ANALYSIS OF THE NOMOTO SHIP MODEL RESPONSE TO COURSE CHANGES USING PID CONTROLLER IN MATLAB/SIMULINK

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ABSTRACT

This paper presents system based on Nomoto model of ship and its response to course changes using regulation loop for course control with PID controller.
Central element of analysis is selecting an appropriate mathematical model of system being analyzed. Equations which describe ship motion are given by Newton laws of motion, considering two coordinate systems: inertial system X0Y0Z0 (ECEF – Earth Centered Earth Fixed) and XOYZ reference ship centered system.
Reference model was created based on linearized equations for surging, swaying and yawing while other motions were ignored.
Using designed system shown in this paper, response to course changes of a Nomoto model ship had been analyzed.
Analysis had been conducted using Matlab/Simulink program package with first order Nomoto model using real parameters of a fully loaded tanker.
Results of an analysis had been used to conclude about characteristics of presented system and its response to course changes along with possible improvements in such system.

Keywords: Nomoto model, ship dynamics, PID controller, ship course control, Matlab/Simulink

1 BASIC SHIP MOTIONS

Equations which describe ship motions are retrieved using Newton's laws of motion, considering two coordinate systems: inertial system X0Y0Z0 (ECEF – Earth Centered Earth Fixed) and XOYZ reference ship centered system as shown in figure 1.

Motion equations can be entirely described with six degrees of freedom of a rigid body in movement using three translation equations (swaying, heaving, surging) and three rotation equations (yawing, rolling, pitching).

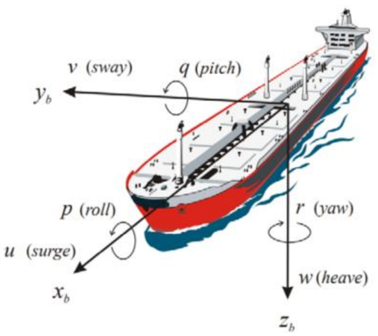

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Figure 1: Ship centered system

Only horizontal motion is taken into consideration for ship, while pitching, rolling and heaving are ignored. Thus, the number of degrees of freedom is reduced to three: yawing in direction of z-axis, surging in direction of x-axis and swaying in direction of y-axis.

According to that, throughout this paper only horizontal plane movements in previously mentioned x, y and z axis were considered while the remaining movements were ignored.

2 NOMOTO SHIP MODEl

Nomoto suggests two simple linear transfer functions. This model is intensively used by engineers for analysis and design of ship auto pilots. This type of model gives accurate description of higher GRT ships motion. Since the speed of navigation is considered constant, model is only applicable for constant thrust and small rudder angle changes.

Nomoto second order model of a ship can be described by following equations:

$$x_l = A_1\dot{x}_1 + B_1 u \tag{1}$$

$$\frac{\Psi(s)}{\delta(s)} = C(SI - A_1)^{-1}B_1\,; C = [0\ 0\ 1] \tag{2}$$

or:

$$\frac{\dot{\Psi}(s)}{\delta(s)} = \frac{b_1 s + b_2}{(s^2 + a_1 s + a_2)} \tag{3}$$

where:

$$a_1 = -a_{11} - a_{22} \tag{4}$$

$$a_2 = a_{11}a_{22} - a_{12}a_{21} \tag{5}$$

$$b_1 = b_{21} \tag{6}$$

$$b_2 = a_{21}b_{11} - a_{11}b_{21} \tag{7}$$

Ψ represents yaw, while δ represents rudder angle.

