-posed quanta. Fifth row: quanta obtained by using the CDF-based algorithm give excellent reconstruction results.

	Fixed P_4	Adapted p_2 (robust 2D LSW)			
		No q.	Sim. q.	CDF q.	oCDF q.
Barbara	10.3	15.8	14.2	5.74	5.73
Lena	2.05	1.44	2.08	1.55	1.54
Goldhill	5.82	4.24	5.61	4.47	4.46
Peppers	2.22	1.73	2.17	1.76	1.73

Table 4. Error norms ($\times 10^6$) for lossy reconstruction of different images. Reconstruction is done for 5 decomposition levels. In each decomposition level 85% of D coefficients were set to zero. Abbreviations used: No q. – original (not quantized) p_2 parameters; Sim. q. – symmetrically distributed quantization values (same in all levels): [-20 -10 -5 0 5 10 20]; CDF q. – quantization values obtained from a CDF-based algorithm; oCDF q. – optimized values of CDF q.

4. Conclusion

In this paper we have used the adaptive filter bank structure that outperforms its fixed counterparts. The robust 2D LSW adaptation algorithm has been employed because of its good properties in the sense of resulting entropy of wavelet coefficients and good tracking of prevalent local image features. To improve the coding gain for the lossy image compression purpose, quantization of the filter parameters has been explored. CDF-based method for finding quantization values of the filter parameters shows very good results. The reconstructed images are perceptually almost as good as those obtained without quantization of the filter parameters. Additional improvements have been obtained by using optimization of quantization levels around the initial CDF-based values. All together, the proposed lossy reconstruction scheme using the adaptive wavelet filter bank combined with the CDF-based quantization of filter parameters gives better perceptual results and lower quadratic error norm when compared to the fixed wavelet filter bank.

5. References

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