Earlier mentioned transfer function (3) is usually written in following form which is known as Nomoto second order model:

$$\frac{\dot{\Psi}(s)}{\delta(s)} = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)} \tag{8}$$

$$K = \frac{b_2}{a_2}\,; T_3 = \frac{b_1}{b_2}\,; T_1 \cdot T_2 = a_2\,; T_1 + T_2 = \frac{a_1}{a_2}$$

Equation (8) can be written in time domain as:

$$T_1 T_2 \dddot{\Psi} + (T_1 + T_2)\ddot{\Psi} + \dot{\Psi} = K(\delta + T_3\dot{\delta}) \tag{9}$$

Equations in time domain are transferred into frequency domain, usually called s-domain, by applying Laplace transform. By transferring equations into frequency domain, calculations are easier since differentiation and integration are replaced by multiplication or division with frequency variable s.

Parameters T_1, T_2 and T_3 are time constants. Value depends on work conditions and they are usually called control quality index. While T_2 and T_3 are usually positive, K and T_1 can be positive or negative value. T_1 is usually positive for ships which maintain originally given course, while it becomes negative for those ships who cannot maintain stable course. K represents static yaw rate gain.

Approximation of equation (8) is given by setting up parameter $T = T_1+T_2-T_3$:

$$T_1 \to T\,; T_2 \to T_3$$

$$\frac{\dot{\Psi}(s)}{\delta(s)} = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)} \approx \frac{K(T_3 s + 1)}{(T s + 1)(T_3 s + 1)}$$

$$\frac{\dot{\Psi}(s)}{\delta(s)} = \frac{K}{(T s + 1)} \tag{10}$$

Equation (10) represents Nomoto first order model, in time domain it is described with following equation:

$$T\ddot{\Psi} + \dot{\Psi} = K\delta \tag{11}$$

Parameters K, T, T_1, T_2 and T_3 can be calculated by using equations mentioned earlier and they differ for different types of ship.

Nomoto first order model equation can be reformulated as following:

$$\frac{\dot{\Psi}(s)}{\delta(s)} = \frac{b}{(s+a)} \qquad b = \frac{K}{T} \quad a = \frac{1}{T} \tag{12}$$

3 SHIP COURSE CONTROL SYSTEM

Automatic course control has its roots in the beginning of last century, after invention of gyrocompass. Generally speaking, ship course control system has one input and one output, so it is SISO (Single input single output) system as shown on figure 2.

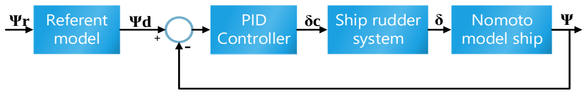

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Figure 2: Ship course control system block diagram

Every block shown on figure 2 represents a subsystem. Therefore, in the following four figures these subsystems will be presented and explained.

Figure 3 presents Nomoto ship model subsystem block used in simulations.

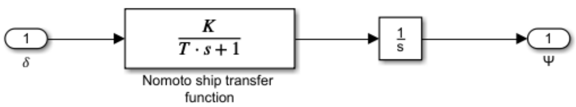

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Figure 3: Nomoto model subsystem block in Simulink

Referent model, shown on figure 4 is considered as a pre-filter which eliminates numerical difficulties attributed to high values of input step signal. Dynamics of reference model should be adjusted to the ship's dynamics regardless of value of desired reference yawing angle. Too sluggish referent model cannot give optimal performances due to ships inability to achieve desired course on time. On the other side, faster referent model, in other words, faster than ships response cannot be used due to rudder actuator saturation and performance degradation. Ψ_r represents referent course, which is a referent value set by an operator, while Ψ_d represents output value of a course.

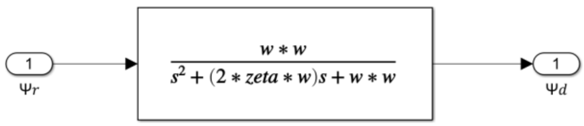

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Figure 4: Referent model subsystem block in Simulink

Second order referent model is usually used, described mathematically by following equation:

$$\frac{\Psi_d}{\Psi_r} = \frac{K_m}{T_m s^2 + s + K_m} \tag{13}$$

Where T_m and K_m are parameters used to describe closed loop system response. T_m represents time constant of the referent model, while K_m represents gain factor of the referent model. For majority of practical uses, these parameters are selected as following:

$$T_m \leq \frac{1}{2}\frac{LT}{U} \qquad K_m = \frac{1}{4\zeta^2 T_m} \tag{14}$$

Where ζ represents closed loop system damping ratio, typically in interval $0.8 \leq \zeta \leq 1$. L represents length of a ship, U represents velocity of a ship, while T represents time constant of a ship.

By comparing equation (13) with general second order system resulting equation is:

$$\frac{\Psi_d}{\Psi_r} = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} \tag{15}$$

T_m and K_m can be expressed through damping ratio and closed loop system natural frequency via equation:

$$T_m = \frac{1}{2\zeta\omega_n} \qquad K_m = {\omega_n}^2 T_m \tag{16}$$

Function of ships rudder system shown on figure 5 is to move rudder to desired position according to instructions from control system or helmsman.

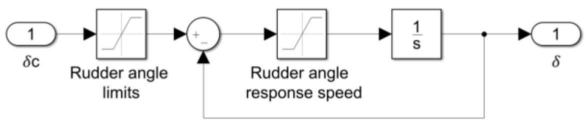

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Figure 5: Ships rudder subsystem block in Simulink

Rudder angle limiter represents mechanical limitation of rudder angles, in this case 35° in both directions. Maximum rate of movement of a rudder is defined by hydraulic pump capacity, usually between 2 and 7 degrees per second. Minimum values are defined by classification societies which demand rudder to be moved from 35° one side to 35° opposite side in less than 30 seconds.

$$\delta_{max} = \pm 35° \qquad \dot{\delta}_{max} = \pm 2°/s \quad to \quad \pm 7°/s \tag{17}$$

Ships rudder system consists of:

- Rudder angle limiter (δ_{max}),
- Rudder movement rate limiter ($\dot{\delta}_{max}$),
- Feedback.

PID controller can be expressed via equation:

$$u(t) = K_p\left[e(t) + \frac{1}{T_1}\int_0^1 e(\tau)d\tau + T_d\frac{de(t)}{dt}\right] \tag{18}$$

Where $T_i = \frac{K_p}{K_i}$ and $T_d = \frac{K_d}{K_p}$ are integration and derivation time constants.

In specific case of ships course control system shown on figure 6, PID controller is expressed via equation:

$$\delta_c = K_p(\Psi_d - \Psi) - K_d\dot{\Psi} + K_i\int_0^t(\Psi_d - \Psi)d\tau \tag{19}$$

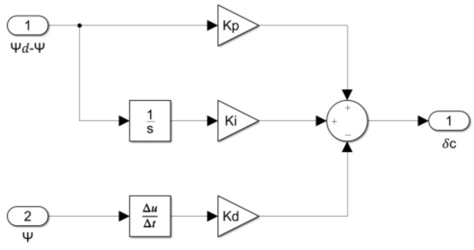

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Figure 6: PID controller subsystem block in Simulink

For the sake of simplicity, PD controller was firstly built, and integration component was added subsequently.

PD controller can be expressed as following:

$$\delta = K_p(\Psi_d - \Psi) - K_d\dot{\Psi} \tag{20}$$

If ship dynamics is expressed by first order Nomoto model, following closed loop system characteristics are obtained by inserting (20) into (11):

$$T\ddot{\Psi} + (1 + KK_d)\dot{\Psi} + KK_p\Psi = KK_p\Psi_d \tag{21}$$

Damping ratio and natural frequency can be calculated by following equation:

$$\zeta = \frac{1 + KK_d}{2\sqrt{TKK_p}} \qquad \omega_n = \sqrt{\frac{KK_p}{T}} \tag{22}$$

Consequently, K_p and K_d can be calculated as following:

$$K_p = \frac{T\omega_n^2}{K} \qquad K_d = \frac{2T\zeta\omega_n - 1}{K} \tag{23}$$

Fossen (2004) recommends following rule for integration component:

$$\frac{K_i}{K_p} \approx \frac{\omega_n}{10} \tag{24}$$

Therefore:

$$K_i = \frac{\omega_n K_p}{10} = \frac{\omega_n^3 T}{10K} \tag{25}$$

4 SIMULATIONS

For simulations, fully loaded tanker of 350 meters length overall is considered. At speed of 8.1 m/s, parameters K and T are -0.019 and -153.7.

Rudder angle limiter is set to ±30° while rudder movement rate is limited to ±2.33°/s. To acquire wanted ship characteristics, second order reference model according to equation (15) was used.

If damping ratio is 1 and natural frequency 0.03 rad/s, then PID controller parameters are calculated according to equation (23) and (25). In this particular case, values are as following:

$K_p = 7.2805$, $K_d = 538$ and $K_i = 0.0218$.

Figure 7 shows control system scheme used for simulation in MATLAB/Simulink.

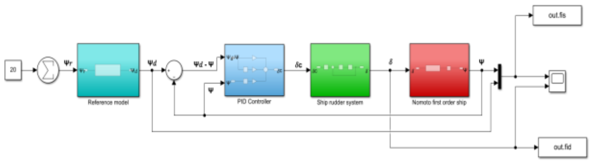

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Figure 7: Ship control system block diagram in MATLAB/Simulink

Using Nomoto model through equations (23) and (25) ideal P, I and D components of PID controller were calculated. System's response to desired 20° course is shown in figure 8.

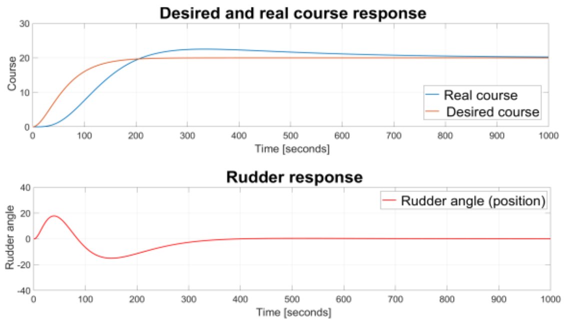

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Figure 8: System's response to desired course of 20°

Figure 9 shows system's response to desired course of 50°.

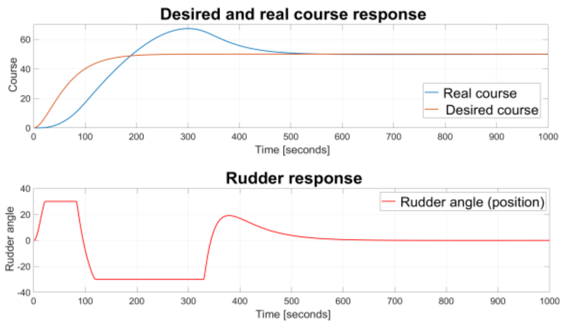

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Figure 9: System's response to desired course of 50°

Figure 10 shows system's response to firstly desired course of 10°. After the desired course is achieved, desired course is set to 50°, and lastly, after achievement of 50° course, desired course of 30° is set.

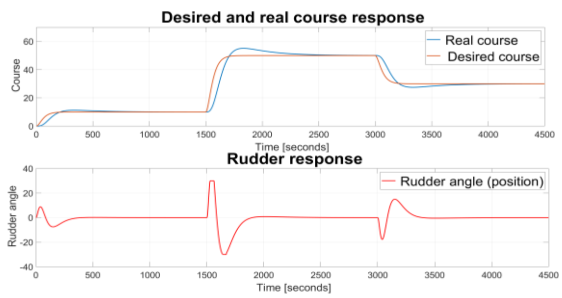

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Figure 10: System's response to desired courses of 10°, 50° and 30° respectively

Since PID controller is designed with respect to Nomoto second order model, it can be further tuned but then the questions of stability, response time and steady state error arises.

Given the aforementioned, following three figures will show system's response with tuned parameters of PID controller.

Figure 11 shows system's response to desired course of 20° with greater proportional (P) component of PID controller, where is obvious that the ship dynamics is faster at the expense of stability. Proportional component is set to 15.

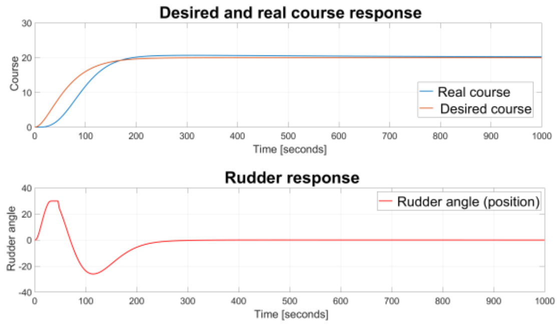

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Figure 11: System's response to desired course of 20° with greater P component, $K_p = 15$

If integral (I) component of a PID controller is increased, while proportional component restored to value already defined by equation (23) it can be concluded that ship's dynamics is faster, and stability is increased as shown in figure 12. K_i is set to 0.1.

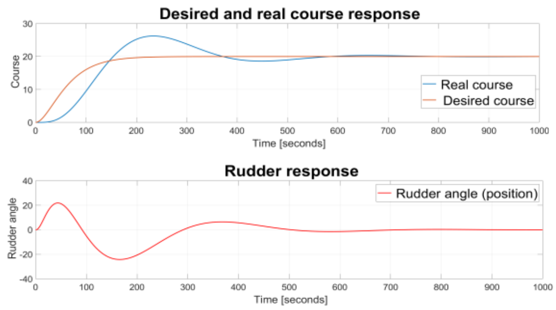

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Figure 12: System's response to desired course of 20° with greater I component, $K_i = 0.1$

Further on, by increasing derivative (D) component of a PID controller, with integral and proportional components defined by equation (23), it is obvious that the response is slower as shown on figure 13. K_d is set to 1000.

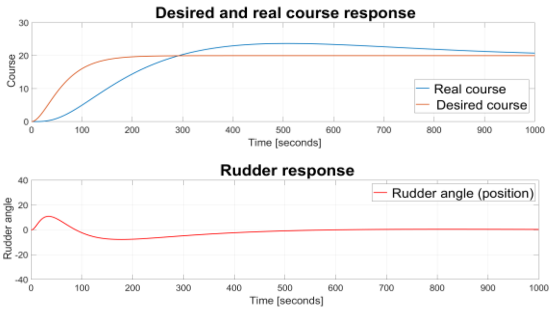

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Figure 13: System's response to desired course of 20° with greater I component, K_d = 1000

5 CONCLUSION

Using Nomoto model ideal values of P, I and D components of a PID controller are calculated via equations (23) and (25). System's response with calculated values for desired course of 20° show that ship does not follow up desired value but after some time value is achieved which was expected. It can be concluded that this does not cause problems as far as the course changes are not sudden.

On the other side, sudden changes of course, in example course change to 50° as simulated and shown in figure 9 where there is an overshoot of 17.28° can be dangerous in case of ship being in the vicinity of other ships or when navigating a narrow channel. Saturation of rudder angle indicates that the performance of the PID controller is not robust, even at constant ship speeds.

Tuning PID controller involved in course control represents great challenge. This paper proved that the equations (23) and (25) provide good enough parameters for PID controller if there are no sudden course changes.

This paper presents complex system which includes ship rudder system because for calculation Nomoto model constants, as well as its use, parameters depend on maximum rudder movement rate and maximum rudder angle.

Furthermore, presented system can be expanded with additional parameters that affect ships course such as wind impact, waves, draft and more.

Such complex system is also more realistic. Also, system can be enhanced by using neural networks or fuzzy logic which would contribute to robustness of a PID controller used as autopilot for any kind of ship.

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